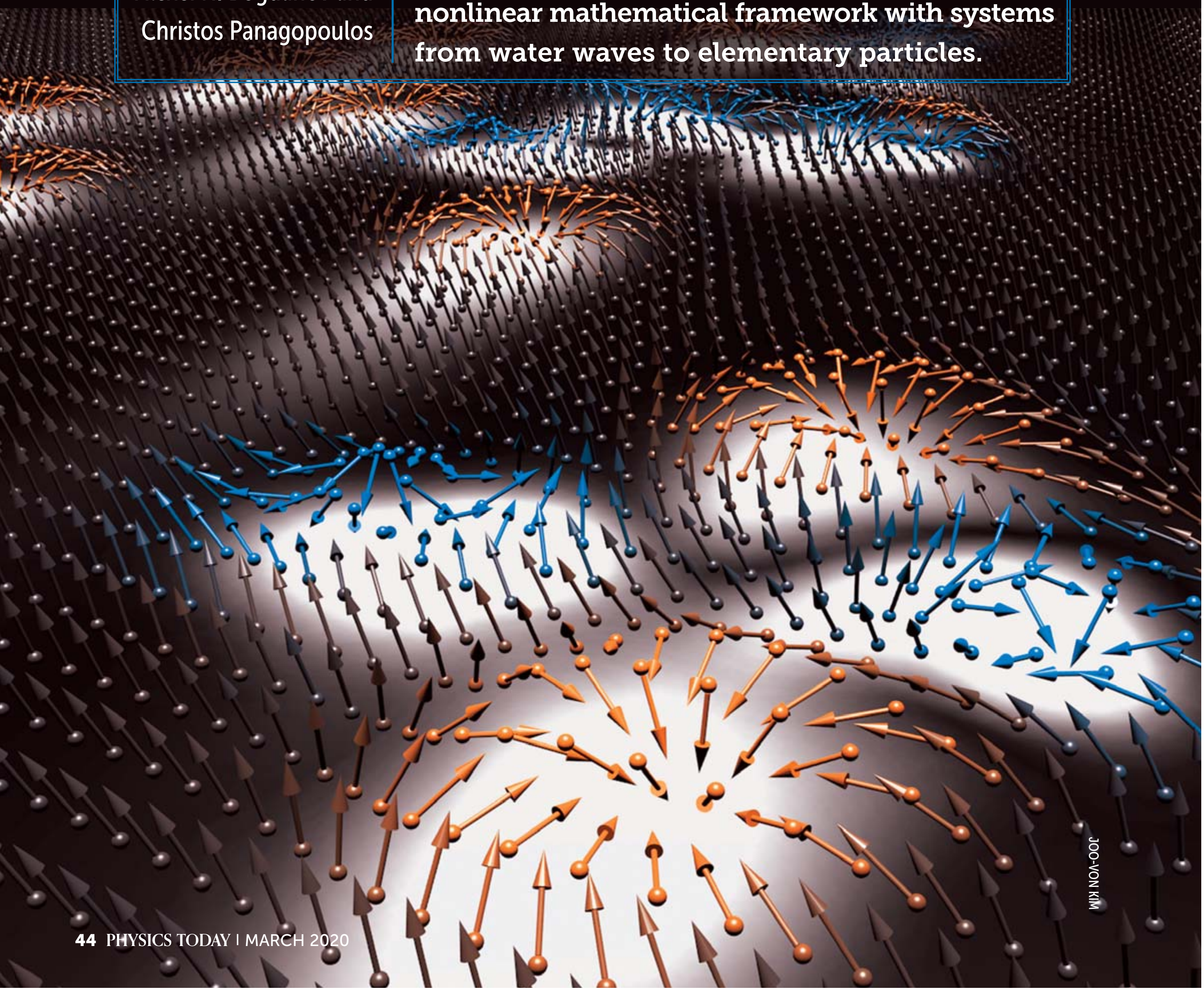
The emergence of MAGNETIC SKYRMIONS

Alexei N. Bogdanov and Christos Panagopoulos

The nanometer-scale localized objects share a nonlinear mathematical framework with systems from water waves to elementary particles.



