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As the world turns—irregularly

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The length of the day varies by milliseconds over the course of weeks, years, and centuries. Conservation of angular momentum explains why.

From noon to noon, the day has long served to define the passage of time. That duration, based on the observed position of the Sun, inherently varies over the course of a year (see the Quick Study on the equation of time by Anna Sajina, Physics Today, November 2008, page 76).

Yet even as measured against the fixed stars (today defined by extragalactic radio sources), Earth's angular velocity $\omega(t)$ is not some constant ω_0; a wide range of geophysical processes cause it to fluctuate slightly. Those variations in $\omega(t)$ allow geophysicists to test models for those processes: The better that a model's predicted fluctuations match the observations, the more likely it is that the model is accurate.

We experience $\omega(t)$ from the solid part of Earth; the changes in $\omega(t)$ come from the fluids that move around on the surface and deep inside. The relevant equation is the definition of the angular momentum L in terms of all Earth's fluid and solid parts:

$$L = C_a\omega_a + C_h\omega_h + C_s\omega + C_c\omega_c, \tag{1}$$

where the C's are the moments of inertia around Earth's spin axis and the ω's are the average angular velocities. The subscripts label the various parts: a is the gaseous atmosphere above the solid surface; h, the liquid above the solid surface—that is, the hydrosphere; s, the solid part (with the subscript for ω_s omitted); and c, Earth's liquid (and, in part, solid) core.

We can rearrange equation 1 to express ω in terms of everything else and do a perturbation expansion in all the variables. When the variations are expressed as normalized fractional changes—defining $\Delta_\omega(t) = (\omega(t) - \omega_0)/\omega_0$ with respect to some reference value ω_0 and likewise for the other variables—the result for variations in ω is

$$\Delta_\omega = (\Delta_L/C_s\omega) - \Delta_{Cs} - \sum_{k=\mathrm{a,h,c}} r_k(\Delta_{Ck} + \Delta_{\omega k}). \tag{2}$$

The factors r_k are the relative moments of inertia, C_k/C_s: $r_a \approx 1.5 \times 10^{-6}$, $r_h \approx 5 \times 10^{-4}$, and $r_c \approx 0.13$. In the summation, $r_k\Delta_{Ck}$ can be viewed as the *mass* terms, from the change in C_k from mass redistribution, and $r_k\Delta_{\omega k}$ the *motion* terms, which originate in fluid flows relative to the solid Earth.

The figure shows, over successively longer times, the past fluctuations in Δ_ω and the dominant contributions that arise from the right-hand side of equation 2. For historical reasons, changes in ω are commonly expressed as variations in the length of day—that is, the number of milliseconds that a clock using Earth's rotation would depart from atomic time over a day. That value is also nondimensional; 1 ms/d is a change in Δ_ω of -1.157×10^{-8}. The figure plots $-\Delta_\omega$ to match the length-of-day sign convention: A decrease in Δ_ω is a longer day.

