Exceptional points make for exceptional sensors

At just the right locations in parameter space, resonant frequencies are ultrasensitive to tiny changes in conditions.

In 1913 French physicist Georges Sagnac showed that when two light beams propagate in opposite directions around a closed optical loop, their interference fringes shift if the whole loop is set spinning. He thought his experiment proved that all space was permeated by a stationary ether.

He was wrong, of course. In fact, Max von Laue had already shown two years earlier that such a result would be entirely consistent with the etherless theory of special relativity. But the Sagnac effect survives today as the basis for the ring-laser gyroscope, an important component of the navigation systems in aircraft, spacecraft, and naval vessels. In a ring-shaped optical cavity with perfect rotational symmetry, clockwise and counterclockwise propagating waves resonate at the same frequencies. A physical rotation of the cavity can be detected by how it breaks that symmetry and lifts the degeneracy.

The same principle applies for a general perturbation, such as a temperature gradient or the presence of a light-scattering particle, of any system with degenerate states. Typically, the resulting frequency splitting is proportional to the perturbation's magnitude, as illustrated in figure 1a for a hypothetical complex-valued perturbation ε (that is, one that can affect both the light's amplitude and its phase). Because the plot's shape resembles a yo-yo-like toy called a diabolo, the degeneracy has been dubbed a diabolic point.

There's another type of degeneracy, called an exceptional point, where not only do resonant frequencies coincide but their resonant modes do too. Perturbing a system about an exceptional point splits the degenerate mode in two, and the frequency splitting scales with the square root of the perturbation magnitude, as shown in figure 1b. For at least half a century, exceptional points have been studied as mathematical curiosities. Then, in 2014, Jan Wiersig of the University of Magdeburg in Germany proposed that the square-root scaling could be put to good use for ultrasensitive measurements of weak signals.[1] Now two groups have created the first proof-of-principle exceptional-point sensors.

Lan Yang and her colleagues at Washington University in St Louis have used a ring resonator tuned to an exceptional point to detect a polystyrene nanoparticle with twice the sensitivity of a diabolic-point sensor.[2] Meanwhile, Mercedeh Khajavikhan and her colleagues at the University of Central Florida in Orlando have used a trio of coupled resonators to create a third-order exceptional point—the coalescence of three modes rather than two.[3] They found, as expected, that the frequency splitting around the degeneracy scales with the cube root of the perturbation magnitude, which opens the door to even better detection sensitivity.

Breaking symmetry

The current understanding of exceptional points owes a lot to quantum mechanics. Conventional quantum theory demands that a Hamiltonian or other operator representing an observable quantity must be Hermitian (that is, when written as a matrix, its complex conjugate must equal its transpose). Hermiticity brings with it a number of favorable qualities. The operator's eigenvalues, which represent possible measurement outcomes, are all real numbers. Probability is conserved as states evolve in time. And a Hermitian operator's eigenstates are all mutually orthogonal and span the space of possible states. It's therefore impossible for a Hermitian system to host an exceptional point: Two eigenstates cannot be both orthogonal and identical.

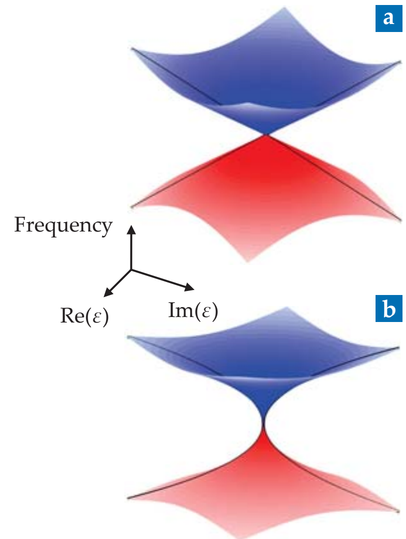

FIGURE 1. THE BASIS FOR ULTRASENSITIVITY. Around a typical degeneracy **(a)**, called a diabolic point, resonant frequencies are split by an amount proportional to the magnitude of the perturbation ε. But near an exceptional point **(b)**, the splitting scales with $\varepsilon^{1/2}$, which for small ε is much greater. (Adapted from ref. 2.)