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Magnetic skyrmions, sometimes teasingly called magic knots or mysterious particles, are nanometer-scale whirling cylinders of magnetization (see figure 1). The spatially localized objects are highly mobile and the smallest possible magnetic configurations in nature. They are thus promising for applications in the emerging field of spintronics, which uses the electron spin as an information carrier in addition to or instead of the electron charge. To explore the unconventional and potentially useful features of skyrmions, research to date has focused on novel classes of bulk and recently synthesized nanolayers and multilayers of magnetic metals.[1,2]

For many years the primary question surrounding skyrmions was whether they even exist. Mathematically, in the majority of physical systems, localized structures similar to skyrmions are unstable and collapse spontaneously into linear singularities. But three decades ago, a surprising mathematical development identified a group of low-symmetry magnetic materials that defy the general rule of instability.[3]

In asymmetric compounds, specific magnetic interactions counteract the collapse and stabilize finite-sized structures, or spin textures, such as magnetic skyrmions.[3,4] Skyrmions are one example of a self-supporting particle-like object known as a soliton. The physics that governs solitons began with a remarkable observation made almost two centuries ago.

The birth of solitons

In August 1834 engineer John Scott Russell noticed that when a boat suddenly stopped in a narrow water canal, a resulting localized wave flowed virtually unchanged for many miles. He noted that "the mass of water in the channel which [the boat] had put in motion … rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed" (reference 5, page 3).

Inspired by the observation, Russell built pools at his house and experimented with such so-called solitary waves. He was convinced of their fundamental importance. But his work was viewed with skepticism, primarily because his findings and conclusions didn't agree with the properties of water in the prevailing theory at the time.

Russell's solitary wave is described by a superposition of harmonic waves. The phase speeds of harmonic waves depend on their wavelengths—a phenomenon known as the dispersion effect. Once formed, the waves gradually spread and decay. But solitary waves show an enigmatic stability. In 1871 physicist Joseph Boussinesq figured out that the seabed stabilizes solitary waves but only in shallow waters. Six years later he introduced an equation to describe the phenomenon:[3]

$$\underbrace{\frac{1}{v_0}\frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x}}_{\text{Transport}} + \underbrace{\frac{h^2}{6}\frac{\partial^3 \eta}{\partial x^3}}_{\text{Dispersion}} + \underbrace{\frac{3}{2h}\eta\frac{\partial \eta}{\partial x}}_{\substack{\text{Interaction}\\ \text{with seabed}}} = 0,$$

in which $v_0 = \sqrt{gh}$. The function η depends on the distance x and time t, and the equation changes with the depth of the water h and gravitation acceleration g. It became known as the Korteweg–de Vries (KdV) equation in honor of the physicists who in 1895 published detailed studies of the localized and periodic solutions. The KdV equation's first two terms, which are labeled as transport, yield solutions for nondispersive waves traveling at speed v_0. The next term describes the wave's dispersion, and the last term accounts for its interaction with the seabed.

When h is a factor of 20 or more smaller than the wavelength L, the equation yields localized solutions with bell-shaped profiles[5] $\eta_s(x, t)$, shown in figure 2a, that describe solitary waves.[6] Contrary to deep-water waves, the balance of dispersion and the interaction with the seabed keeps shallow solitary waves stable and preserves their bell-shaped profile.

Despite that mathematical description, researchers ignored solitary waves for decades. But in the 1960s, scientists discovered localized states in various physical systems. Since then researchers have used modern mathematical methods and numerical simulations to study solitary waves and other self-supporting localized states, which are referred to as solitons, a term coined by Martin Kruskal of Princeton University and Norman Zabusky, who worked at Bell Labs in New Jersey at the time.[7]

Physics beyond the linear world

In the KdV equation, solitary waves are stabilized because of the quadratic term in the function η; the other terms are linear and do not support the formation of localized states. Nonlinearity is the characteristic feature of soliton mathematics, but most physical systems have linear models from Maxwell's equations to the Schrödinger equation. In a linear model, the net response from two or more stimuli is the sum of the isolated responses. But sufficiently strong distortions make a system become nonlinear—that is, the net response is not the sum of its parts. (For more on localized states in nonlinear systems, see the article by David Campbell, Sergej Flach, and Yuri Kivshar, PHYSICS TODAY, January 2004, page 43.)