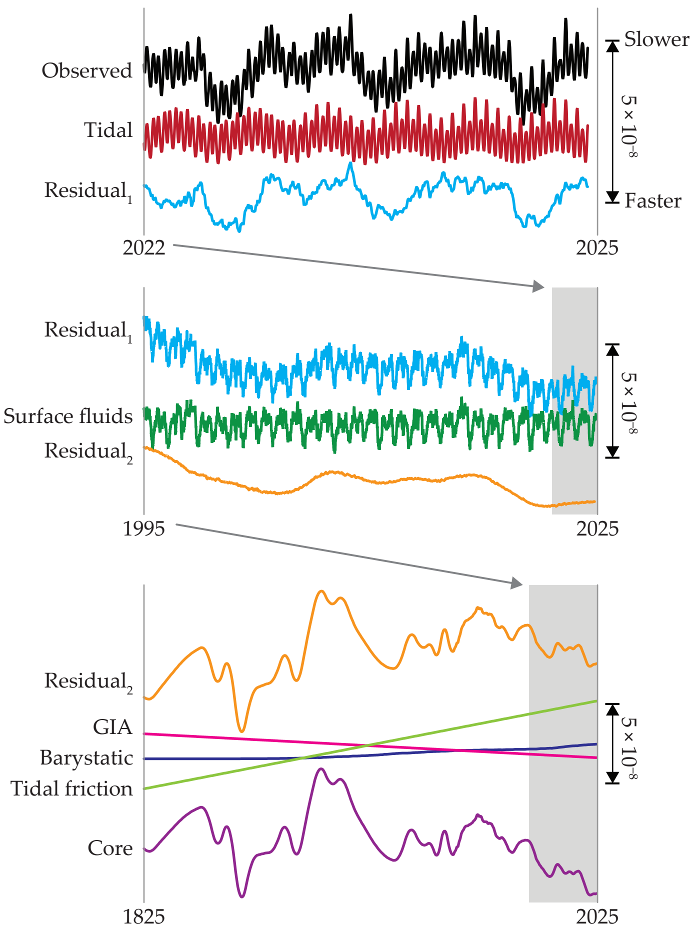
Years and decades

The top frame of the figure shows the past three years of changes in Δ_ω (black line). On that time scale, the dominant contributors are changes in Δ_{Ch} and Δ_{Cs} caused by the tidal deformations of the ocean and solid Earth; the deformations can be viewed as bulges aligned with the lunar and solar gravitational fields. Because of the inclinations of Earth's rotation axis and the Moon's orbit, the tidal contributions to the moments of inertia vary. They peak when the corresponding body is over Earth's equator: twice per year for the Sun and every 14 days for the Moon. Given models of the tidal response of the ocean and Earth, we can compute the expected changes (red line) and subtract them; that the residual (Residual$_1$, blue line) has no remaining tidal fluctuations validates the models at those time scales.

The middle frame looks at the past three decades. The blue line extends Residual$_1$—that is, Δ_ω with tidal effects removed—from the top frame (corresponding to the gray region). It shows a clear, though irregular, seasonal change and other fluctuations. Below it are the variations (green line) expected from observation-based models of Earth's atmosphere and ocean. The largest contribution, especially at seasonal time scales, comes from changes in $\Delta_{\omega a}$, which are due largely to variations in the winds of the upper atmosphere. Fluctuations in the air-mass distribution affect Δ_{Ca} and must also be included to match the observed Δ_ω. The resulting residual (Residual$_2$) is shown in orange.

Longer variability

The bottom plot again repeats the process of subtracting known sources of variability, this time over two centuries. The data extend to well before the advent of atomic clocks in 1955; the reference clock used instead is the motion of the



THE RATE OF EARTH'S ROTATION is constantly fluctuating. Plotted here are the fractional changes in the rotation rate over different time scales, from the past three years (**top**) to the past 200 (**bottom**). Many geophysical processes contribute to the fluctuations, as discussed in the main text; at the bottom of each frame are the residual variations left over after accounting for the factors above it. The scale bars denote a fractional change in rotation rate of 5×10^{-8}, or a 4.3 ms change in length of day. Upward on the plot corresponds to longer days (slower spin).

Moon—the lunar occultations of stars can be timed with great accuracy.

There are three long-term effects that change Δ_ω. The first is glacial isostatic adjustment (GIA). During the last glacial period, which ended roughly 11 000 years ago, large ice sheets covered Hudson Bay and the Baltic Sea, and their weight caused the ground surface to drop. When they melted, the load was removed, and the surface rebounded toward its elevation with no load—so-called isostatic equilibrium. Because Earth's mantle is not perfectly elastic, the rebound is still going on; over the time period shown, it can be regarded as steady. Because the rebound is transforming Earth into a less oblate, more spherical shape, it decreases C_s and causes Earth to spin faster (magenta line).

A second effect, termed barystatic, comes from changes in Δ_{Ch} as water is redistributed between higher and lower latitudes. Since 1900, and recently at an accelerating rate, melting of the polar ice caps and the Greenland ice sheet has redistributed mass from those areas to the global ocean. That increases Δ_{Ch} by an amount that over the past century has been large enough (dark blue line) to cancel out the decrease from GIA.

The third effect was the one first detected and identified: tidal friction, another consequence of the tidal deformation of the ocean and Earth. Because tidal bulges are slightly offset from the gravitational potential—high tide is slightly delayed from the Moon being straight overhead—the tidal mass distribution exerts a torque on the Moon (and likewise on the Sun). There's an opposite torque on Earth that causes Δ_L and hence Δ_ω to decrease with time (light green line). Because the total angular momentum of the Earth–Moon system is conserved, the Moon accelerates and recedes from Earth. Measurements of that recession rate, currently 40 mm/yr, give the best estimate of tidal friction: It dissipates about 3.5 TW of energy, mostly into the ocean. Extrapolating the recession rate backward in time implies that the Moon must be 1.5 Gyr old. Its age is known to be much greater, approximately 4.5 Gyr, which means that over most of geological time, tidal friction must have been smaller.

Subtracting those three long-term effects leaves the fluctuations in purple at the bottom of the figure. The only possible source for them is motion in Earth's liquid core. Such motion produces Earth's magnetic field, which varies irregularly. The changes in Earth's field and the residual changes in Δ_ω provide much of our information about the core's complex magnetohydrodynamic behavior.

Over the past 50 years, $\Delta_{\omega c}$ has steadily decreased and the solid Earth has spun faster—with significant implications for global time standards. Unlike the tides and the weather, motions in the core cannot be predicted with any confidence, so beyond a year in the future, Earth's exact spin rate becomes more and more uncertain.

Additional resources

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- M. K. Shahvandi et al., "The increasingly dominant role of climate change on length of day variations," *Proc. Natl. Acad. Sci. USA* **121**, e2406930121 (2024).
- H. Daher et al., "Long-term Earth–Moon evolution with high-level orbit and ocean tide models," *J. Geophys. Res. Planets* **126**, e2021JE006875 (2021).
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