bigger than about 600 μm in diameter, biofilm growth began to periodically slow down and speed up.[2] The researchers surmised that those oscillations reflected an internal conflict within the colony between long-term survival and short-term growth.

Cells at the film's edges protect the interior cells from external crises, such as a sudden change in pH or the presence of antibiotics. The long-term maintenance of interior cells is a critical hedge against such threats. But for the colony to expand, cells at the periphery, where film growth largely occurs, often use up a lion's share of available nutrients. That overconsumption of nutrients—glutamate in the case of *B. subtilis*—at the edges can starve the interior.

Süel and his colleagues realized that the biofilm colony balanced those two requirements by developing a metabolic codependence between the interior and peripheral cells. The cells break down some of the glutamate in the nutrient broth into ammonium. They then combine the ammonium with glutamate to make the amino acid glutamine, a necessary ingredient for bacterial growth.

When the interior cells are starved of glutamate, they stop producing ammonium. The reduction in available ammonium is enough to halt the growth of the peripheral cells; glutamate then becomes available in the interior.

Sending out an SOS

Although the ammonium–glutamate balancing act is sufficient to explain the growth oscillations, the high degree of synchronization in those oscillations suggested that additional coordinating mechanisms must be at play. Because a cell's ability to absorb glutamate or retain ammonium depends on its membrane potential, Süel and his colleagues suspected that the mechanism was electrochemical.

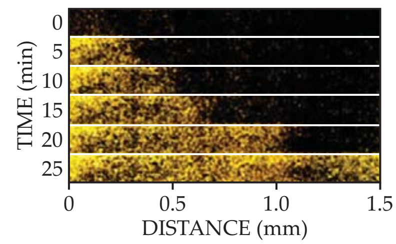

FIGURE 3. ACTIVE PROPAGATION of potassium ions. This time series of fluorescence microscope images shows that cells in the interior of a biofilm release K^+ (yellow) as a metabolic distress call to cells at the periphery. Neighboring cells actively amplify and relay the signal by releasing their own K^+. (Adapted from ref. 1.)

Using a voltage-sensitive fluorescent dye, the researchers found periodic fluc-

PHYSICS UPDATE

These items, with supplementary material, first appeared at www.physicstoday.org.

A QUANTUM DERIVATION OF A CLASSIC MATH FORMULA

When a quantum mechanical Hamiltonian cannot be solved exactly, one can estimate system energies with a technique called the variational method. The idea is to calculate the energy expectation value for a trial wavefunction with one or more tunable parameters and determine the minimum energy that results from varying the parameters. As an exercise to help his students get a feel for the approach, the University of Rochester's Carl Hagen applied it to a solvable system—the hydrogen atom. With the help of Rochester colleague Tamar Friedmann, he found to his surprise that the exercise yielded a representation for π published in 1655 by mathematician John Wallis:

$$\frac{\pi}{2} = \frac{2\cdot 2}{1\cdot 3}\,\frac{4\cdot 4}{3\cdot 5}\,\frac{6\cdot 6}{5\cdot 7}\cdots.$$

Hagen and Friedmann considered a trial wavefunction that had the same angular behavior as the hydrogen atom but different radial behavior. They calculated how the minimum variational energy of their trial form depended on the angular momentum quantum number ℓ and divided that energy by the exact energy of a hydrogen atom with angular momentum ℓ and principal quantum number $n = \ell + 1$. According to the variational principle, the ratio of energies

$$\frac{(\ell+1)^2}{(\ell+\frac{3}{2})}\left[\frac{\Gamma(\ell+1)}{\Gamma(\ell+\frac{3}{2})}\right]^2$$

must be ≤ 1. (The Γ function is the famous generalization of the factorial.) In fact, as ℓ approaches infinity, the ratio approaches 1. The Wallis representation then follows from the recursion property of the Γ function, $\Gamma(z+1) = z\Gamma(z)$, and the specific values $\Gamma(1) = 1$ and $\Gamma(1/2) = \pi^{1/2}$. Given the different radial behavior of the trial and exact wavefunctions, it may seem surprising that the ratio of variational to exact energies tends to 1 for large ℓ. The authors note, however, that in the infinite-ℓ limit, both wavefunctions describe electrons on sharply defined trajectories; the quantum fuzziness that distinguishes the electron orbits for finite ℓ goes away. (T. Friedmann, C. R. Hagen, *J. Math. Phys.* **56**, 112101, 2015.)

—SKB

ATMOSPHERIC WAVES ABOVE NEW ZEALAND

When air blown across a sea or a plain encounters a mountain range, it's pushed upward into the cooler air above. The difference in buoyancy between the two air masses sets up a standing gravity wave—a mountain wave—leeward of the range. Mountain waves, in turn, engender other gravity waves that lift energy and momentum into the stratosphere and mesosphere. (See Backscatter in PHYSICS TODAY, June 2006, page 96.) To characterize those waves, Bernd Kaifler of the German Aerospace Center and his collaborators developed a compact, mobile light and detection ranging (lidar) experiment. Between June and November 2014, they installed it in one of the world's strongest sources of mountain waves: New Zealand's Southern Alps. (The accompanying photo shows waves above the site.) The experiment sent light pulses upward into the atmosphere and measured the echoes' travel time, which yields the altitude, and their intensity, which is proportional to the local atmospheric density. Thanks to the lidar's power, the experiment could quantify gravity waves up to 80 km, which corresponds to the top of the mesosphere. As expected, the gravity waves were most active during southern winter, when the prevailing westerly winds are strongest. Yet even during the summer, the gravity waves pervaded the stratosphere and mesosphere. The biggest surprise came when the researchers correlated the speed of the surface winds with the altitude of the gravity waves: Moderate, not strong, winds are the most effective