In 1998 Carl Bender of Washington University and Stefan Boettcher of Los Alamos National Laboratory proposed that a quantum Hamiltonian might not actually need to be Hermitian to make physical sense.[4] Specifically, they found that a non-Hermitian Hamiltonian can have all real eigenvalues if it is invariant under the combination of time reversal (T) and spatial inversion (parity, P). Furthermore, tuning the parameters of a PT-symmetric system can induce a phase transition in which the symmetry of the eigenstates is spontaneously broken. The

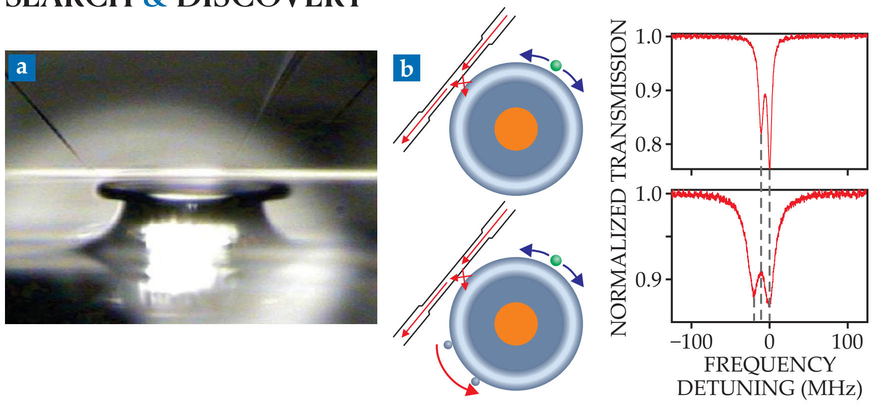

FIGURE 2. AN EXCEPTIONAL-POINT NANOPARTICLE SENSOR. (a) The optical image shows the 80 μm silica ring, nearby coupled linear waveguide, and silica nanotips that set up an exceptional point by scattering light from clockwise to counterclockwise propagation. **(b)** In the schematics, the tips are represented by the light blue spheres in the bottom panel; they scatter light in just one direction, as indicated by the red arrow. Without the tips (top panel), a third scatterer (green) that scatters light in both directions (blue arrows) splits the resonant frequencies slightly; with the tips, the splitting is larger. (Adapted from ref. 2.)

transition threshold, it turns out, is an exceptional point.

The physical implications, if any, of non-Hermitian quantum theory are still not clear. In optics, however, a non-Hermitian Hamiltonian has a straightforward physical meaning: It represents a system with optical gain or loss. In recent years, spearheaded by the work of Central Florida's Demetrios Christodoulides and collaborators, the framework of *PT* symmetry breaking has been taken up in the photonics community as a new tool for the unconventional manipulation of light.[5]

To see how the symmetry breaking works, consider two identical coupled optical elements—such as waveguides or resonators—one of which is subject to optical gain and the other to loss of equal magnitude. The configuration is *PT* symmetric: Parity interchanges the two elements, and time reversal converts gain to loss and loss to gain. When the gain (and loss) level g is small compared with the coupling constant κ, the system's eigenmodes also have *PT* symmetry.

But when $g \gg \kappa$, the gain-endowed element acquires new photons faster than it can transfer them to the lossy element, and a qualitatively different spectrum of asymmetric eigenmodes emerges. At the exceptional point between the two regimes, some eigenmodes coalesce. Because the set of eigenmodes no longer spans the space of possible states, some states—the details of which depend on the specific system—are disallowed entirely.

Features of *PT*-symmetric photonic systems have been explored for various optical effects. For example, the disallowal of states can be exploited to make a waveguide that transmits light in one direction but not the other. In 2014 Khajavikhan led one of two groups—the other was led by Xiang Zhang of the University of California, Berkeley—to use *PT* symmetry breaking to compel microring lasers, whose output typically comprises several optical modes spanning tens of nanometers in wavelength, to operate at a single optical mode.[6]

Two tips

Whereas Khajavikhan's work has been guided by the principles of *PT* symmetry, Yang and her colleagues found their exceptional point by accident. The serendipitous discovery was born out of the group's efforts to develop a compact sensor to detect, count, and measure the size of nanoparticles.[7] Such an instrument could be useful for characterizing environmental pollution, newly discovered large biomolecules, and more.

Like the ring-laser gyroscope, Yang and colleagues' sensor was based on the mode splitting of a ring-shaped resonator at a diabolic point. When a particle approaches the surface of the sensor's solid silica ring, it couples to the resonator's evanescent field. As a result, some photons are scattered out of the ring entirely, while others remain in the ring but change their direction, from clockwise to counterclockwise or vice versa. The scattering lifts the degeneracy of the resonator's eigenmodes—essentially the symmetric and antisymmetric combinations of the clockwise and counterclockwise waves—which are split in frequency by an amount related to the particle size.

