



Emmy Noether, pictured here circa 1900, was one of the first German women to receive a PhD in mathematics.

Celebrating Emmy Noether

The mathematician Emmy Noether (1882–1935) is best remembered by physicists for the systematic connection between symmetries and conservation laws, which she proved in her groundbreaking 1918 paper “Invariante Variationsprobleme” (Invariant variation problems). *The Philosophy and Physics of Noether’s Theorems: A Centenary Volume*, edited by James Read and Nicholas Teh, is a celebration of Noether’s 1918 paper and the ongoing importance of and fruitfulness of the theorems therein. It contains chapters by historians, physicists, and philosophers covering material ranging from biographical details to present-day applications of her theorems.

The decision to produce this collection is enormously welcome. In the de-

cade after the publication of Noether’s paper, it was rarely cited and even less frequently read. Indeed, physicists who made use of “Noether’s theorem” were, until recently, often surprised to discover that her paper contained *two* theorems, both of which are important for the field.

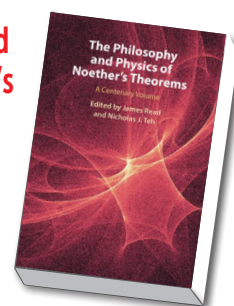
An example of the more famous first theorem is the connection between rotational symmetry and conservation of angular momentum. The second theorem underlies important mathematical relationships, such as the Bianchi identities in general relativity. Work on Noether’s theorems has blossomed over the past quarter century and, as the volume editors point out, includes an increasing number of applications across a wide variety of physics subfields. The paper’s

The Philosophy and Physics of Noether’s Theorems

A Centenary Volume

James Read and
Nicholas J. Teh, eds.

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centenary saw several commemorative conferences, but the present volume is, to my knowledge, the only one dedicated to the 1918 paper.

The Philosophy and Physics of Noether’s Theorems is not an introduction to her theorems, nor is it designed to be. It is a collection of papers presenting the state of the field. For that reason, readers with different backgrounds will find certain papers more—or less—attractive and accessible. Although it is unlikely that many people will read the book from cover to cover, anyone with an interest in Noether’s theorems will find something to draw them in. Each contribution stands on its own, and the editors’ introduction is a succinct and helpful guide to finding one’s way around the 14 chapters.

The opening chapter, by Yvette Kosmann-Schwarzbach, provides an accessible overview of the context, content, and reception of Noether’s 1918 paper, and it is nicely complemented by chapters 2 and 3, which are primarily historical. We learn about the great mathematicians Felix Klein and David Hilbert inviting Noether to join them at the University of Göttingen in 1915, shortly before Albert Einstein visited. She became involved in their investigations into the as-yet incomplete general theory of relativity, which led her to formulate her two theorems. The biographical details of Noether’s 1933 dismissal from Göttingen by the Nazis and the efforts of many colleagues to find a safe place for her outside Germany—in the USSR, the UK, and, successfully, the US—remind us of the importance of our academic community and our support for one another in difficult times.

Together, the first three chapters introduce themes that run through multiple later contributions. One of those is energy conservation. It played a role in the puzzles that led Noether to formulate

her theorems and continues to pose questions in the context of general relativity today. Readers are given multiple angles from which to see how Noether's theorems can be used to probe the mathematical structure of various theories and the ways in which energy conservation is encoded in that structure.

A second theme running throughout the book is the subtleties that arise once we move beyond the familiar—and simplified—derivations of Noether's theorems. The issues discussed include the relationship between variational symmetries and other types of symmetry, especially those to which we attribute physical significance; the converse theorems; how to state the most general formulation of the theorems; and whether symmetries are more fundamental than conservation

laws. In chapter 7, Harvey Brown argues that the alleged primacy of symmetries over conservation laws is because of their heuristic power and pragmatic utility rather than any physical (or even meta-physical) priority. And that brings us to a third theme, which is the wide range of contexts in which Noether's theorems find useful application. Examples in the book include general relativity, classical particle mechanics, quantum electrodynamics, algebraic quantum field theory, the theory of defects in elastic media, and the heat equation.

Because of how these and other themes recur, there are extra benefits for someone reading the volume from beginning to end. But readers who encounter a chapter that is not to their taste or for whom the mathematical demands

outstrip their expertise should simply move on to the next chapter.

In 2003 Elena Castellani and I collected a set of papers on the general topic of symmetry in our edited volume *Symmetries in Physics: Philosophical Reflections*. Noether's theorems were just one theme among many discussed in our book. Almost two decades later, Read and Teh have produced a volume devoted entirely to Noether's theorems. It simultaneously serves as a comprehensive demonstration of the past 20 years of progress in philosophy of physics, an invaluable reference for physicists and philosophers alike, and a superb springboard for future research.

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How the justices of the US Supreme Court will vote on cases is an example of a complex system and one that can be predicted using the maximum-entropy method.

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Exploring complex systems through applications

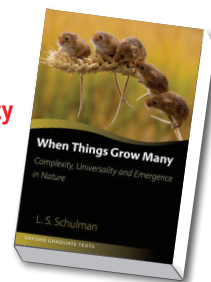
There is no universally accepted definition of a complex system, but it is often characterized as a composition of many interacting components that display emergent properties, such as self-organization, power-law distributions, and phase transitions. The complex-

systems community is highly interdisciplinary, and the systems studied usually originate from disciplines outside of physics.

Because complex systems share some of the characteristics of condensed-matter systems—namely, many elementary

When Things Grow Many
Complexity, Universality and Emergence in Nature

L. S. Schulman
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components, multiple scales, and phase transitions—it is natural to apply statistical mechanics techniques to them. In his new textbook, *When Things Grow Many*: