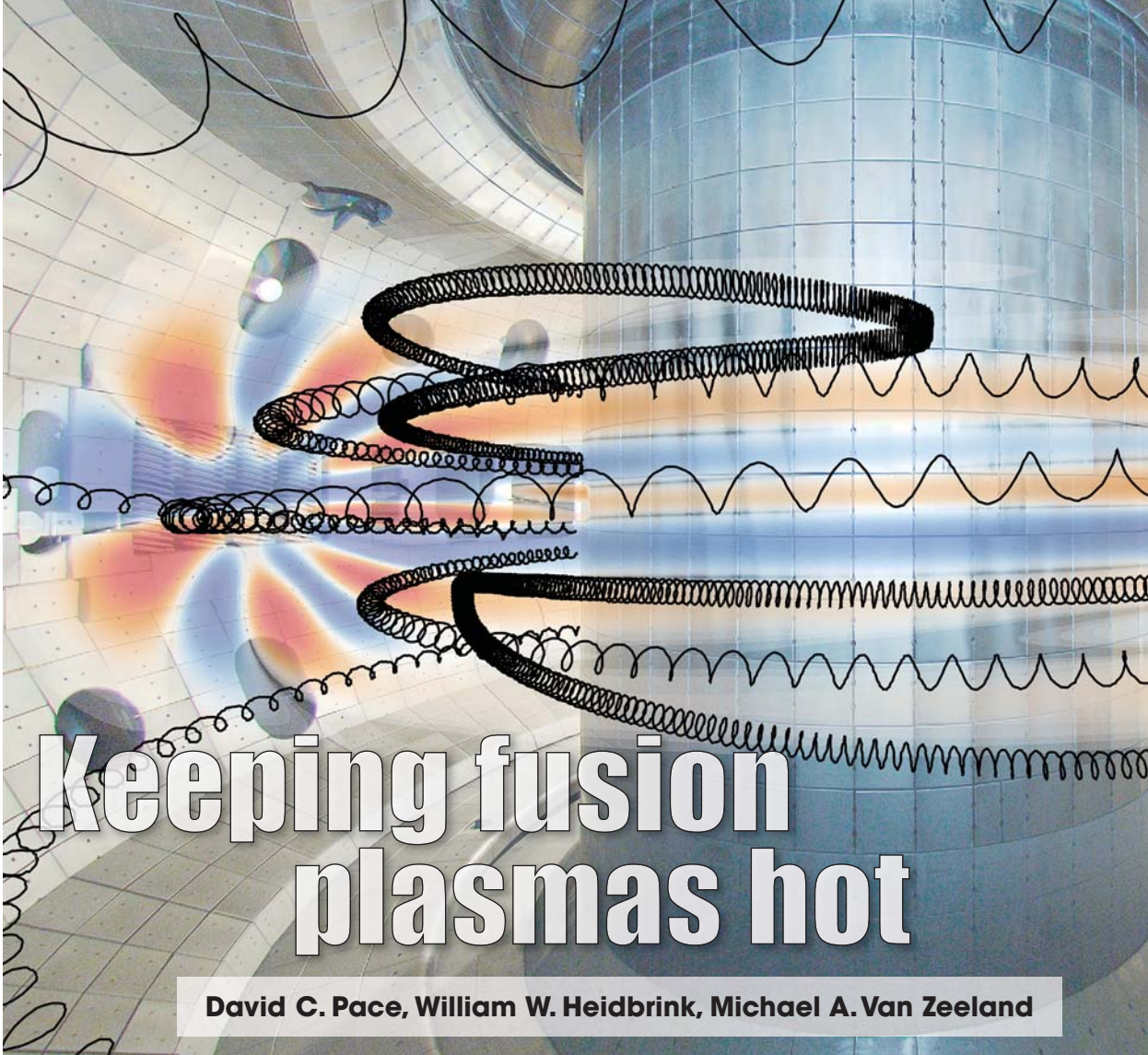




Keeping fusion plasmas hot

David C. Pace, William W. Heidbrink, Michael A. Van Zeeland

Interactions between electromagnetic waves and the most energetic ions in a plasma can perturb the orbits of those ions enough to expel them from the confining magnetic field.

Igniting and sustaining the fusion of light nuclei requires an astonishingly high temperature, on the scale of 100 million kelvin. No material can tolerate such an environment, and for some 60 years scientists have been studying plasmas bound within a cage of magnets to insulate the walls of a reaction vessel from the intense heat inside it. Equally important, the confinement helps avoid cooling the bulk plasma to the point of quenching the reaction.

In a tokamak—an acronym derived from the Russian *toroidal'naya kamera s magnitnymi katushkami*, meaning toroidal chamber with magnetic coils—the charged particles circulate within a

helical magnetic field that winds around and delimits a bagel-shaped confinement region. The helical field arises from two components: a toroidal field whose lines extend the long way around the torus and are produced by a set of ring-shaped electromagnets, and a poloidal field whose lines form circles orthogonal to the toroidal field and are produced by the current of electrons and ions in the plasma. That current can be, for example, induced by a large central solenoid, effectively making the plasma the secondary winding of a transformer. (For a detailed primer on magnetic confinement in a tokamak, see the article by Don Batchelor, *PHYSICS TODAY*, February 2005, page 35, and the news story from June 2009, page 18.)

