



Single-electron cyclotron radiation

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Experiments that track the radiation emitted by a lone electron orbiting a magnetic field may, in time, reveal the effects of neutrino mass.

Students of electromagnetism learn a lot about electromagnetic wave propagation long before they learn about electromagnetic wave sources; indeed, they often see their first source problems in graduate school. Nevertheless, undergraduate students will almost certainly hear that a classical orbiting electron emits radiation, as if that fact were obvious. Let's look more closely. How does a classical orbiting electron emit radiation? Is that radiation something physicists can observe?

Collecting radiation from a large ensemble of electrons is as easy as building a loop antenna. But single-electron cyclotron emission evaded detection until 2014, when the Project 8 collaboration measured it in preparation for a neutrino-physics experiment.

Cyclotron versus synchrotron

In a uniform magnetic field $\mathbf{B}$, an electron with mass m , charge magnitude e , and speed v follows a circular or helical path whose circular component has a "cyclotron" frequency $f_c = eB/2\pi\gamma m$. Here $\gamma = (1 - v^2)^{-1/2}$ is the relativistic Lorentz factor. For low speeds, $\gamma \approx 1$, and f_c is nearly the constant value $f_0 = eB/2\pi m$, familiar from elementary electrodynamics classes.

Note that f_c depends on speed, not individual velocity components; for a given value of γ , the frequency is the same for helical and circular orbits. That invariance suggests that f_c , because of the Lorentz factor in its denominator, might be a tool for measuring particle energies.

Is a measurement of an orbiting electron's radiation frequency the same thing as a measurement of the cyclotron frequency? Not necessarily. After all, when the electron's orbit is helical, the detected radiation may be Doppler shifted away from f_c . Another counterexample is synchrotron radiation. In that phenomenon, electromagnetic radiation of high frequency can emerge from an electron orbiting at relatively low frequency. For example, electrons orbiting at megahertz frequencies in an electron storage ring can emit exahertz (exa = 10^{18}) x-ray radiation.

One can understand the difference between cyclotron and synchrotron radiation by means of a laborious classical calculation, but a quantum mechanical picture is actually more intuitive. A magnetically trapped electron—what Nobel laureate Hans

Dehmelt called a geonium atom—has quantized energy and angular momentum states, just like a regular atom. If one ignores fine structure, geonium's energy levels are the Landau levels, $E_n = nhf_c$, and the n th energy level has angular momentum $L = nh$. Classical cyclotron radiation corresponds to emission of the smallest possible quantum—that is, an energy-level jump with $\Delta n = 1$. Such transitions carry away an angular momentum $\Delta L = \hbar$ and are always allowed.

Synchrotron radiation, though, is the emission of high-energy photons, $\Delta n \gg 1$, which, in geonium, implies $\Delta L \gg \hbar$. That's a highly forbidden transition, inasmuch as the emitted photon must carry away a large amount of orbital angular momentum. The x rays emitted from a synchrotron pull off the feat because they're emitted tangentially to a large-radius, ring-shaped orbit. Intuitively, then, the difference between cyclotron radiation, with its allowed $\Delta L = \hbar$ transitions, and synchrotron radiation, for which $\Delta L \gg \hbar$, is that synchrotron radiation begins as the growing radius of an orbit permits easier coupling to photons with large orbital angular momentum.

Message in a bottle

The multi-institutional Project 8 collaboration, for which I am co-spokesperson, set out to detect single-electron cyclotron radiation and thus measure the electron's energy via the Lorentz γ factor. We started with a comparatively easy-to-use isotope, krypton-83m. The excited nucleus of the isotope (signified by the "m") de-excites and, in a process called internal conversion, ejects one of the atom's orbiting electrons at one of several energies between 7 keV and 32 keV.

We wanted the electrons to orbit in a strong magnetic field and stay there long enough to be detected. So we let the ^{83m}Kr decays occur in a shallow magnetic bottle—a region where a $\mathbf{B}$ field is slightly weaker in the center and stronger in either direction along the field lines. That might seem like an odd choice: If we're trying to interpret $f_c = eB/2\pi\gamma m$ as an energy measurement, we'll want to keep the B in the numerator constant, or so one would think. Unfortunately, in a constant magnetic field, most electrons would spiral along the straight magnetic field lines and exit the system too quickly for us to get a good frequency measurement. How big an effect does the nonuniform field have on the cyclotron frequencies?

It's complicated. An electron that starts with a small velocity component parallel to the field will spend some of its time in the higher-field areas; the oscillating trajectory is a source of uncertainty in γ . For shallow traps—that is, if $\mathbf{B}$ doesn't vary

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too much—only a narrow range of pitch angles leads to observable signals, and the impact of the nonuniform field is thus minimized. For the deeper traps needed in larger experiments, the result of the oscillatory electron motion is a frequency modulation of the cyclotron radiation. Just as for an FM radio signal, the oscillating trajectory generates power in frequency sidebands located at $f_c \pm n f_z$, where f_z is the frequency of oscillations back and forth in the magnetic bottle and n is a positive integer. Note that the modulation accounts for both the Doppler shift and the nonuniform $\mathbf{B}$. In future experiments, we expect to measure both f_c and f_z ; it's possible we'll need both to constrain the electron energies.