The transformation from linear to nonlinear is clear in the simplified model of waves shown in figure 2b. In deep water, although waves propagate, particles below the surface orbit in circles. The orbits' diameters decrease with increasing distance from the surface and become zero at a distance $L/2$ below the surface. In shallow water with h less than $L/2$, the orbits are increasingly elliptical with distance from the surface and become horizontal oscillations at the seabed. The distortion of the particle orbits increases with decreasing total depth of the water. In extremely shallow water with h less than $L/20$, the distortions are strong enough to suppress dispersion and stabilize solitary waves.

As a local energy minimum of the system, solitary waves are extremely stable. They preserve their shape not only while in motion but during interactions with other solitary waves; in other words, they behave similarly to solid particles. The shallow-water mathematical model represents the extended class of self-supported and particle-like solitons that emerge in nonlinear physical systems—for example, magnetic skyrmions.

Stability of magnetic skyrmions

Unlike the one-dimensional solitons described by the KdV equation, magnetic skyrmions are two-dimensional axisymmetric configurations of the magnetization (see arrows in figure 1). Solutions for 1D solitons can be derived from established mathematical methods for many nonlinear equations.[5] But theorists have proven that 2D and 3D solitons are unstable in most models. The Derrick–Hobart theorem, for example, states that skyrmions in ferromagnets should shrink and collapse to a linear singularity.

The Derrick–Hobart theorem is generally true, but it doesn't apply to models in which instabilities are eliminated by terms with higher-order spatial derivatives. In 1961 mathematical physicist Tony Skyrme, for whom skyrmions are named, introduced solutions for one case of 3D solitons—the subatomic particles called mesons and baryons.[8] Whether those multidimensional solutions described any real physical phenomena wasn't yet clear. In condensed-matter physics, for example, no obvious physical interactions are described by higher-order degrees of spatial derivatives.

Theorists later established that magnetic skyrmions emerge

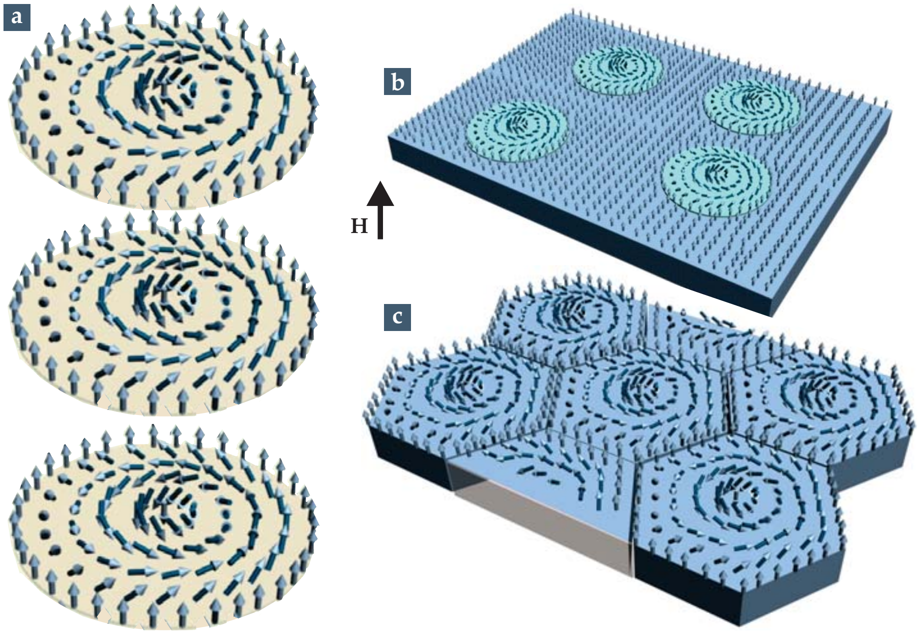

FIGURE 1. MAGNETIC SKYRMIONS are nanoscale cylinders of spinning magnetization **(a)** embedded in a ferromagnet **(b)** under an applied magnetic field **H**. The skyrmions' magnetization is antiparallel to **H** at their centers, is parallel to **H** at distances far from the centers, and rotates radially with a fixed chirality for distances between. They're known as Bloch-type skyrmions, one of the five possible configurations in uniaxial ferromagnets. For small values of **H**, isolated skyrmions condense into a hexagonal lattice **(c)**. (Courtesy of A. N. Bogdanov and C. Panagopoulos.)