But what if more than one particle is

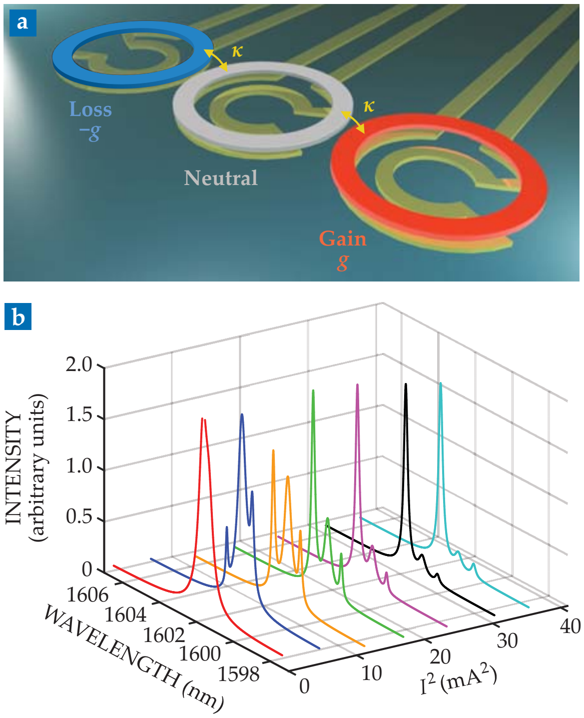

FIGURE 3. A PARITY–TIME SYMMETRIC SENSOR at a third-order exceptional point. Three semiconducting rings **(a)** are optically pumped to produce gain *g* in one, loss −*g* in another, and neither in the third. They're evanescently coupled with coupling constant *κ*, and each can be tuned with a separately controllable gold microheater. When one of the rings is heated with a current *I* **(b)**, the symmetry is broken and the resonant frequencies split. (Adapted from ref. 3.)

present? Yang and colleagues quickly realized that the contributions are not simply additive but depend on the relative positions on the ring. By mimicking the nanoparticles with more easily controlled fine silica nanotips, the researchers found that for just the right combination of placements, they could scatter light from the clockwise wave into the counterclockwise wave but not the other way around. In that case, the ring has a single resonant mode, the counterclockwise wave; neither the clockwise wave nor any linear combination that includes it is resonant. That is, the system is at an exceptional point.

It was Wiersig who recognized the implications of the results: If a ring resonator can serve as a nanoparticle detector, and if two nanotips can tune the ring to an exceptional point, then the exceptional-point resonator should be an even better nanoparticle detector. He published his idea[1] and got in touch with Yang to put it into practice.

The proof-of-principle sensor is shown in figure 2, with a side-view optical image in figure 2a and top-down schematics in figure 2b. The resonant modes of the 80-μm-diameter ring resonator are measured through the transmission spectra of an evanescently coupled waveguide. For the diabolic-point sensor without the two extra nanotips (figure 2b, top panel), the mode splitting in the presence of a scatterer is moderate; for the exceptional-point sensor (bottom panel), it's much greater.

The splitting enhancement—which was observed whether the scatterer being detected was a nanoparticle or a third nanotip—can be understood from the qualitative difference between the modes being probed. At and around the diabolic point, the eigenmodes are orthogonal linear combinations of the clockwise and counterclockwise waves. But around the exceptional point, they're almost parallel: the counterclockwise wave with a small admixture of the clockwise wave. At the exceptional point itself, the mode-space dimensionality is reduced. With a small perturbation, it struggles to recover its lost dimension, and the space remains highly skewed, which results in a fundamentally different behavior of mode splitting.

The spectra in figure 2b do hint at a potential problem: The exceptional-point sensor produces greater splitting but also broader peaks that are harder to resolve. The overall sensitivity, therefore, is enhanced by only a factor of two. That limitation is by no means fundamental—after all, as the perturbation magnitude goes to zero, the ratio of exceptional-point to diabolic-point splitting goes to infinity. The researchers were able to partially mitigate the broadening by doping the resonator with erbium ions that produce optical gain, and they're working on other improvements. "Exceptional-point sensors should be able to do much better than what's demonstrated in our paper," says Yang, "and we're eager to see even larger enhancements."

Three rings

Khajavikhan was also inspired by Wiersig's proposal. She and her colleagues had experience in using *PT* symmetry to find second-order exceptional points, so they decided to try for a third-order degeneracy. An exceptional point of order higher than two had never before been seen in an optical system.

The setup is shown in figure 3a. Instead of two coupled ring resonators, one with gain and one with loss, the researchers used three, with the middle one subject to neither gain nor loss. Each pair of adjacent rings exchanges energy with the same coupling constant κ. Although it's experimentally more complicated to work with, the configuration is *PT* symmetric for the same reason the two-resonator system is.

The rings, each 20 μm across, are made of indium gallium arsenide phosphide. Like other III–V semiconductors, InGaAsP is inherently lossy but exhibits gain when optically pumped. To set up their desired gain–loss profile, the researchers carefully shaped a pump beam to shine strongly on the rightmost ring and weakly on the middle one. By adjusting the ratio of the gain to the coupling constant, they tuned the system to an exceptional point.