Project 8's first single-electron detections were from $^{83\text{m}}\text{Kr}$ decays occurring in a waveguide. We created a uniform 1-T magnetic field within the waveguide and introduced trapping nonuniformities by winding a small coil of copper wire around it. When an electron is born in the trap, it radiates about 10^{-15} W of power into one of the waveguide modes. The waveguide itself carries the low-power signal away from the trap region, to cryocooled amplifiers. To minimize noise, we would have liked to cool our system to about 15 K, but we couldn't do that because the Kr would freeze onto the waveguide walls.

Electrons revealed

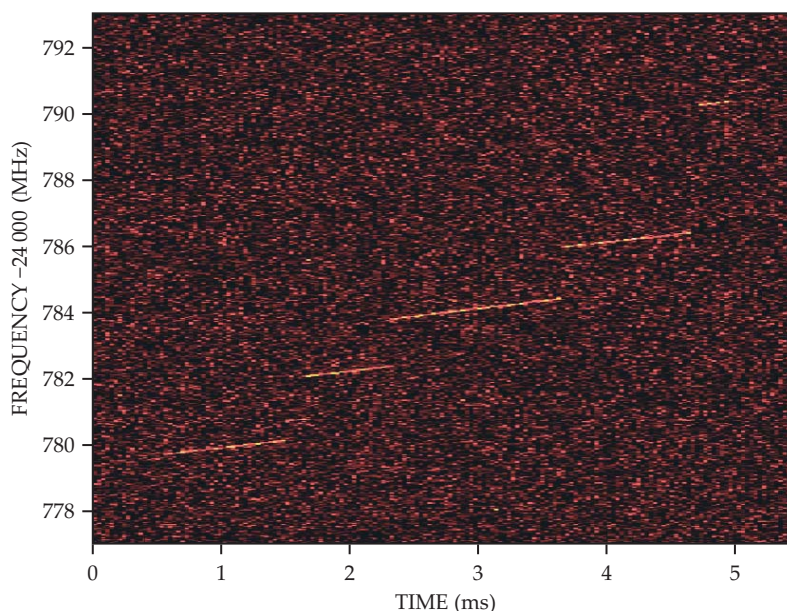
We finally saw electrons. To be more precise, we were the first to see cyclotron radiation from a single electron. Physicists had long before observed cyclotron radiation from electron ensembles, synchrotron radiation from single electrons, and single-electron energy losses attributed to unseen cyclotron radiation.

The electrons we spotted had many of the features we anticipated. Most importantly, f_c was as expected, given the known decay energies of $^{83\text{m}}\text{Kr}$. The gradual increase in frequency seen in the figure is due to radiative energy loss—that is, due to the emission of the very radiation we're detecting. Indeed, we can even check energy conservation: The observed rate of change of the electron's energy is fairly close to the electromagnetic wave power we detect.

The sudden frequency jumps visible in the figure are evidence of electron scattering off of a residual gas atom, probably hydrogen. The characteristic 10-eV scale of the jumps is determined by the energy necessary to electronically excite the atom.

In fact, we were surprised to see the jumps at all. We had originally imagined that a trapped electron would orbit for a while, then scatter and usually escape due to its direction change. Contrary to our expectations, the typical direction change is small, and except in the shallowest traps, the electron usually remains in the trap after many visible scattering events. The resulting spectrograms are packed with information—and they're also pretty to look at!

In measuring a γ -dependent frequency, the Project 8 experiment is making a precise determination of an electron energy by means of frequency-domain methods. We hope that our ap-



THE PROJECT 8 EXPERIMENT measured the cyclotron radiation emitted by single electrons. This spectrogram shows that as an electron radiates energy away, its emission frequency increases due to relativistic effects. In addition to that gradual increase, Project 8 scientists were able to observe a series of 10-eV jumps due to collisions of electrons with residual gas atoms. Brighter colors indicate more detected power.

proach will prove to be a useful technique, not just a curiosity. Most notably, precise measurements of electron energies in beta-decay reactions may allow physicists to get a handle on the neutrino mass scale.

Beta decay is a nuclear transformation that emits an electron and an antineutrino. Because those two particles have to share the energy available from the nucleus, a measurement of the electron energy is also a measurement of the neutrino energy. When the electron has nearly all of the energy, the neutrino has nearly none of it. In extreme cases, the neutrino's total energy may be so low that a nonzero neutrino mass begins to have an effect on the shape of the electron energy probability distribution near the high-energy endpoint. Exactly how the distribution is affected depends on the still-unknown neutrino mass scale, but the distribution itself is a straightforward observable quantity.

Tritium is the best isotope for a Project 8 investigation, because the maximum electron energy in tritium beta decay occurs at around 18.6 keV, near what seems to be our sensitivity sweet spot. The Project 8 team is working on incorporating a small tritium source into its current apparatus and designing the future large experiment that will use cyclotron radiation for a high-statistics, high-precision measurement of the neutrino-mass effect near the endpoint.

Additional resources

- B. Monreal, J. A. Formaggio, "Relativistic cyclotron radiation detection of tritium decay electrons as a new technique for measuring the neutrino mass," *Phys. Rev. D* **80**, 051301 (2009).
- D. M. Asner et al. (Project 8 collaboration), "Single-electron detection and spectroscopy via relativistic cyclotron radiation," *Phys. Rev. Lett.* **114**, 162501 (2015).
- P. Huber, "Viewpoint: Cyclotron radiation from one electron," <https://physics.aps.org/articles/v8/36>.

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