in magnetic compounds without inversion symmetry—that is, noncentrosymmetric crystals.[3] Chiral magnetic materials, or those with broken mirror symmetry, are a common and well-investigated group of noncentrosymmetric ferromagnets that host magnetic skyrmions. A chiral object and its mirror image have two distinguishable forms of the crystal: left- and right-handed enantiomers. The enantiomeric pair of cubic iron monogermanide, FeGe, is shown in figure 3.

In chiral ferromagnets, the underlying structure induces magnetic Dzyaloshinskii–Moriya interactions,[4] and those forces give rise to helical spatial modulations of the magnetization perpendicular to the direction of propagation **p**. Such helical strings are known as Bloch-type magnetic skymions, as shown in figure 1. In polar noncentrosymmetric ferromagnets, Dzyaloshinskii–Moriya interactions stabilize strings with magnetization rotation along the propagation direction. Those so-called Neel-type skyrmions, shown in figure 4a, are one of the five possible configurations.[3]

Dzyaloshinskii–Moriya interactions result in energy contributions composed of Lifshitz invariant functionals $\mathcal{L}_{ij}^{(k)} = M_i(\partial M_j/\partial x_k) - M_j(\partial M_i/\partial x_k)$. Those invariants[4] are linear in the first spatial derivatives of the magnetization $\mathbf{M}$, which produce rotating magnetization in the ij-plane and propagation along the k-axis. The $\mathcal{L}_{xy}^{(z)}$ invariant, for example, stabilizes helices propagating along the z-axis with magnetization rotating in the xy-plane. In a cubic chiral ferromagnet, the Dzyaloshinskii–Moriya energy is

$$w_D(\mathbf{M}) = D(\mathcal{L}_{yx}^{(z)} + \mathcal{L}_{xz}^{(y)} + \mathcal{L}_{zy}^{(x)}) = D(\mathbf{M} \cdot \nabla \times \mathbf{M}).$$

The equation favors helices propagating along three spatial axes with the rotational direction determined by the sign of a constant D, which depends on the symmetry of the material. Mathematically, Lifshitz invariants violate the Derrick–Hobart theorem and stabilize 2D and 3D localized skyrmions.[3,9]

Although the term skyrmion was introduced in nuclear physics, the term has spread, and now it describes various physical phenomena in condensed matter, string theory, and particle and nuclear physics.[10] The term's wide use creates issues similar to that explained by Umberto Eco in the postscript of his novel *The Name of the Rose*: "because the rose is a symbolic figure so rich in meanings that by now it hardly has any meaning left." For skyrmions, such a wealth of meanings sometimes leads to the misunderstanding that they are a fundamental entity, similar to electrons or quarks, with shared common physical properties.

In condensed matter, skyrmions are specifically nonsingular, localized, and topologically stable configurations and distinct from singular localized states, such as disclinations in liquid crystals. Their nontrivial topology protects them from unwinding into homogeneous states, but they can collapse. The nature of magnetic skyrmions is best understood by picturing them as particle-like objects—that is, as solitons.

