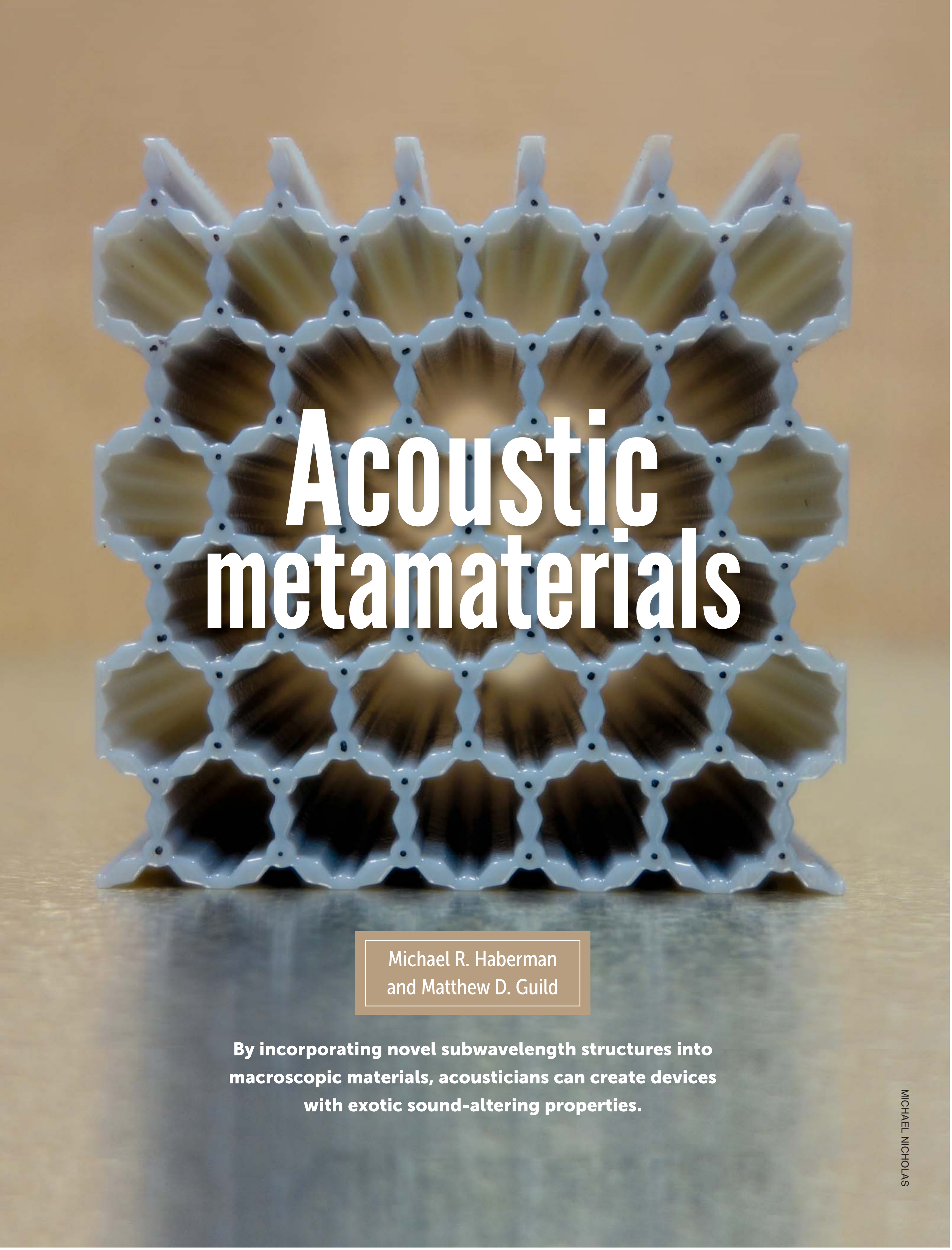

Acoustic metamaterials

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By incorporating novel subwavelength structures into macroscopic materials, acousticians can create devices with exotic sound-altering properties.

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We humans have never been simply passive listeners to the sounds around us. Indeed, we have invented ways to modify and manipulate our environment to generate and receive acoustic signals for specific purposes. Acoustic applications are wide ranging enough to encompass language and other forms of communication, the detection of danger, nondestructive evaluation of precision components, and the creation and appreciation of music.

A striking example is the Cadet Chapel pipe organ at the US Military Academy in West Point, New York. The instrument contains 23 511 meticulously crafted and arranged open-ended pipes, each designed to produce an acoustic resonance at a specific frequency, or note. As many as 380 pipes could be used to play a single note, which makes the organ capable of generating a tremendously rich variety of sounds. The tone produced by each individual pipe is quite ordinary; it's not much different from the sound made by blowing across an empty glass bottle, which acts like an acoustic element known as a Helmholtz resonator. However, when a musician simultaneously plays several carefully chosen pipes in a particular order, a collection of ordinary resonators becomes an instrument capable of creating beautiful music.

Like an organ builder, researchers in the field of acoustic metamaterials seek to achieve extraordinary acoustical characteristics with a carefully chosen arrangement of ordinary acoustical elements. In fact, many acoustic metamaterials use acoustic resonators resembling those found in a pipe organ. However, the aim in fabricating acoustic metamaterials is not to make music but rather to generate material behavior that often exceeds the behavior of naturally occurring materials. Those new material properties can then be used to control acoustic wave propagation in unprecedented ways. The etymology of the word *metamaterial* reflects that goal: *Meta-* is a Greek prefix meaning beyond. In this article we describe the fundamental attributes of acoustic metamaterial elements and what makes acoustic metamaterials extraordinary. We also hope to provide some insight into the current and future potential of those fascinating structures. Along the way we'll discuss metamaterials designed to interact with electromagnetic or elastic waves, though we will not consider those arrangements in detail.

Beyond materials

The advent of acoustic and elastic metamaterials was preceded by a significant research effort in the related field of composite materials. Indeed, composite materials have revolutionized our notions of what conventional materials can achieve, and they are now essential ingredients in modern aircraft, thermal-management components, automobiles, and even such elements of modern infrastructure as underground pipes. Consider fiberglass, which is composed of small glass fibers embedded in an epoxy matrix. As is typical of composites, fiberglass has mechanical properties that are a mixture of those of its two parent materials. The result is a material whose properties are preferable to that of either constituent: a lightweight material with a stiffness approaching that of the glass fibers and a damage tolerance that approaches the performance of the epoxy.

Unfortunately, the material properties of traditional composites are constrained to lie somewhere between the properties of their constituents. One would not expect the strength of fiberglass to exceed that of either the glass fibers or the epoxy matrix. That intuitive limitation has been extensively studied and rigorously formalized; upper and lower bounds on the effective properties of a composite have been well established for specified combinations of materials with a wide variety of microstructures.[1] In the past few years we've learned that precise control of microstructures can produce materials with extraordinary combinations of stiffness and density (see reference 2 and PHYSICS TODAY, January 2012, page 13) and elastic cloaks that could hide a pea from even the most discerning of fairy-tale princesses.[3] Their remarkable properties notwithstanding, such materials do not violate the known bounds on the effective elastic properties.

Unlike traditional composites, acoustic metamaterials can exceed known bounds on conventional material properties. That is accomplished by exploiting subwavelength microstructure that has been fabricated from ordinary materials and embedded in a background medium. (For air- or waterborne acoustic waves, the background medium is simply the surrounding fluid.) Acoustic metamaterials can be used to obtain

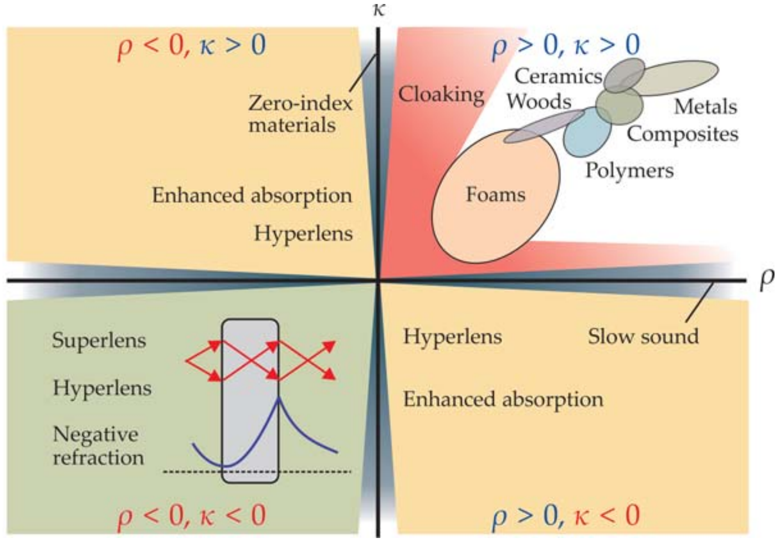


FIGURE 1. AN EXPANDED RANGE OF MASS DENSITY ρ and bulk modulus (stiffness) κ can be accessed with acoustic metamaterials. The upper-right quadrant, for which ρ and κ are positive, is the realm of conventional materials and metamaterial cloaking devices. The remaining three quadrants and their boundaries correspond only to metamaterials, some of whose applications are shown. Zero-index and slow-sound materials, respectively, have vanishing ρ and κ. The lower-left quadrant illustrates some properties of superlenses and hyperlenses, both of which, as described in the text, defeat the diffraction limit. The bending of acoustic rays by those lenses is governed by a negative index of refraction, which leads to the simultaneous focusing of propagating waves (red arrows) and magnification of evanescent waves (blue curve), which together mitigate the effects of diffraction.

effects similar to those obtained with traditional composites, but their novel properties can also enable them to manipulate acoustic wave fields in ways that are impossible to achieve with naturally occurring materials or traditional composite materials.

Negative mass and stiffness

The material properties of interest for acoustic metamaterials are the effective mass density ρ and the effective bulk modulus κ, which is analogous to a spring's stiffness. Figure 1 shows the regions of parameter space appropriate for common materials such as polymers, metals, and ceramics. Of course, all those materials lie in the upper-right quadrant of the ρ–κ plot. The parameter space occupied by acoustic metamaterials goes well outside the area occupied by conventional materials and even includes regions in which one or both of the effective material properties are zero or negative.

Of particular interest for acoustical applications is the ability to manipulate an acoustic wave's speed. Because acoustic metamaterials have a broad range of effective properties, they can produce propagating waves with extremely high, zero, or even negative speeds, not to mention the purely imaginary values that correspond to nonpropagating evanescent waves. Negative phase speed—a backward-propagating phase front—requires that both the effective mass density and the bulk modulus be negative. A material with that combination of properties has been referred to as double negative or left-handed. Materials with either negative mass density and positive bulk modulus or positive mass density and negative bulk modulus are called single negative. Single-negative materials cannot support propagating waves, so any acoustic wave in those materials will exponentially decay. That makes them superior sound absorbers.

The expanded range of acoustic material parameters and phase speeds has inspired concepts and applications that have captured the attention of both the scientific community and the general public.[4,5] Some of those applications and their associated parameter regimes are indicated in figure 1. Acoustic cloaks, for example, bend acoustic waves around their interior and hide any object inside from an incident acoustic field. They do not require single- or double-negative materials, but some types of cloaks do require extremely large, anisotropic parameters that lie outside the range of ordinary materials.[6] The properties necessary for acoustic cloaking, therefore, are only achievable through a properly designed acoustic metamaterial. Phenomena made possible by extremely low material properties include slow sound and substances with a vanishing index of refraction.

Double-negative acoustic metamaterials enable significantly improved refractive properties through negative refraction. They permit us to construct superlenses that amplify evanescent waves and provide subwavelength resolution for acoustic imaging devices. Double-negative materials can also be fabricated into hyperlenses, which use a hyperbolic dispersion relation between frequency and wavenumber, rather than the elliptical dispersion found in traditional anisotropic materials, to produce negative group velocity and a negative index of refraction. Hyperlenses provide a different means to improve the resolution of acoustic imaging devices. They can also be constructed from single-negative anisotropic acoustic metamaterials, but those hyperlenses lack certain advantages arising from negative refraction.

Dynamic microstructure

What enables acoustic metamaterials to have properties that exceed the bounds of traditional materials? The key lies in the metamaterial's microstructure—specifically in its hidden degrees of freedom—which is not directly detectable by an external observer.[7] Those hidden degrees of freedom are often realized through a careful arrangement of dynamic microstructural elements. The simplest and most widely used are small, resonant elements (often called inclusions) such as Helmholtz resonators, resonant scatterers, and elastic membranes. Even though each individual element is small, the contribution of the many elements leads to a large net effect, reminiscent of the way unspectacular-sounding individual organ pipes lead to a majestic sound when correctly played together. In particular, a repeated structure leads to a pronounced overall response, even when each individual element of the microstructure is tiny compared with a wavelength in the background medium.

Furthermore, at some frequencies, the response of the dynamic inclusions can be large and out of phase with the incident acoustic wave. In such a case, an acceptable interpretation of the overall response of the acoustic metamaterial is that the background medium contains homogeneous inclusions whose effective material properties are negative. The resulting effective properties for the metamaterial are necessarily frequency dependent and are often markedly different from the static properties of the metamaterial.

Figure 2 illustrates the notion of hidden degrees of freedom with simple representative mechanical systems comprising masses and springs.[8] Those systems highlight how the presence and arrangement of hidden degrees of freedom can produce the expanded material-parameter space of figure 1. As depicted in figure 2a, the different arrangements of coupled masses and springs act as single effective mass and spring elements, with the mass displaced by a distance $x(t)$ when subject to an applied time-dependent force $F(t)$. By rearranging Newton's second law and Hooke's law, one can obtain the effective mass and spring constant of the system: $m_{eff} = F(t)/\ddot{x}(t)$ and $k_{eff} = F(t)/x(t)$, where the double dots denote the second derivative with respect to time.

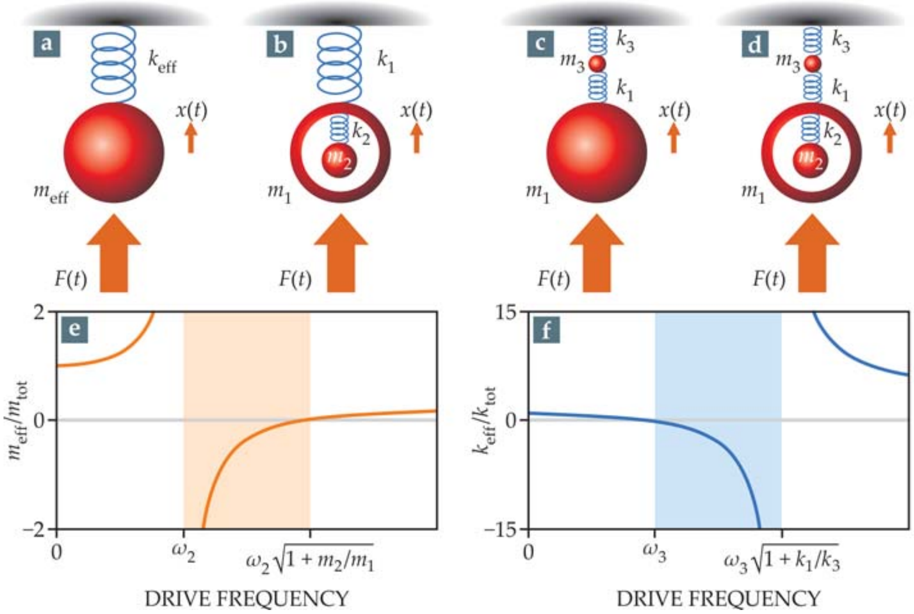


FIGURE 2. SYSTEMS WITH MASSES AND SPRINGS illustrate the idea of hidden degrees of freedom in an acoustic metamaterial. **(a)** The various arrangements of masses (m_i) and spring constants (k_i) in panels b–d combine to give effective mass and spring constants that describe each system's displacement $x(t)$ in response to a driving force $F(t)$, as depicted here. We assume that m_3 is much lighter than the other two masses. Specific arrangements of masses and springs enable **(b)** negative mass, **(c)** negative spring stiffness, and **(d)** negative mass and stiffness. **(e, f)** Normalized effective mass and spring stiffness are shown as a function of drive frequency. The resonance frequencies ω_2 and ω_3 are, respectively, $\sqrt{k_2/m_2}$ and $\sqrt{k_3/m_3}$. In panel e, which corresponds to the mass–spring setup of panel b, the orange band shows a region of negative dynamic mass. In panel f, which corresponds to the arrangement in panel c, the blue band shows a negative-stiffness region. The normalizations are determined by the quasistatic mass $m_{tot} = m_1 + m_2$ and the quasistatic spring constant $k_{tot} = (1/k_1 + 1/k_3)^{-1}$.

To understand how negative effective properties result from the dynamic motion of ordinary masses and springs, consider the frequency responses shown in figures 2e and 2f. Under quasistatic conditions (that is, a driving frequency well below any resonance frequency of the system), the resulting effective mass and spring stiffness correspond to the familiar positive values one would obtain for masses and springs in series. As the driving frequency increases and approaches resonance, the system deviates from its familiar behavior. That response can correspond to a dramatic change in the magnitude of the effective mass or spring stiffness.

As the frequency passes through resonance, the phase of the movement of the effective mass relative to that of the applied force can shift by 180°. As a result, the applied force vector flips its orientation relative to the acceleration or displacement of the composite object. The equivalent system shown in figure 2e replaces the composite object with an effective mass that is negative over some frequency range. Likewise, the system shown in figure 2f replaces the composite object with a spring whose effective stiffness is sometimes negative. Although the simple mass–spring systems in figure 2 might seem trivial, their fundamental behavior is at the root of many acoustic metamaterial microstructures. The true challenge associated with acoustic metamaterials has therefore been to translate physical principles such as those illustrated by the figure into realizable acoustic metamaterial microstructures.

Inception of a 21st-century field

The ground for acoustic metamaterial development may have been fertilized by earlier work with composite elastic materials, but the origin of acoustic metamaterials actually has a more direct connection to the field of electromagnetism. (See the article by John Pendry and David Smith, PHYSICS TODAY, June 2004, page 37.) Developments in electromagnetism at the turn of the 21st century inspired scientists to look at ideas—cloaking, negative refraction, superlenses, and more—that were previously the stuff of science fiction. Although such ideas have been around for many years, their realization is possible only with metamaterials. The enabling metamaterial element in the field of electromagnetism was the split-ring resonator, first proposed by John Pendry and colleagues in 1999 and experimentally demonstrated by David Smith and company the following year.[9]

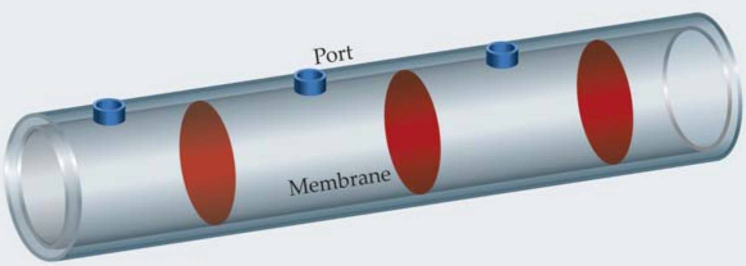
At the same time, researchers were using phononic and sonic crystals (figure 3a shows an example) to study bandpass and bandgap behavior associated with elastic and acoustic waves propagating in periodic media. Those efforts built on extensive work in the related field of photonic crystals. The objective of phononics research is often to use the band structure of phononic crystals to achieve such extraordinary results as guiding waves along specified paths in three-dimensional space, frequency filtering, and focusing or slowing down waves propagating through the crystal.[10] Unlike phononic crystals, which derive their unique properties from the mutual interaction of elements whose size and spacing are on the order of a wavelength, acoustic metamaterials rely on subwavelength structure—both in the geometry of the individual elements and in the spacing between them—and self-interaction to generate novel behaviors. Acoustic metamaterials do not require periodic microstructure to achieve their extraordinary properties, though effects of periodicity are often exploited and can lead to a simpler mathematical analysis.

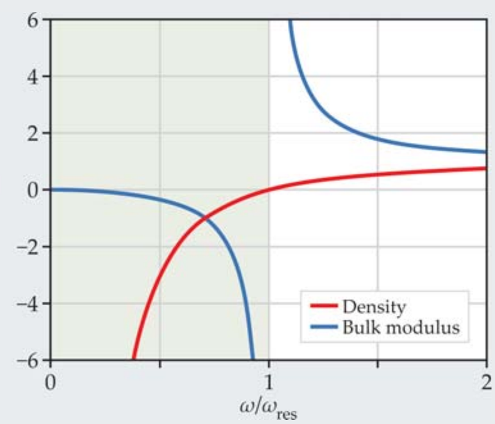
In 2000 Zhengyou Liu and colleagues found negative effective properties in what is now acknowledged as the first experimentally demonstrated acoustic metamaterial: a sonic crystal constructed from locally resonant elements whose spacing is much less than the relevant acoustic wavelength.[11] The sonic-crystal inclusions used by Liu and colleagues exploit the same simple physical mechanisms as the mass–spring resonators illustrated in figure 2. Their sonic-crystal embodiment replaces the masses with concentric, dense, nearly rigid spherical shells and the springs with highly compliant layers.

Acoustic waveguides provided another robust means to realize acoustic metamaterials. Long before the existence of acoustic metamaterials, acoustic waveguides were used to demonstrate extreme or exotic effects. In its simplest form, an acoustic waveguide can be created from a rigid-walled tube. One example is the acoustic muffler, which has been used for generations to suppress the unwanted sound propagating in ducts and pipes. In fact, some of the earliest demonstrations of acoustic Bloch waves (waves in periodic media) and their band structure were achieved with acoustic waveguides having a periodic arrangement of side-branch cavities.[12] When the walls of the waveguide are not rigid, a cutoff frequency exists below which acoustic waves are evanescent and cannot propagate. Acoustic waves are also evanescent in the stop band of a periodic structure such as a waveguide with equally spaced membranes along its length; for some frequencies, the periodic guide is a single-negative metamaterial.

Furthermore, a periodic arrangement of acoustic elements in a long acoustic waveguide is the acoustic equivalent of an electromagnetic transmission line. That analogy has enabled designs for electromagnetic metamaterials to be leveraged for the construction of acoustic transmission-line metamaterials. Research with acoustic waveguides has thus led to the realization of acoustic metamaterials that display single-negative or double-negative properties over a broad range of frequencies.[13]

TRANSMISSION-LINE ACOUSTIC METAMATERIALS





Elastic membranes and ports, or side holes, in an acoustic waveguide can act like the inductive and capacitive components in an electromagnetic transmission line. Based on that principle, Sam Hyeon Lee and colleagues fabricated a double-negative acoustic transmission-line metamaterial by periodically arranging membranes and ports in a manner similar to that illustrated above.[13]

The nonintuitive negative effective mass density ρ_{eff} and bulk modulus κ_{eff} arise due to dynamic microstructure. As the plot of normalized parameters shows, the effective properties of acoustic metamaterials are frequency dependent; the desired response occurs within a specific frequency band.

Negative effective density below resonance can be achieved by placing transverse elastic membranes in a waveguide. With such a configuration, the effective frequency-dependent density has the form

