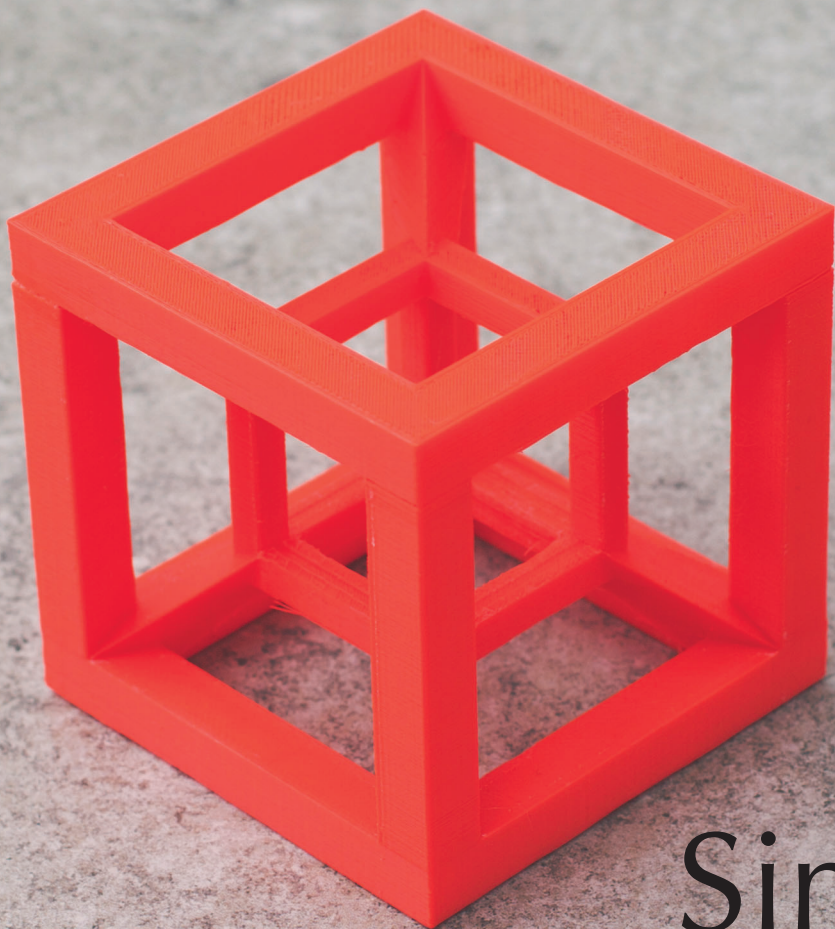




Simulating **FOUR-DIMENSIONAL PHYSICS** in the laboratory

Hannah Price

Experimental methods to imitate extra spatial dimensions reveal new physical phenomena that emerge in a higher-dimensional world.

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What would the universe be like if it had four spatial dimensions instead of three? Experimentalists are starting to explore the physics of higher dimensions with the help of recently developed tricks that synthetically mimic an extra fourth dimension in platforms such as ultracold atoms, photonics, acoustics, and even classical electric circuits. Although any such trick necessarily has limitations, as the fourth spatial dimension is always artificial, those approaches have proven that they can simulate some four-dimensional effects in controlled experimental systems.

But what is a fourth spatial dimension? In nonrelativistic physics, in which space and time are distinct, a spatial dimension is simply a direction along which objects can move both forward and backward (unlike time, which always flows from past to future). The number of relevant spatial dimensions in a system is defined by the directions along which spatial motion can take place or, alternatively, the number of spatial coordinates—for example, (x, y, z) —that must be specified to define where an object is at a particular moment in time.

The number of spatial dimensions can be reduced by constraining a system. For example, threading a bead onto a long, straight wire limits the bead to move in only one spatial dimension: either forward or backward along the wire. A single coordinate gives the bead's position along the wire at any given moment.