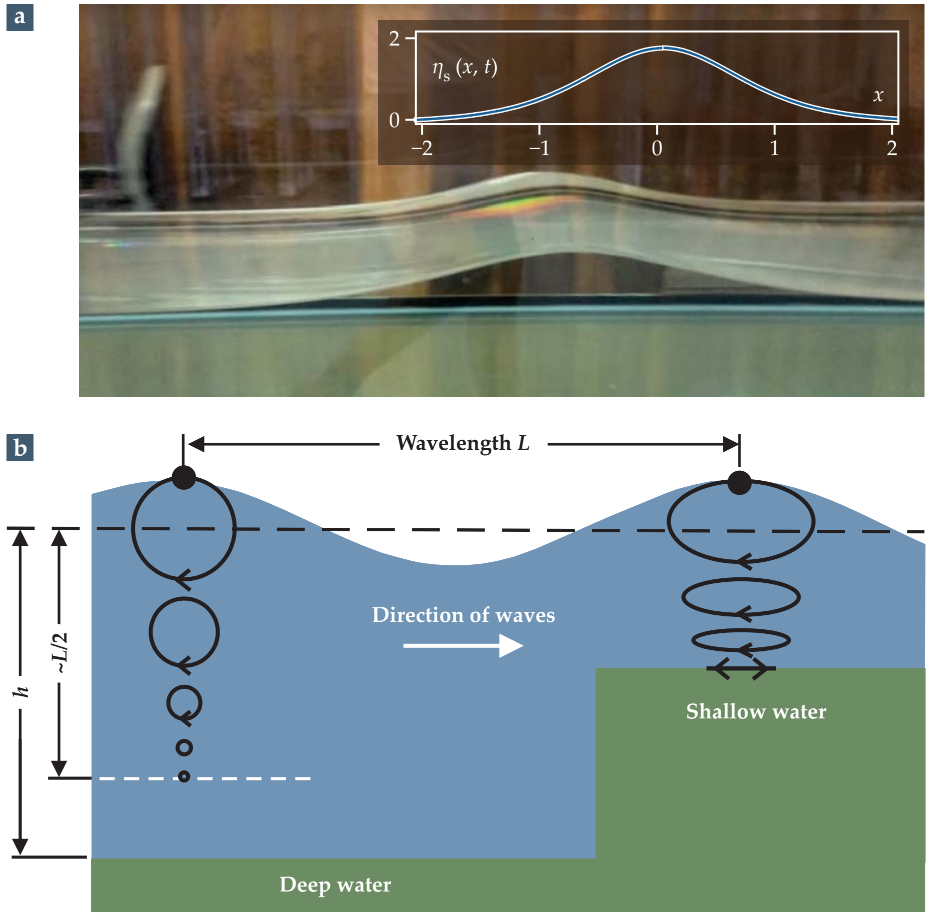

FIGURE 2. JOHN SCOTT RUSSELL'S SOLITARY-WAVE SOLITONS occur in shallow water channels, as shown in a laboratory reconstruction **(a)** of his original observation. (Adapted from ref. 6.) The inset is a typical localized solution of the Korteweg–de Vries equation for extremely shallow water. (Adapted from ref. 5.) **(b)** In deep water, waves propagate from left to right, whereas water particles orbit in circles around their average position. The diameters of the circular orbits shrink gradually with increasing distance from the surface down to half the wavelength L. In shallow waters with depth h less than L/2, the orbits become elliptical and essentially flat at the bottom of the tank or seabed. (Image by A. N. Bogdanov and C. Panagopoulos, adapted by Donna Padian.)

Isolated skyrmions and lattices

Experimentalists have discovered Bloch-type skyrmions in many chiral cubic helimagnets, including FeGe, manganese silicide, and copper selenium oxide.[1,2,11] Neel-type skyrmions have appeared in polar noncentrosymmetric ferromagnet gallium vanadium sulphide and in nanolayers with engineered Dzyaloshinskii–Moriya interactions.[1,11–13]

A simplified model for the energy w_0 of a chiral cubic ferromagnet in an applied magnetic field $\mathbf{H}$ is

$$w_0(\mathbf{M}) = \underbrace{A(\nabla\cdot\mathbf{M})^2}_{\text{Exchange}} - \mathbf{M}\cdot\mathbf{H} - \underbrace{D(\mathbf{M}\cdot\nabla\times\mathbf{M})}_{\text{Dzyaloshinskii–Moriya interaction}}.$$

The equation has three energy components, which contribute to the formation of magnetic skyrmions. The first ferromagnetic exchange energy term, with constant A, imposes parallel ordering of the magnetic moments. The second term, which is the interaction with the applied magnetic field, favors magnetization oriented along $\mathbf{H}$. The final Dzyaloshinskii–Moriya term induces helical modulations. The model introduces two fundamental parameters: the helix period $L_D = 4\pi A/|D|$ and the saturation field $H_D = D^2M/(2A)$ that suppresses chiral modulations.[1,9] For the two most studied cubic helimagnets, MnSi and FeGe, $L_D = 18$ nm and 70 nm and $\mu_0 H_D = 620$ mT and 359 mT, respectively, in terms of the vacuum permeability μ_0.

