Evolution of Density Perturbations in a Slowly Contracting Universe

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Abstract. One focus of research in cosmology regards the growth of structure in the universe, i.e., how we end up with stars, galaxies, galaxy clusters, and large-scale structure in a universe that appears homogeneous and isotropic on large scales. Using cosmological perturbation theory, we investigate the evolution of density perturbations corresponding to a universe that is slowly contracting as proposed in 2019 by A. Ijjas and P. J. Steinhardt, testing with and comparing different values for the equation-of-state parameter in the context of Newtonian gravity. This allows for the comparison of the growth of large-scale structure in scenarios including a matter-dominated expanding universe, a dark-energy-dominated expanding universe, and now, an ekpyrotic scalar-field-dominated contracting universe. These predictions become observationally useful in the context of two-point correlation functions to describe clustering. It is valuable to discriminate between various cosmological models to understand both the distant past and the ultimate fate of our universe.

INTRODUCTION

Most versions of the prevailing cosmological model, the big bang lambda cold dark matter ΛCDM model, involve an ever-expanding universe which began at infinitesimal size at a finite time in the past. However, since the early days of modern cosmology, back to the 1930s, cyclic models of universal evolution have been proposed to alleviate the need for such a finite beginning of time. In this work, we investigate the consequences of one such model. The model considered here transitions from contraction to expansion before a singularity is reached and therefore avoids concerns surrounding singularities at the bounce associated with many cyclic models. It is additionally constructed such that over many cycles, the scale factor exhibits de Sitter–like expansion. We will specifically study the evolution of the large-scale structure of the universe and determine how the predictions made by this cyclic universe model differ from those of the standard ΛCDM model.

The ΛCDM model invokes inflation to produce the seeds of large-scale structure formation. While inflation remains the consensus description of the early universe, alternatives have been proposed, invoking an ekpyrotic or contracting phase governed by a scalar field with an equation-of-state parameter that varies significantly from that of matter [1–7]. It is therefore expected that matter density in a universe dominated by such a scalar field will differ in its evolution from that in a matter- or cosmological-constant-dominated state.

Employing cosmological perturbation theory, we investigate the evolution of density perturbations corresponding to an ekpyrotic contracting phase, comparing different values for the equation-of-state parameter. This allows for the comparison of the growth of large-scale structure in scenarios including a matter-dominated expanding universe, a dark-energy-dominated expanding universe, and now, a scalar-field-dominated contracting universe. We then consider timescales on which one could discriminate between the leading cosmological model, ΛCDM, and the cyclic universe examined in this work.

PERTURBATIVE ANALYSIS

First-Order Density Perturbations

We begin by solving for the first-order density perturbations, starting with the partial differential equation for density perturbations given in [1]:

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G \rho_0 \delta. \tag{1}$$

A first-order approximation can be achieved by first noting that for a perfect fluid in a flat universe with equation-of-state $\varepsilon \equiv \frac{3}{2}(1+\frac{p}{\rho})$ [2], the scale factor varies with time as $a \propto t^{1/\varepsilon}$. Thus, $H \equiv \frac{\dot{a}}{a} \propto \frac{1}{\varepsilon t}$. Then, invoking the first Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G \rho_0 \ , \tag{2}$$

we find

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{\varepsilon t}\frac{\partial \delta}{\partial t} - \frac{3}{2\varepsilon^2 t^2}\delta = 0 \ . \tag{3}$$

Assuming a solution of the form $\delta \propto t^{\lambda}$, solving this partial differential equation yields

$$\lambda = \pm\sqrt{\frac{5}{2\varepsilon^2} - \frac{1}{\varepsilon} + \frac{1}{4}} - \frac{1}{\varepsilon} + \frac{1}{2} \ , \tag{4}$$

giving a generic equation for $\delta(t)$ into which a choice of ε can be substituted. For example, the accepted result for a matter-dominated universe is recovered by choosing $\varepsilon = 3/2$. Now we begin to diverge from standard cosmology by considering a contracting phase of this cyclic universe, dominated by a scalar field with $\varepsilon \gg 1$ [2]. We consider three test cases: $\varepsilon \to \infty$, $\varepsilon = 100$, and $\varepsilon = 10$. For $\varepsilon \to \infty$, we find

$$\delta_{1,\infty} = At + B. \tag{5}$$

For $\varepsilon = 100$, we have

$$\delta_{1,100} = At^{0.980} + Bt^{-0.000153}, \tag{6}$$

and for $\varepsilon = 10$, we have

$$\delta_{1,10} = At^{0.818} + Bt^{-0.0183}. \tag{7}$$

In each of the above, A and B are arbitrary functions of position describing density conditions at some initial time, and the subscript 1 indicates a first-order result.