What would happen, then, with an increase in the number of spatial dimensions to four or more? Theoretical physicists can simply extend familiar physical equations to an enlarged set of spatial coordinates—for example, (x, y, z, w) . Often that extension leads to no new phenomena. But in certain fields of physics, new effects are predicted to emerge, such as so-called topological insulators, which are the primary source of inspiration for efforts to simulate 4D physics experimentally. This article delves into what 4D physics is and how experimental tricks to mimic 4D space work.

Topological insulators

The transfer of topological concepts from mathematics to physics has deepened researchers' understanding of states of matter and led to the discovery of a plethora of exotic topological materials. In mathematics, topology is most famously a framework to classify different surfaces. For example, donuts belong to the family of surfaces with one hole, whereas oranges belong to the family with no holes. If one smoothly squishes an orange, its shape deforms, but it cannot take the shape of a

donut without tearing a new hole and thereby changing the topology, in that case quantified by an index known as the genus. Other mathematical problems have many other sorts of topological indices, such as the family of so-called Chern numbers, which are discussed later.

In physics, topological indices lie at the heart of electrical, optical, and other behaviors in many materials.¹ In particular, they often classify electronic energy bands in a crystal. When nontrivial, those indices guarantee

special properties, such as the existence of currents circulating around the edge of a material despite the bulk remaining insulating—as in the aptly named topological insulator. Similar to the genus of a squishable orange, the indices are hard to change, so topological properties, such as those special edge currents, can be remarkably robust even in the face of disorder, as long as the bulk remains insulating.

Spatial dimensionality changes the nature of topological insulators and their edge currents. As depicted in figure 1, a 2D topological insulator has effectively 1D conducting edge channels, whereas a 3D topological insulator is covered with 2D conducting surfaces. Similarly, a 4D topological insulator should be an unusual material with robust 3D conducting surface volumes. What's more, not only the edge behavior but also the underlying physics and the definitions of topological indices depend on the spatial dimensionality and symmetries of the system.¹

Quantum Hall effects

The story of 4D topological insulators starts with the 2D quantum Hall effect, discovered in 1980 by Klaus von Klitzing of the Max Planck Institute for Solid State Research in Stuttgart, Germany. That research earned him the 1985 Nobel Prize in Physics.

As the name suggests, the 2D quantum Hall effect is intrinsically a 2D phenomenon, first observed in an effectively 2D electron gas moving in a high-quality semiconductor heterostructure.¹ In his seminal experiment, von Klitzing exposed silicon-based heterostructures to low temperatures and high out-of-plane magnetic fields. He then flowed a current through his device and measured the voltage across it to find the Hall conductance. What he found was unexpected: The conductance exhibited robust plateaus that were precisely quantized by integer multiples of e^2/h , where e is the electron charge and h is Planck's constant. In fact, that quantization is so robust and precise that it became part of the 2019 redefinition of the

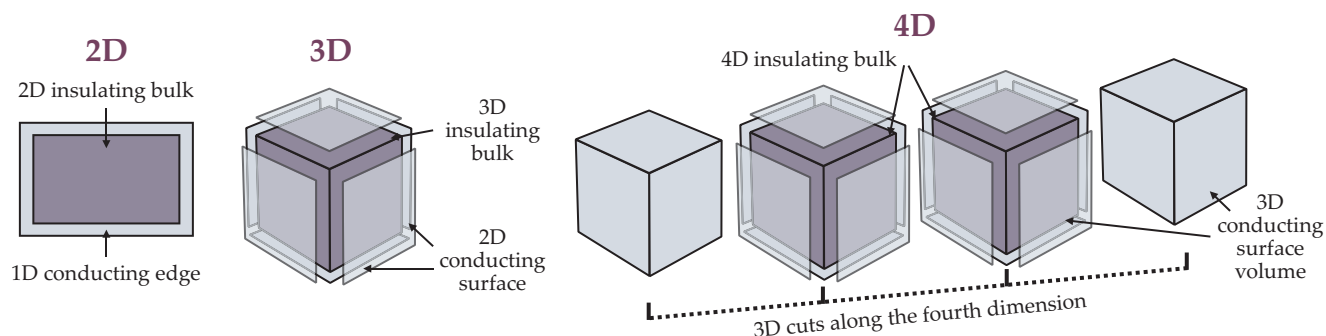


FIGURE 1. TOPOLOGICAL INSULATORS in two, three, and four dimensions conduct on their edges or surfaces (light gray) despite an insulating bulk (violet). That unusual behavior results from the topology of the electronic band structure. In 3D and 4D systems, the conducting surfaces are depicted lifted off the bulk to show it and the surface simultaneously. The 4D topological insulator is shown as several separate 3D cuts along the fourth dimension.

kilogram in SI units. (See the article by Wolfgang Ketterle and Alan Jamison, *PHYSICS TODAY*, May 2020, page 32.)