At high magnetic fields, minimizing the energy functional $w(\mathbf{M})$ yields isolated skyrmions in the form of weakly repulsive localized states in an otherwise uniformly magnetized state, as seen in figure 1b. They arise from a subtle balance of the competing magnetic forces. Similar to solitary waves and other solitons, isolated magnetic skyrmions manifest as internally stable, particle-like objects in a continuous medium: Numerical simulations demonstrate colliding and scattering magnetic skyrmions in narrow channels.[14]

Figure 4b plots the evolution of the skyrmion core diameter L_S as a function of an applied magnetic field. The antiparallel magnetization at the skyrmion center is energetically unfavorable in the presence of an applied magnetic field, so the

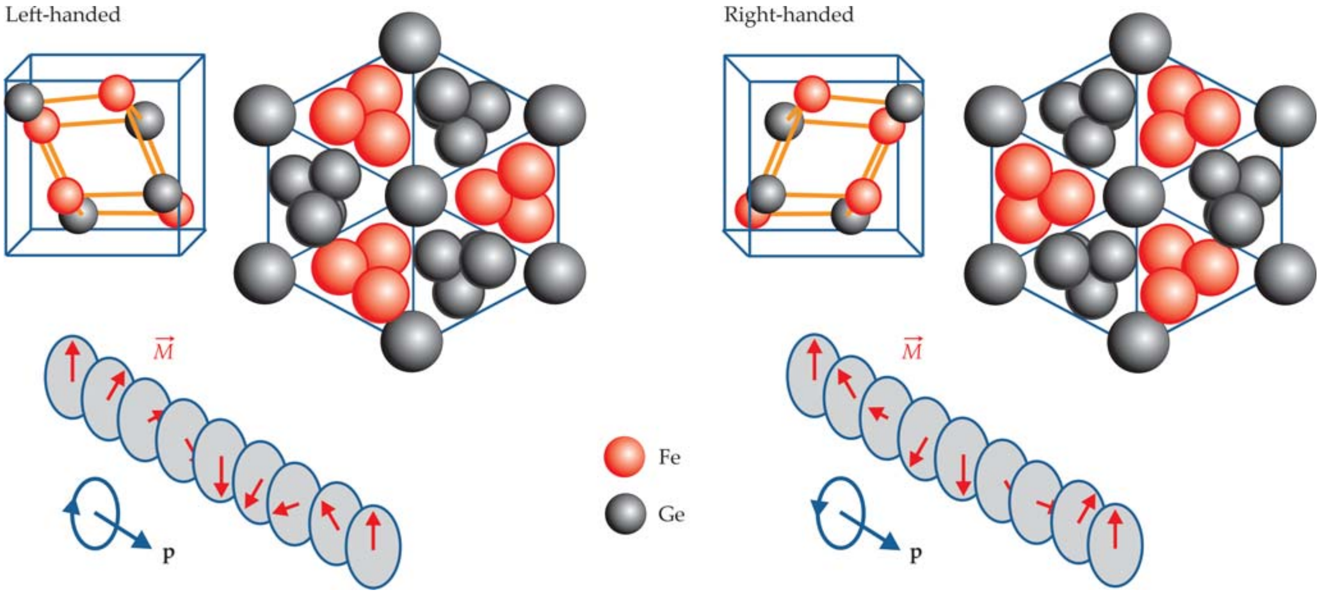

FIGURE 3. LEFT- AND RIGHT-HANDED ENANTIOMERS of chiral cubic crystal iron monogermanide do not have mirror symmetry. Magnetic interactions in FeGe stabilize Bloch-like strings of magnetization **M** with a fixed helicity relative to the direction of propagation **p**. (Courtesy of A. N. Bogdanov and C. Panagopoulos.)