Second-Order Density Perturbations

For consideration of the second-order density perturbations, we let $B = 0$, as it is the decaying mode for all $\varepsilon < \infty$. Using equations given in [3], it can be shown that

$$\dot{\delta}_2 + \frac{1}{a}\nabla \cdot v_2 = -\frac{1}{a}\nabla \cdot (v_1 \delta_1) \tag{8}$$

and

$$\frac{\partial}{\partial t}\left(\frac{v_2}{a}\right) + \frac{v_2}{a}\frac{2}{\varepsilon t} + \frac{3}{2}\frac{1}{\varepsilon^2 t^2}\nabla\nabla^{-2}\delta_2 = -\left(\frac{v_1}{a}\cdot\nabla\right)\frac{v_1}{a}, \tag{9}$$

into which we can substitute our previous findings. In the limit where $\varepsilon \to \infty$, this gives a second-order partial differential equation for $\delta_{2,\infty}$:

$$\ddot{\delta}_{2,\infty} + (\nabla \cdot F) - E = 0, \tag{10}$$

which has the solution

$$\delta_{2,\infty} = \frac{1}{2}t^2(E - \nabla \cdot F), \tag{11}$$

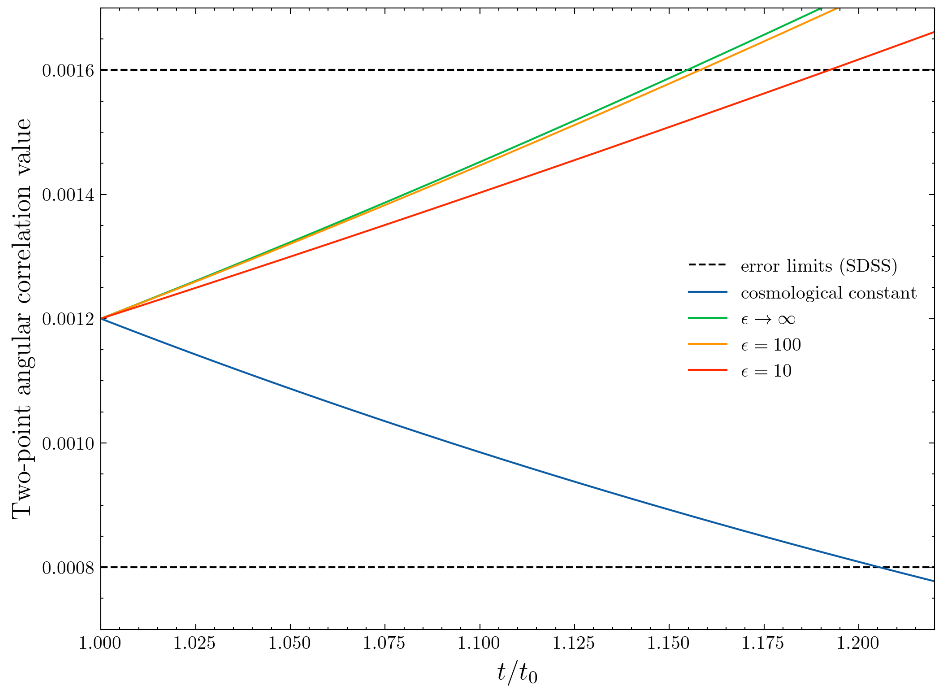

FIGURE 1. The predicted growth or decay of angular two-point correlation values for a cosmological constant-dominated universe (blue) compared with our analytical results. $\varepsilon \to \infty$ is shown in orange, $\varepsilon = 100$ is shown in green, and $\varepsilon = 10$ is shown in red. The black dashed lines correspond to the angular two-point correlation measurement $\pm$ errors given in [4] for an angle of 4.120 degrees.

where

$$E = \nabla \cdot (A \nabla \nabla^{-2} A) - \nabla \cdot (CA) \tag{12}$$

and

$$F = -(\nabla \nabla^{-2} A) \cdot \nabla (\nabla \nabla^{-2} A) + (\nabla \nabla^{-2} A \cdot \nabla) C + C \cdot \nabla (\nabla \nabla^{-2} A) - (C \cdot \nabla) C. \tag{13}$$

Repeating the process for $\varepsilon = 100$ and $\varepsilon = 10$, we find

$$\delta_{2,100} = 0.521[0.9604 E_1 + 0.98 E_2 - \nabla \cdot (0.9604 F_1 + 0.98 F_2 + F_3)] t^{1.96} \tag{14}$$

and

$$\delta_{2,10} = t^{1.636}(0.5055 E_1 - 0.4947 \nabla \cdot F_1) + t^{1.618}[0.625(E_2 - \nabla \cdot F_2)] + t^{1.6}(-0.7905 \nabla \cdot F_3), \tag{15}$$

where the subscripts of E refer to their term in Eq. (12). F_1 refers to the first term in Eq. (13), F_2 refers to the second and third terms in Eq. (13), and F_3 refers to the final term in Eq. (13).