In 1982 David Thouless of the University of Washington in Seattle and his colleagues showed that the origins of the 2D quantum Hall effect lie in the topological nature of the electronic energy bands. That realization was, in part, why Thouless was awarded a share of the 2016 Nobel Prize in Physics. The integer in the Hall conductance is related to a 2D topological index called the first Chern number, which guarantees the existence of topological currents around the edge of the material¹ (see figure 1 and box 1). In other words, a 2D quantum Hall system is an example of what would now be called a topological insulator, with the robustness of the Hall conductance being one of its key experimental signatures.

After the discovery of the 2D quantum Hall effect, theorists suggested that certain 3D materials would also have bands characterized by first Chern numbers, except in that case a triad of them: one for each of the three Cartesian planes of the 3D material. The theorized 3D quantum Hall effect was indeed observed experimentally in 2019 in bulk zirconium pentatelluride crystals.² But the 3D quantum Hall effect is what's often referred to as a weak topological phenomenon because key properties, such as the first Chern numbers, remain essentially 2D concepts even though the system is 3D. The resulting topological behavior can thus sometimes be less robust.

In four spatial dimensions, however, a fundamentally different type of quantum Hall effect was proposed in the early 2000s by Jürg Fröhlich and Bill Pedrini from ETH Zürich in Switzerland and independently by Shou-Cheng Zhang and Jiangping Hu of Stanford University.³ That 4D quantum Hall effect has a different form of quantized Hall conductance from its 2D cousin and is instead related to a 4D topological invariant called the second Chern number, which creates 3D conducting surface volumes, as shown in figure 1.

To date, various 4D quantum Hall models have been proposed.^{3–5} Some, similar to the 2D quantum Hall effect, describe charged particles in magnetic fields. Others, such as that of Zhang and Hu, exploit the physics of a Yang–Mills gauge field, as explained in box 2, and take inspiration from particle physics.

The 4D quantum Hall effect is not the end of the story. Over the past 20 years, other quantum Hall effects have been predicted in 6D and 8D systems, while many other families of 2D and 3D topological insulators have been discovered that require

topological invariants other than Chern numbers.¹ Mathematical classifications categorizing topological phases of matter up to arbitrary numbers of spatial dimensions also suggest other higher-dimensional phenomena waiting to be uncovered.⁶

Not just electrons

Bringing the physics of higher dimensions into the laboratory requires thinking beyond solid-state materials—where the 2D and 3D quantum Hall effects were observed—to other more controllable platforms.

Although originally associated with electronic transport, many topological properties are now understood instead to stem from band theory and the general physics of waves.⁵ In other words, a topological index, such as the first Chern number, also applies to ultracold atoms, classical waves of light, mechanical oscillations, and waves on the ocean surface, to name just a few possibilities.

Intuitively, classical waves or noninteracting bosons shouldn't be called topological insulators, because without the Pauli exclusion principle or other effects to fill up the states in an energy band, those systems will not be insulating in the usual sense. The current convention, however, is to use the term topological insulator whenever the physics derives from energy bands with well-defined topological indices.⁵

Probing the topological physics of nonelectronic systems requires different experimental methods because those systems no longer have robust quantized plateaus in the Hall conductance. For wave-based systems, the most important experimental signature is instead typically the existence of robust modes localized on the system's surface at frequencies forbidden to penetrate the bulk. In those cases for a given frequency, waves can propagate on the surfaces but not in the bulk, as sketched in figure 1. Such topological protection may someday be useful for applications such as photonics devices because it provides a way to robustly guide light around any disorder and imperfections introduced during device fabrication.⁵