skyrmion core gradually shrinks with increasing magnetic field. Below a transition field $H_S = 0.801\ H_D$, isolated skyrmions condense into a lattice, although the scenario is realized only if they nucleate. Otherwise, isolated skyrmions persist below H_S, elongate, and expand into a 1D band below an external magnetic field $H_{el} = 0.534\ H_D$, known as the elliptic instability field. Experimentally, skyrmions gradually elongate from circular to elliptic to 1D modulations in iron-palladium-iridium nanolayers in a decreasing magnetic field,[11,12] as shown in the spin-polarized scanning tunneling microscopy images in figures 4c–4g.

In model w_0, isolated skyrmions exist only in applied magnetic fields larger than the elliptic instability field. But in some highly anisotropic ferromagnetic bulk crystals and artificially synthesized magnetic nanolayers and multilayers, internal magnetic interactions can stabilize skyrmions even in the absence of an applied magnetic field,[3,11] as has been observed in FePd/Ir nanolayers.[13]

With decreasing magnetic field, isolated skyrmions (indicated by the right white square in figure 5) often transform into a lattice, usually with hexagonal symmetry. A demonstration on FeGe nanolayers, as shown in figure 5, reveals transitions between a skyrmion lattice and competing 1D phases.[9,12,15]

The flexibility to tune spin textures has led to intensive experimental and theoretical investigations in noncentrosymmetric ferromagnets and magnetic nanolayers and multilayers. A broad spectrum of unique capabilities of magnetic skyrmions has already emerged—for example, their creation, deletion, and motion using a magnetic tip.[12]

Perspective and potential technology

Magnetic skyrmions constitute a promising new direction for data storage and spintronics. They are countable objects that can be created and manipulated in, for example, layers of magnetically soft materials to develop versatile nanoscale magnetic patterns.[2,16] Skyrmions thus offer a route to localized magnetic inhomogeneities in low-coercivity materials, which are not viable for traditional magnetic recording.

Interactions similar to Dzyaloshinskii–Moriya arise in a wide range of condensed-matter systems with broken inversion symmetry. The introduction of chiral interactions in those systems provides the stabilization mechanism for solitonic states analogous to magnetic skyrmions. For example, axisymmetric solitons appear in chiral liquid crystals,[17] a state of matter thermodynamically located between an isotropic liquid and a 3D ordered solid. In ferroelectrics, which display spontaneous electric polarization, layered oxide architectures host emerging magnetic solitons.[18] Noncentrosymmetric condensed-matter systems broadly form a class of materials with multidimensional solitonic states.[9]

For condensed-matter physicists, the Dzyaloshinskii–Moriya interaction in particular offers a playground for investigations that only require a lack of inversion symmetry. In addition to naturally occurring systems, such as chiral cubic helimagnets, researchers can engineer a structure that cannot be inverted—for example, the interface between two materials. The interface between a ferromagnet and a strong spin-orbit metal gives rise to tunable-strength Dzyaloshinskii–Moriya interactions, which vary with the material selection.[1,2,16] The interfacial effects can even dominate if the ferromagnet is sufficiently thin.

The flexibility to design the host and tune the skyrmion properties offers versatility for technological applications. Skyrmion-based devices have the potential to store and process information in unprecedentedly small spaces.[1,2,14,16] The presence or absence of a skyrmion could serve as a 1 or 0 in a data bit for racetrack memory, and multiple skyrmions could aggregate to form storage devices. The states of such devices could be modulated by an electric current that drives skyrmions in and out of the devices, analogous to biological synapses. The