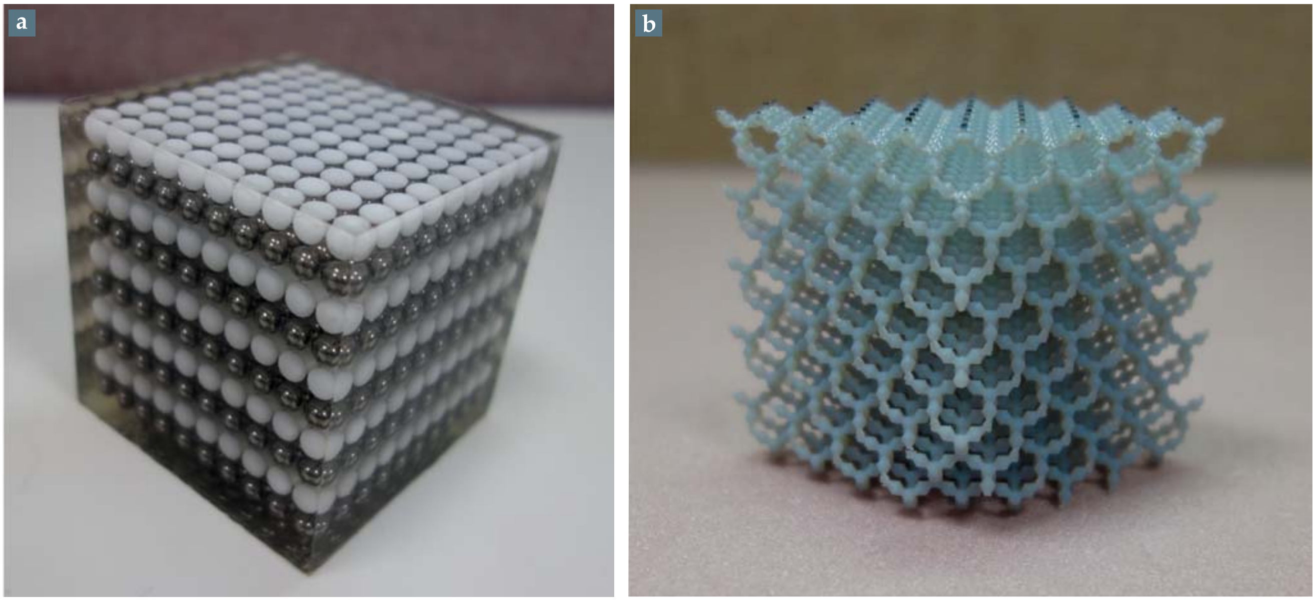
$$\rho_{eff} = \rho \left[1 - \left(\frac{\omega_{res}}{\omega}\right)^2\right].$$

Transverse elastic membranes are typically thought of as stiffness elements: Like a spring or trampoline, they provide a restorative force proportional to the displacement. But in the transmission-line acoustic metamaterial, the membrane functions as a frequency-dependent mass element at frequencies well below the flexural resonance ω_{res} of the membrane. That nonintuitive characteristic is a consequence of the acceleration given to the membrane by the force of the incident acoustic wave, as described by the Newtonian relation $m_{eff} = F/a$. A negative effective mass results when the net force due to the interaction with the acoustic wave is out of phase with the acceleration.

On the other hand, a parallel arrangement of acoustic ports in the waveguide can be used to achieve negative effective stiffness below resonance; such a configuration leads to an effective bulk modulus

$$\kappa_{eff} = \kappa \left[1 - \left(\frac{\omega_{res}}{\omega}\right)^2\right]^{-1}.$$

In brief, in an acoustic transmission-line metamaterial, it is the acoustic ports that produce negative stiffness and the membranes that produce negative mass density. For ease of presentation, we have assumed that the resonance frequencies for density and stiffness are equal. In general, and for the device developed by Lee and colleagues in particular, the frequencies are different. As a result, the combination of ordinary elements in an acoustic transmission line can produce extraordinary single- and double-negative effective acoustic properties.



The box on page 46 discusses in detail a representative double-negative acoustic transmission line.

FIGURE 3. REAL-WORLD ACOUSTIC METAMATERIALS look like this. **(a)** Sonic crystals are the acoustic analogues of photonic crystals: In both cases, periodic lattices of scatterers can create extreme changes in wave propagation characteristics at certain frequencies. Sounds of certain frequencies and directions cannot propagate through them. In this structure, a periodic arrangement of elastic spheres is embedded in a polymer matrix. **(b)** Pentamode materials, which support stress in only one of six possible modes, have inspired many acoustic metamaterial designs. Pictured here is an example with three-dimensional structure. The image on page 42 shows an analogous material with structure in just two dimensions. (Photographs courtesy of Michael Nicholas.)