The expansion into nonelectronic platforms has also been advantageous for the study of topological phenomena (see “Topological insulators: from graphene to gyroscopes,” *PHYSICS TODAY* online, 27 Nov 2018). Many of those platforms are easier to engineer than real materials and have thus allowed scientists to explore beyond what is currently accessible in solid-state physics.⁵ As part of the push, researchers have developed ex-

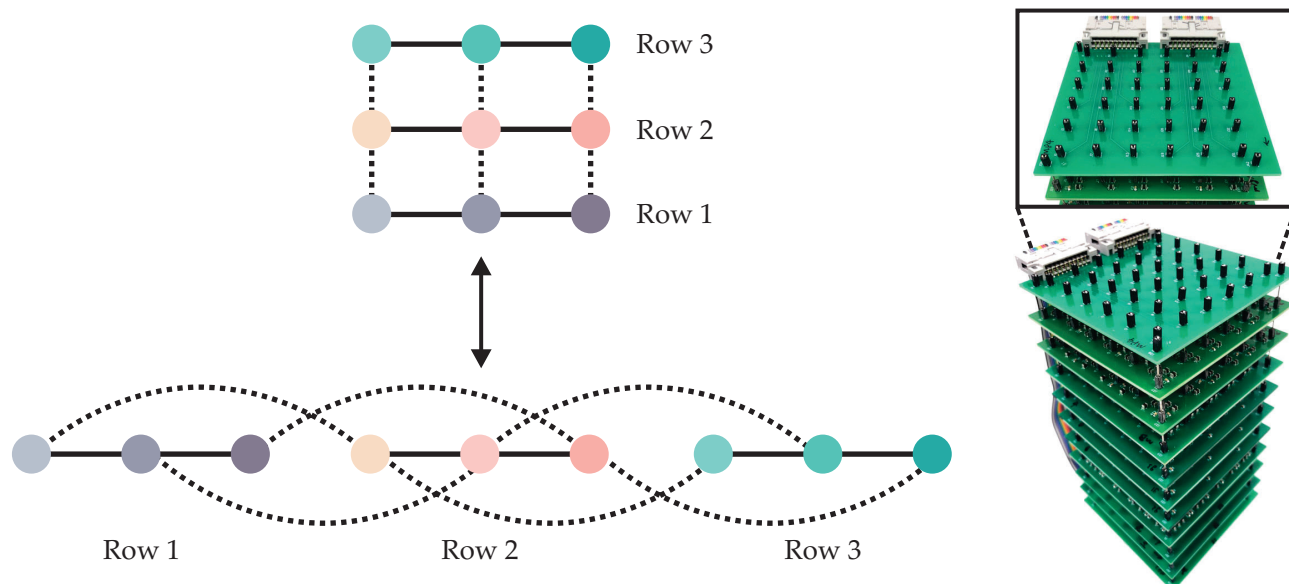


FIGURE 2. HIGHER-DIMENSIONAL LATTICES can be constructed in lower-dimensional systems. On the left, a two-dimensional discrete lattice model is composed of lattice sites (circles) with connections (lines). That same lattice can be effectively embedded into one dimension provided the same connectivity is maintained. On the right, that embedding trick was used to encode a 4D lattice into this 3D stack of circuit boards. (Photo from ref. 9.)

perimental tricks to mimic extra dimensions, in part to probe higher-dimensional topological insulators. Three main approaches are topological pumping, connectivity, and synthetic dimensions, although other schemes are also under development.

Topological pumping

One of the earliest but perhaps most abstract tricks to mimic higher dimensions is topological pumping, which Thouless first proposed in 1981 as a method to realize the 2D quantum Hall effect. He predicted that slowly tuning the parameters of certain types of 1D quantum systems could robustly pump particles across the system.¹

The simplest example starts with an insulator in which particles occupy every minimum of a 1D chain of periodic potential wells. If the overall potential's location in space is then slowly tuned such that the entire crystal slides along the chain, the resulting motion of the minima drags the particles along with it. Thouless calculated not only that such robust particle transfer was a product of a topological invariant but that the invariant

was the same 2D index—the first Chern number—as in the 2D quantum Hall effect. The result suggested that, in a sense, a 1D topological pump is a dynamic version of the 2D quantum Hall effect, as has since been explored experimentally.

