

Two's a crowd in quantum theory

Ilya G. Kaplan's new book, *The Pauli Exclusion Principle: Origin, Verifications, and Applications*, is an impressive review of the most mysterious law in quantum theory. Kaplan is well known for contributions to modern quantum theory, including his work on spin-free quantum chemistry. His other books, *Symmetry of Many-Electron Systems* (1975) and *Intermolecular Interactions* (2006), are written with remarkable pedagogic mastery and perfect accuracy of theoretical detail.

The Pauli Exclusion Principle touches on a broad range of ideas, from the basic principles of quantum theory to the new developments in applied group theory that have benefited from Kaplan's contributions. "Yet another book on group theory," a reader might say. But Kaplan's monograph goes far beyond standard group theory. It incorporates concepts that are relatively new in theoretical physics: holon gases, fractional statistics, parafermions, parabosons, and parastatistics, to name a few. The book does briefly introduce applied group theory in four appendices at the end. Those appendices help the reader better understand the book's main chapters, and they make *The Pauli Exclusion Principle* self-contained. But it is clear that for Kaplan, group theory is just a convenient tool.

From the very beginning of the quantum era, such figures as Satyendra Nath Bose, Albert Einstein, Enrico Fermi, Paul Dirac, and Werner Heisenberg realized the critical importance of permutation symmetry. It was Wolfgang Pauli who introduced the concept of spin and formulated the formal criterion of permutation symmetry that every wavefunction must satisfy in a many-particle system. According to Pauli, with respect to permutations, particles can be either bosons or fermions, and their transformations correspond to one-dimensional representations of the permutation group—symmetric for bosons and antisymmetric for fermions.

After a historical survey in chapter 1, chapter 2 introduces the standard technique of Young diagrams for building multiparticle wavefunctions. It covers a

variety of important cases and is useful for treating weak spin-orbit coupling as a perturbation. Still, even in zeroth order, the Pauli principle cannot be ignored.

Kaplan was the first to ask an important question about the permutation symmetry of identical particles: Is it possible for permutation symmetry to be more general than symmetric or antisymmetric? Can we expect a future discovery of exotic elementary particles that are neither bosons nor fermions? In chapter 3 Kaplan gives a rigorous proof that we cannot. Allowing states with a generalized permutation symmetry leads to a contradiction with the concept of particle identity and independence.



WOLFGANG PAULI lectures in Hamburg in 1955.

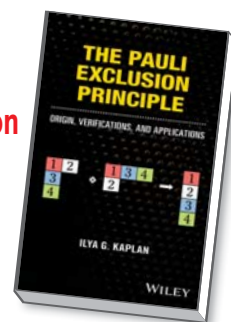
But the fundamental problem of the connection between spin and statistics still awaits its solution. Unfortunately the attempts thus far to make that connection, including the ones by Pauli and, decades later, Richard Feynman, have been unsatisfactory, for reasons Kaplan details in sections 1.2 and 3.1 of the book.

Although elementary particles and their composite particles do not possess exotic symmetry, that may not be true of quasiparticles and excited states. The key is the relative strength of the "inner coupling" of elementary particles, which ultimately leads to the emergence of quasiparticles as separate entities. If the inner interaction is weak compared with the interaction between quasiparticles, the "gas" of quasiparticles follows the laws of Fermi-Dirac or Bose-Einstein statistical physics.

In several cases, however, the inner coupling is strong enough to compete with the coupling between quasiparti-

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cles. In such cases, the emergent particles are neither bosons nor fermions, and the standard formulas of quantum statistics do not apply. That motivated Kaplan to develop modified para-Fermi statistics in 1976. The details of Kaplan's statistics are discussed in chapter 5 of his book, where they are applied to excitons, magnons, and some other quasiparticles

in periodic lattices. Although para-Fermi statistics seems to be at the margins of modern quantum theory, recent data suggest it has important applications for understanding the fractional quantum Hall effect.

In a group of outstanding creators of quantum theory, Pauli was arguably the brightest personality. Max Born, a close friend of Einstein, believed Pauli's intellectual potential was even

greater than Einstein's. Yet Pauli was also controversial—warm to his close friends, but often arrogant and obnoxious in his criticism. Although a couple of interesting facts about Pauli are mentioned in the preface, they are not enough to feed readers' hunger for information about the great quantum revolution and its revolutionary figures. Admittedly, Kaplan's book is a scientific monograph, and bringing together the modern view of the Pauli exclusion principle and its historical development is a challenge. Additional historical details about the quantum discoveries would enrich the book and make it more engaging.

To my knowledge, Kaplan's book has no equal in subject or clarity. It is therefore needed on the bookshelf of every theoretical physicist and computational chemist. It can also serve as a useful supplemental source in a graduate course on applied group theory and quantum mechanics.

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