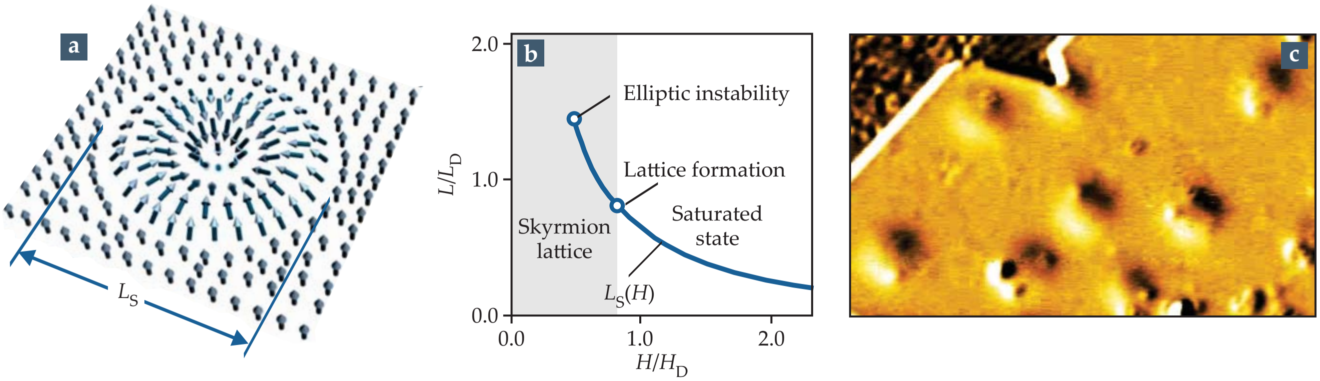

FIGURE 4. ISOLATED NEEL-TYPE SKYRMIONS (a) have magnetization rotating along the direction of propagation. **(b)** The calculated skyrmion diameter L_S shrinks as the applied magnetic field increases. The characteristic parameters L_D and H_D are the helix period and saturation field, respectively. **(c)** Skyrmions in an iron-palladium-iridium nanolayer are isolated at an applied magnetic field $\mu_0 H = 1280$ mT. With decreasing magnetic field, a skyrmion (circled) deforms into an ellipse and terminates as a one-dimensional modulation **(d–g)**. (Adapted from ref. 11.)

devices could thus potentially perform neuromorphic pattern-recognition computing.

Researchers have already engineered interfacial skyrmions in magnetic multilayers at up to room temperature.[16] Those skyrmions offer an opportunity to bring topology into consumer-friendly nanoscale electronics. In the magnetic multilayers, exotic skyrmion configurations also form under the combined influence of chiral interactions and magnetodipolar effects. Such skyrmion hybrids present a novel class of localized states that have yet to be explored in depth.

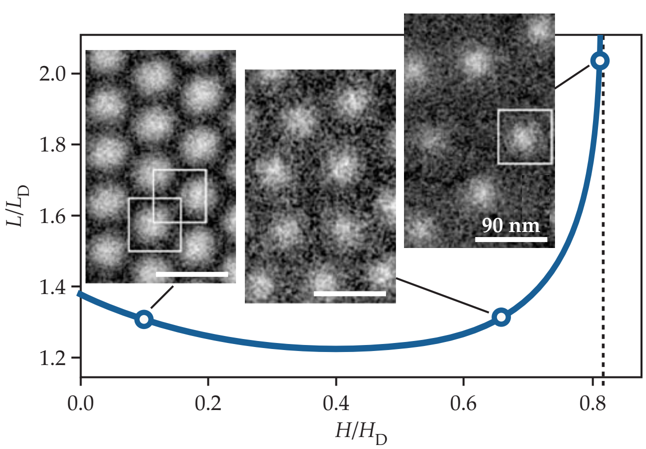

FIGURE 5. A HEXAGONAL SKYRMION LATTICE evolves in the presence of weaker applied magnetic fields in materials that allow nucleation. The calculated size L of the skyrmion cell varies with the reduced applied magnetic field, H/H_D. The characteristic parameters L_D and H_D are the helix period and saturation field, respectively. The insets are three electron holography images of skyrmion lattices in a thin layer of iron monogermanide for different applied magnetic fields—from left to right, 100 mT, 350 mT, and 400 mT. The white squares indicate individual skyrmions. (Adapted from ref. 15.)

But there are still fundamental properties to investigate, such as the wave–particle duality of skyrmions, their interaction with other magnetic textures, and skyrmion lattices as magnonic crystals. The interaction between magnetic skyrmions and light or other topological excitations—for example, superconducting vortices—could also lead to new exotic states of matter. Investigating those topics will require new material architectures and advancements in characterization techniques.

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