Going from one dimension to two dimensions may seem quite far from higher-dimensional physics. But in 2013 Yaakov Kraus of the Weizmann Institute of Science in Israel, Zohar Ringel of Oxford University in the UK, and Oded Zilberberg of ETH Zürich predicted that a 2D topological pump would be related to the 4D topological index—the second Chern number—of the 4D quantum Hall effect.⁴

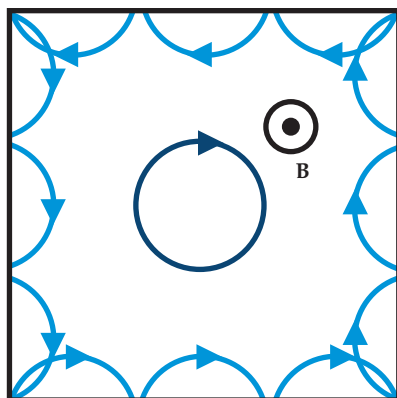
The prediction proved correct in 2018 in two complementary experiments led by Zilberberg. One he conducted in photonics with the team of Mikael Rechtsman at the Pennsylvania State University; the other was in cold atoms with the team of Immanuel Bloch at the Max Planck Institute of Quantum Optics and the Ludwig-Maximilians University Munich in Germany and my collaboration at the University of Birmingham.⁷ Those experiments identified signatures of the 4D quantum

Hall effect in the propagation of light around the edge of a waveguide array and in the net motion of atoms across a system, respectively, and have since been extended by other groups to acoustic platforms.

Topological pumping has many intrinsic limitations because it is essentially a mathematical trick based on slicing up a higher-dimensional model in a clever way. In reality, the particles are only ever able to move in the lower-dimensional system and do not have the full

BOX 1. SKIPPING ALONG THE EDGE

What is the origin of the two-dimensional quantum Hall edge current? Classically, when a charged particle confined to 2D motion experiences an out-of-plane magnetic field $\mathbf{B}$, it executes closed cyclotron orbits in the bulk (dark blue circle), but one-way skipping orbits along the boundary of the box (light blue arrows). Even if the boundary is deformed, those skipping orbits keep moving in the direction dictated by the orientation of the magnetic field. Quantum mechanically, that behavior translates to the characteristic insulating bulk energy bands and robust conducting edge states of a topological insulator.



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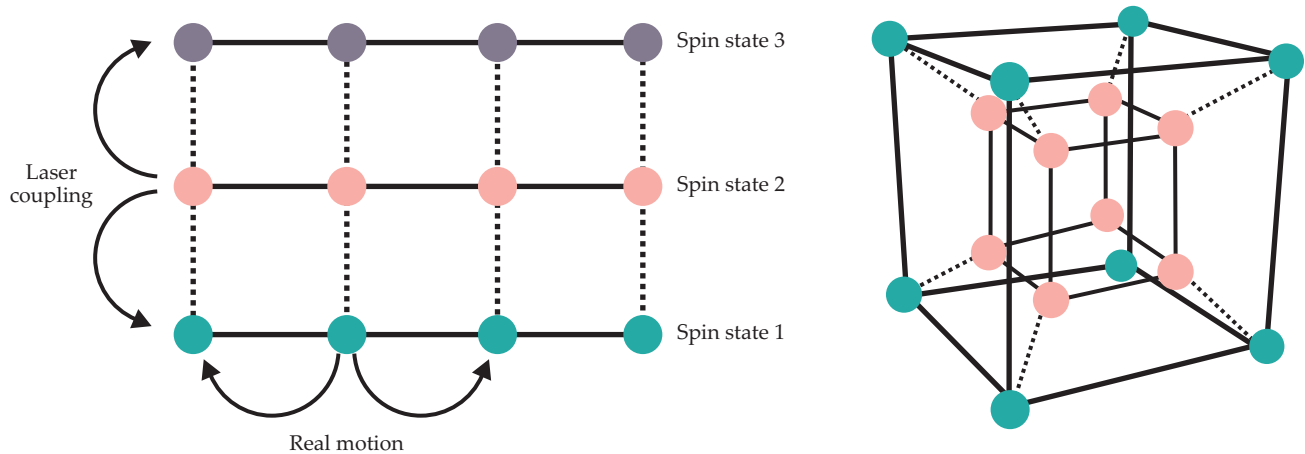