Their perturbation was temperature, and each ring was placed atop a separately controllable gold microheater. Heating InGaAsP changes its refractive index by an amount proportional to the dissipated power, or the square of the current *I*. Figure 3b shows how the mode spectrum evolves as one ring is heated to perturb the system away from the exceptional point. The mode splitting increases quickly for small perturbations and more slowly for larger ones, and it's a good fit to the cube-root function.

The Florida researchers chose to create a temperature sensor because it was easy to implement. But they're now exploring whether it could actually be a practical technology for detecting microscale temperature gradients. They're also looking into whether a *PT*-symmetric setup can be adapted into an ultrasensitive optical gyroscope. "The application of exceptional points in optics is still in its infancy," says Khajavikhan, "and still very few groups are working on it. I think exceptional points will eventually be used for all sorts of things."

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How squid build their graded-index spherical lenses

Gelation maintains aberration-preventing protein-density gradients in the cephalopods' eyes.

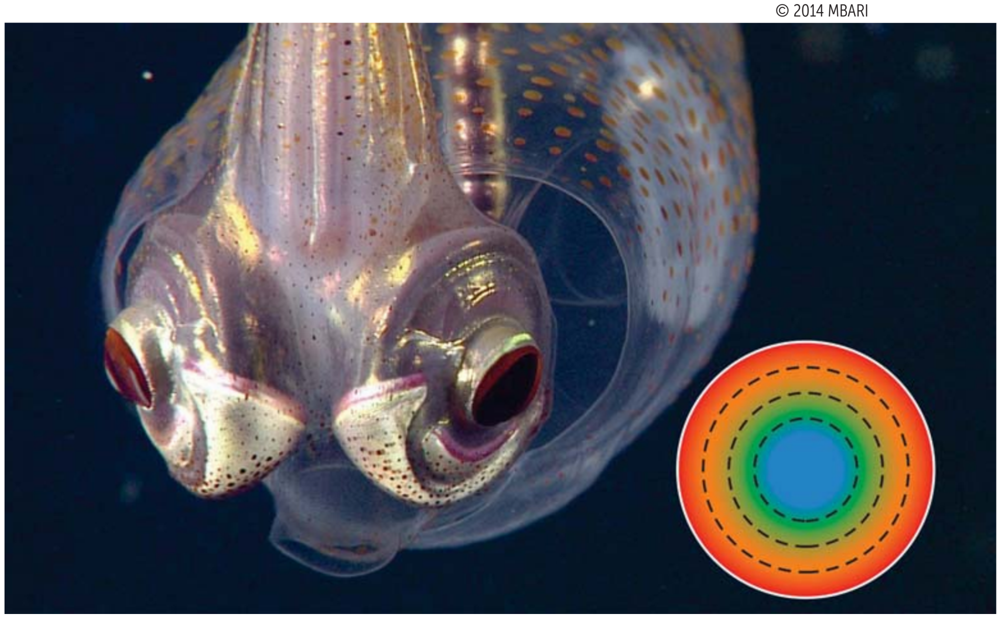

FIGURE 1. THE SQUID EYE up close. To optimize the refractive power of their eyes without suffering spherical aberration, squid have evolved sphere-shaped lenses with a radial gradient in the protein density and, thus, the refractive index. The inset shows schematically the continuous protein-density gradient in the spherical squid lens. The dashed circles at relative radii of 40%, 60%, and 80% separate the spherical layers into which lenses were sectioned for experiments.

The deep ocean is a dim place. The amount of sunlight that reaches a depth of 300 m is roughly 1/10 000 of that at the surface. Squid have adapted to those low-light aquatic conditions by evolving eyes with lenses that are nearly spherical in shape. A spherical lens has the shortest possible focal length for a given lens radius. It thus optimizes the size of the eye opening relative to the size of the image it makes on the retina. Unlike the oblong human lens, which changes shape to change focal length, the squid lens has a fixed focal length. Squid focus by moving the lens. But a spherical lens can suffer spherical aberration, and the greater the focusing power, the more serious the aberration.

In 1854 James Clerk Maxwell showed theoretically that a spherical lens could be made aberration free if its refractive index varied parabolically as a function of the distance from the lens's center. Evolution beat Maxwell to the discovery by millions of years: Squid, such as the one seen in figure 1, and certain fish employ exactly such graded-index lenses. In squid, the index gradient is achieved through a radial variation in the density of lens-forming S-crystallin proteins—high density at the center and low density at the edge.

Alison Sweeney and her colleagues at the University of Pennsylvania now re-