TWO-POINT CORRELATION

The two-point correlation function gives the probability that two galaxies will be found within a given distance of each other. Thus, it gives the evolution of clustering, which can be observed. However, this evolution varies for different regimes: linear, quasilinear, and nonlinear. These different evolutions are given in [5]. Linear is given by

$$\xi(t) \propto [\delta(t)]^2. \tag{16}$$

The linear regime corresponds to the largest scales, so that is the equation we wish to use. By substituting our results for the density perturbations, we can predict how the values of the correlation function will evolve with time. This is an observable and could be used to discriminate between these various models of early universe cosmology in the future. We now seek to estimate the time at which this discrimination becomes feasible. To begin, we square our equations for the density perturbations to obtain $\xi(t)$:

For $\varepsilon \to \infty$,

$$\xi_{1,\infty}(t) = A^2 t^2 + 2ABt + B^2. \tag{17}$$

For $\varepsilon = 100$,

$$\xi_{1,100}(t) = A^2 t^{1.96} + 2ABt^{0.981053} + B^2 t^{-0.000306}. \tag{18}$$

Finally, for $\varepsilon = 10$,

$$\xi_{1,10}(t) = A^2 t^{1.636} + 2ABt^{0.7997} + B^2 t^{-0.0366}. \tag{19}$$

For a cosmological-constant-dominated universe, the density perturbations are expected to decay and eventually reach a constant value [6]. The two-point correlation function is given by

$$\xi_{1,\Lambda} = G_1^2 + 2G_1G_2 e^{-2Ht} + G_2^2 e^{-4Ht}. \tag{20}$$

The spatial dependence of these two-point correlation functions is contained in the prefactors. We can similarly decompose the angular two-point correlation into spatial and temporal components. Since the difference between these is effectively a change of basis, we can maintain our treatment of the time evolution when considering angular two-point correlation function observations, such as those given in [4]. Under the approximation of large times, we only consider the leading-order terms in the above equations and fix the values of the remaining prefactors to match the value reported in [4].

Figure 1 shows the expected change in angular two-point correlation value as a function of time for different choices of ε per age of the universe. At the present time, both the cyclic universe and standard ΛCDM cosmology are compatible with observations. Analytical calculations using $H_0 = 7.158 \times 10^{-5}/\text{Myr}$ yield $t = 1.15t_0$ as a lower bound on time ($\varepsilon \to \infty$), $t = 1.16t_0$ for $\varepsilon = 100$, $t = 1.19t_0$ for $\varepsilon = 10$, and $t = 1.21t_0$ for a cosmological constant-dominated universe, providing perspective on the time needed to make a distinction between the cyclic model in [2] and inflationary ΛCDM cosmology in terms of the Hubble time.

DISCUSSION

Using our results and $t_0 = 13.8 \times 10^3$ Myr, the most extreme scenario gives us a lower bound on time of 2.07 billion years from now. This means that using current measurements and errors, we would need to wait 2.07 billion years to distinguish between the model in [2] and a cosmological-constant-dominated universe through two-point correlation function measurements. Perhaps this large value of time is unsurprising, given that any serious cosmological model should produce the same observables we have measured at the present time, such as current two-point correlation values and cosmic microwave background anisotropies. As a general trend, the distinguishing time increases for decreasing values of ε, making it more difficult to distinguish this model from the cosmological constant scenario.

Though unlikely to produce results corresponding to a distinguishing large-scale structure measurement in our lifetimes, improvements could be made to this analysis. For example, a similar analysis could be conducted using more sophisticated techniques that push beyond the linear regime.

A more timely approach would be considering other possible observables. Another feature of the model proposed in [2] is secondary tensor modes or scalar-induced gravitational waves. As such, one could calculate the associated gravitational wave spectrum. Even if the spectrum is beyond current detector sensitivities, it may be within the scope of future or proposed detectors such as the Laser Interferometer Space Antenna (LISA), the Decihertz Interferometer Gravitational wave Observatory (DECIGO), Cosmic Explorer (CE), or the Big Bang Observer (BBO).

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