FIGURE 3. SYNTHETIC DIMENSIONS turn atomic spin states—or other internal states or intrinsic properties—into something similar to a spatial dimension. A two-dimensional discrete lattice model (**left**) comprises one real spatial dimension and one synthetic dimension composed of atomic spin states. Hopping along the real dimension (solid lines) corresponds to real atomic motion, whereas hopping along the synthetic dimension (dashed lines) corresponds to laser-induced transitions between spin states. The unit cell of a 4D hypercubic lattice (**right**) is a tesseract. Such shapes can be crafted with a suitable combination of real and synthetic spatial dimensions.

freedom of higher dimensions. Something closer to real higher dimensions may be possible through other types of experimental schemes.

Electrical circuit connectivity

The second method to simulate higher spatial dimensions is based on the idea of connectivity, which can be understood by starting with discrete lattice models. In those models, particles can exist only on a set of lattice sites. Those sites can be represented as a set of discrete points distributed in space, as shown in figure 2. Depending on the model specifics, particles can hop between pairs of lattice sites, as indicated by the dashed lines. Such discrete lattice models are common approximations for real systems, including electrons moving through a solid-state material and an electrical current moving around a circuit. They can also identify and isolate the essential ingredients of phenomena.

The key point for understanding higher-dimensional simulations is that a discrete lattice model is essentially a network of nodes, as in lattice sites, and connections, as in allowed hops. That perspective reveals that it does not matter where the nodes are physically located in real space, provided that all the connections are the same.

For example, the 2D square lattice in figure 2 can transform to a 1D chain if each row of sites is laid out end to end. So long as the same types of connections exist between sites, the system will obey the same mathematical equations as before. In a sense, the process embeds the 2D model into a 1D scheme—albeit a strange 1D scheme, in which some short-range connections are absent, while other long-range connections appear.

The same idea extends to higher-dimensional lattices too—for example, creating a 4D lattice model with a 3D or 2D scheme. The embedding trick therefore offers a recipe for realizing a 4D lattice model in a real physical system, but with the challenge of engineering complicated connections between sites.

In an early proposal from 2013, Dario Jukić and Hrvoje Buljan from the University of Zagreb in Croatia envisioned simulating a discrete 4D lattice with photonic waveguides.⁸ Since then, research interest has focused on more flexible systems, such as electrical circuits, with various proposals for how lattice sites composed of inductors, capacitors, and resistors can be wired together to realize 4D topological models.

In 2020 You Wang, Baile Zhang, and Yidong Chong of the Nanyang Technological University in Singapore and I applied the approach for the first time experimentally, as shown in figure 2. We created a small 4D topological lattice of 144 sites embedded in an electrical circuit.⁹ In the experiment, we designed a stack of 3D circuit boards and wired them together to match a 4D discrete lattice model for the 4D quantum Hall effect. As predicted for a 4D topological insulator, we observed that currents flowed through the sites that would be on the surface of the 4D topological insulator but not through the bulk.

Those electrical circuit experiments do have limitations be-

BOX 2. EXOTIC MONOPOLES

One way to think about topological pumping is that it replaces some of the real spatial dimensions in the Hamiltonian with externally controlled parameters. But if all the spatial dimensions are swapped for externally controlled parameters, then no real spatial degrees of freedom are required to simulate higher dimensions.