Transformation acoustics

Experimental demonstrations of extraordinary behavior in sonic crystals and waveguides showed that acoustic metamaterials could be realized, but it was an apparently unrelated mathematical concept—coordinate transformations—that sparked intense interest in metamaterials and drove much of the first decade of research in the field. In the context of acoustic waves, the associated technique for deducing metamaterial properties is known as transformation acoustics. A one-to-one map connects a target space exhibiting acoustic behavior of interest—for example, routing acoustic waves around an enclosed object to shield it from prying ears—to a physical space with conventional materials and a mundane acoustic field. The map is then used to determine the material properties in the target space that produces the desired effect. Transformation acoustics is an elegant and extraordinary technique. The material properties it determines are also extraordinary and in practice require metamaterials for their realization.

Before coordinate transformations had been applied to the manipulation of acoustic waves, they had been applied to electromagnetic waves (see PHYSICS TODAY, February 2007, page 19). Indeed, the concept of transformation acoustics was inspired by a direct analogy that exists in two dimensions between Maxwell's equations and the equations describing acoustic wave propagation.[6] Slight differences in the electromagnetic versus acoustic coordinate transformations arise in three dimensions, because 3D electromagnetic waves are transverse but acoustic waves are longitudinal.[14] In both cases, the coordinate transformations typically demand highly anisotropic materials. In the electromagnetic context, such materials could theoretically be realized with alternating layered structures, each having isotropic properties. However, for elastic materials, in which compressional and transverse shear waves coexist, coordinate transformations require far more exotic stuff—pentamode materials—with a fundamentally new type of microstructure. Pentamode materials are characterized by stiffness tensors that have only one non-null eigenvalue out of a possible six.[15] That description is a mathematical way to state that pentamode materials can support only one stress; therefore, five deformation modes do not store energy in the material. Figure 3b shows a pentamode metamaterial created using additive manufacturing.

Leveraging technology

History is replete with examples of new technologies facilitating scientific advances. Notable cases include the printing press, which accelerated communication and enabled the permanence of scientific concepts; microscopes that allowed us to study biological cells; telescopes that we turned to the heavens; and the technique of x-ray diffraction, which enabled the discovery of DNA structure. Importantly, such technological advances not only help scientists confirm hypotheses and discover key concepts, they also subtly influence how we formulate questions about the world and the universe. A more concrete picture of DNA structure, for example, allowed scientists to make hypotheses about genetic encoding that could not have been imagined prior to the knowledge gains achieved with x-ray diffraction.

Progress in acoustic, elastic, and electromagnetic metamaterials is the product of a combination of technological advances that have been made in the past 20–30 years. Indeed, the field of acoustic metamaterials is a prototypical example of novel concepts in physics converging with advances in technology, primarily relating to the exciting field of additive manufacturing and widespread access to robust computational tools. (See the article by Martin Wegener and Stefan Linden, PHYSICS TODAY, October 2010, page 32, and the news item, September 2015, page 26.) Those and other technologies allow researchers to rapidly simulate, build, and test the elaborate structures for acoustic wave manipulation that follow from rigorous mathematical predictions such as those of transformation acoustics.

In the past 15 years, the field of acoustic metamaterials has grown from a fledgling discipline at the intersection of acoustics, phononics, and composite materials to a dedicated and thriving field at the cutting edge of science and engineering. Researchers have now successfully developed proof-of-concept designs and experimentally demonstrated a wide range of extreme effective acoustic properties. Still, significant technical challenges must be overcome if acoustic metamaterials are to be brought to their full potential. After all, designing and fabricating a dynamic microstructure tailored to create a desired effective material response requires great precision and attention to structural detail, potentially across orders of magnitude in length scale.

We are optimistic. The confluence of additive manufacturing, advanced computation, and the new ideas that they help generate hold significant promise for continued advancement in acoustic metamaterial research. With those tools, scientists who had previously worked in the fields of phononic materials, composite materials, mathematics, electromagnetics, and solid-state physics are rapidly changing the possibilities for manipulating acoustic and elastic waves.

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