In 2018 Seiji Sugawa, Ian Spielman, and their colleagues at the Joint Quantum Institute and the University of Maryland in College Park used that type of approach. Inspired by the work of Shou-Cheng Zhang and Jiangping Hu on the 4D quantum Hall effect,⁴ the researchers experimentally simulated what's known as a Yang monopole in an effective 5D parameter space created by coupling

four internal states of an atomic quantum gas.¹⁶ Similar to how Paul Dirac postulated the hypothetical magnetic monopole as a source for the magnetic field, the Yang monopole is proposed as the source of a Yang–Mills gauge field in five dimensions. Sugawa, Spielman, and their colleagues mapped out the properties of the simulated monopole and verified that it is characterized by the second Chern number, as predicted.

More recently, in 2020, similar experimental approaches have simulated so-called 4D tensor monopoles, which are postulated as the sources of tensor gauge fields and are characterized by an exotic topological index called the Dixmier–Douady invariant.¹⁴

cause they typically cannot access the whole energy spectrum of states at once. They are also classical systems, which cannot exhibit quantum effects. Nevertheless, the simplicity of manufacturing electrical circuits and their flexibility make them a fruitful avenue to explore 4D physics.

Synthetic dimensions

The final trick—synthetic dimensions—gets closest to genuinely simulating particles moving in four dimensions. The method interprets some set of a system's internal states or intrinsic properties as lattice sites along an imaginary extra dimension.⁵ By combining that strategy with other real or synthetic dimensions, it has the potential to realize high-dimensional lattice models.

To get a feel for how the trick works, consider the example of a gas of identical atoms trapped in a vacuum chamber and cooled close to absolute zero. Each atom has various possible internal atomic spin states, which correspond to different configurations of its constituent electrons and nucleus. Shining suitable lasers onto an atom can stimulate a sequential transition between those internal states, as sketched in figure 3. As those transitions happen, the atom's spin-state label changes step-by-step, similar to how a discrete spatial coordinate changes when particles hop between lattice sites. That analogy is powerful and effective, and it reframes different spin states as spanning a synthetic dimension.

The idea of synthetic dimensions of atomic spin states originated in 2012 in work by Octavi Boada and José Ignacio Latorre at the University of Barcelona in Spain and Alessio Celi and Maciej Lewenstein at the Institute of Photonic Sciences in Barcelona.¹⁰ The same idea was extended three years later to a discrete 2D quantum Hall lattice model with one real dimension and one synthetic dimension realized in cold-atom experiments, explained in box 3. In the future, the approach may be pushed even further to realize 4D topological models.⁵

Since 2015 the field of synthetic dimensions has expanded dramatically. One prominent innovation was to swap spin states for atomic momentum states in cold atoms. The momentum states can be coupled into a synthetic dimension by pulsing a standing wave of light, which kicks the atoms and changes their momenta by quantized amounts along the wave's direction.⁵ Ulrich Schneider's group at the University of Cambridge in the UK recently extended that approach to four separate standing waves of light at once, each one pointing along a different direction in the 2D plane. The feat engineered up to four synthetic dimensions simultaneously.¹¹ Although not yet topological, the results of the experiment could be interpreted in terms of atoms hopping on a 4D hypercubic lattice, as shown in figure 3, that is composed of momentum states.

Photonics has also undergone significant recent developments in synthetic dimensions. Most notable are two schemes: one in which the synthetic dimension is formed from the

frequency modes of a ring cavity and the other in which it's formed from the lattice modes of a waveguide array. Shanhui Fan at Stanford University and his colleagues demonstrated two simultaneous independent synthetic dimensions based on frequency modes in a single photonic cavity.¹² Mordechai Segev's group at the Technion–Israel Institute of Technology in Haifa proposed and developed experiments based on lattice modes, which have already revealed both 2D and 3D topological edge physics with a synthetic dimension.¹³ Both approaches may someday lead to realizations of 4D topological insulators.

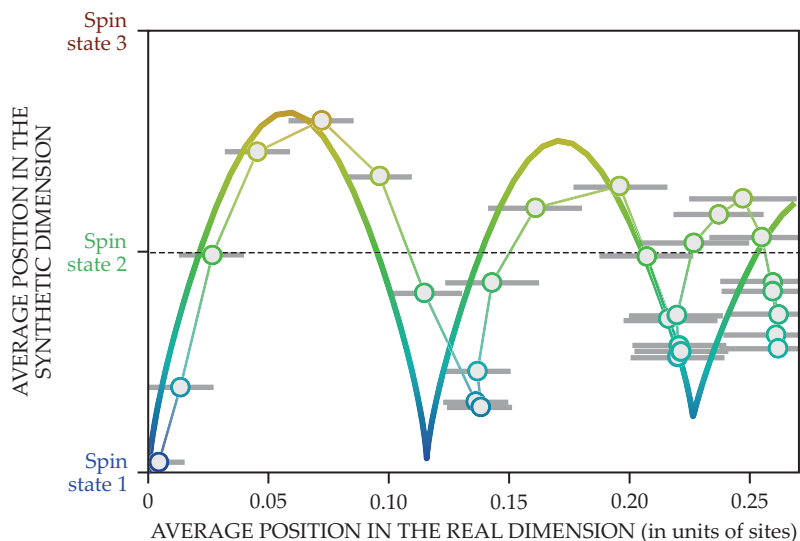
Opening up higher dimensions

Despite so much progress over the past few years, experiments simulating 4D physics are still in their early stages. Topological pumps have successfully employed mathematical tricks to observe signatures of 4D effects, but they cannot completely capture 4D dynamics. Electrical circuits can capture the full connectivity of a 4D topological lattice, but they have not yet provided full access to 4D physics. In the future, all those limitations will hopefully be overcome by synthetic dimensions, in which particles may be able to move as if in 4D space.

Synthetic dimensions may also reveal new ways to think about the 3D world. After all, a synthetic dimension consists of coupling together existing physical degrees of freedom. For example, creating a synthetic dimension of optical frequency modes involves controlling the frequency of light, whereas finding topological edge currents in such a setup is about identifying a new mechanism to robustly channel light or convert

BOX 3. SKIPPING IN A SYNTHETIC DIMENSION

In 2015 the groups of Leonardo Fallani and Massimo Inguscio at LENS (the European Laboratory for Nonlinear Spectroscopy) and the University of Florence and of Ian Spielman at the Joint Quantum Institute and the University of Maryland in College Park both realized a two-dimensional quantum Hall system made of a real spatial dimension and a synthetic dimension of three atomic spin states,^{17,18} similar to those in figure 3. As shown here (adapted from reference 17), the systems exhibited the key signature of Hall physics: skipping orbits along the edge of the system, analogous to those of a charged particle in a magnetic field, as explained in box 1.



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its frequency. By giving an alternative viewpoint for understanding and designing complex systems, synthetic dimensions may, in the long term, lead to applications in optical isolators or the spectral manipulation of light, for example.^{5,12}

In terms of fundamental science, much more 4D physics is left to explore. The topics in this article are all single-particle physics, in that particle–particle interactions are negligible. Only a few steps have been taken in the theoretical understanding of 4D phenomena, such as Zhang and Hu’s proposed generalization³ of the 2D fractional quantum Hall effect to four dimensions. Understanding what many-body physics might emerge in higher dimensions and whether those phenomena can be accessed with current experimental tricks requires further work.

From the experimental point of view, a future challenge is that particle–particle interactions naturally depend on the particle separation in the real 3D world rather than in the synthetic 4D system.⁵ In the case of synthetic dimensions, for example, two atoms in different spin states often interact strongly so long as they occupy the same physical location. Those interactions correspond to strange nonlocal interactions along the synthetic dimension. Researchers are developing various approaches to understand and tackle such problems.

Finally, although the simulation of 4D physics started with the 4D quantum Hall effect, the field should flourish far beyond that effect in the future. Recent experiments have already shown other topological effects, such as the exotic 4D tensor monopoles¹⁴ described in box 2. Other experimental tricks are also in development, including schemes based on using mul-

terminal Josephson junctions to replace spatial degrees of freedom with superconducting phases.¹⁵ In the near future, more 4D physics will be simulated in the laboratory.

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