The goal of most efforts worldwide is to create a self-sustaining fusion energy device that hosts the nuclear reaction with the largest cross section: the fusion of deuterium and tritium to produce an alpha particle, or helium nucleus, and a neutron. Because alpha particles are more energetic than the back-



David Pace is a research scientist in the magnetic fusion energy (MFE) division at General Atomics in San Diego, California.

Bill Heidbrink is a professor in the department of physics and astronomy at the University of California, Irvine. **Michael Van Zeeland** is also a research scientist in the MFE division at General Atomics.

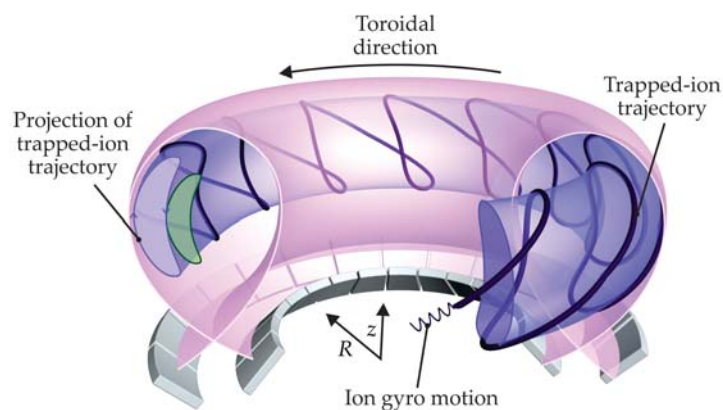
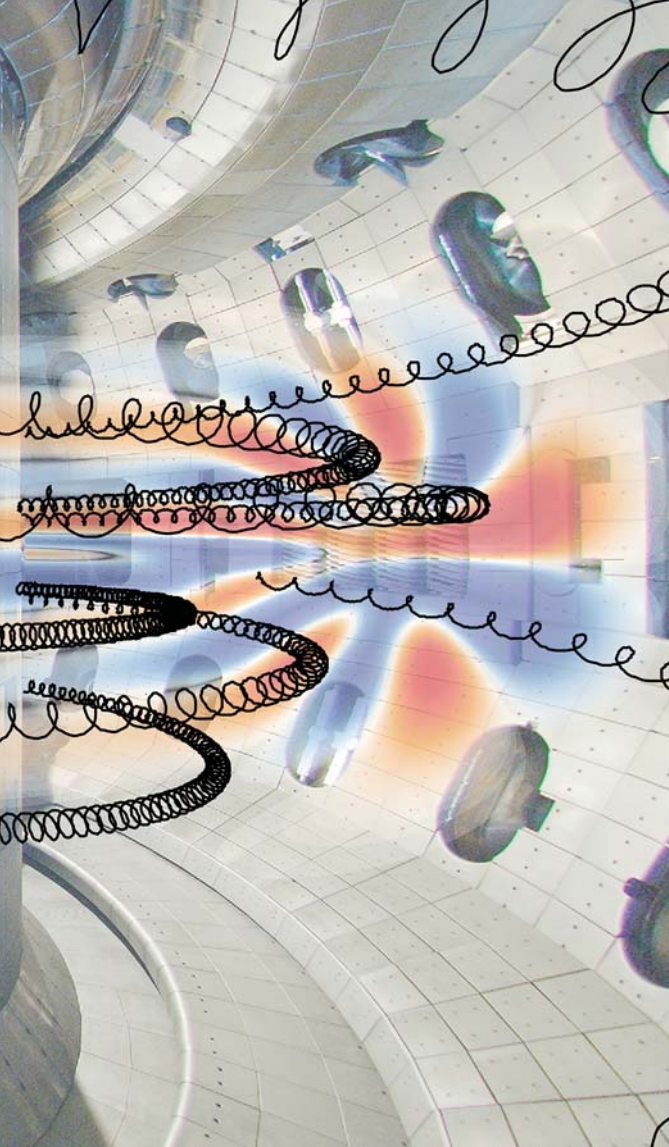


Figure 1. A tokamak is a bagel-shaped vessel that confines a hot plasma (bounded by the pink shell) within a magnetic field that twists around the torus in a gentle helix. The field arises from a combination of external magnets (not shown) and guides electrically charged particles into so-called passing and trapped-ion orbits. In the case of the latter, because the magnitude of the net applied field falls off with radius R from the axis of the torus, a trapped ion follows a banana-shaped trajectory (black, in the purple shell). While tracing that path, the ion also experiences small-scale oscillations, or gyro motion, about the magnetic field. Projections of two ion trajectories (purple and green) are shown at left. The silver rails below the plasma collect helium ash and other debris from the fusion reaction. (Courtesy of EUROfusion.)

ground plasma, they heat the plasma and help maintain thermonuclear temperatures. Unfortunately, ions in the plasma can leak out of it, which lowers the reaction yield and has long stymied efforts to drive the fusion process from its own internal heat. Central to the leaking problem are the complicated interactions between plasma waves, which are produced by oscillating ions and electrons, and the fastest-moving ions themselves.

The wave-particle interactions may perturb the ions' orbits enough to kick them out of the plasma. The loss doesn't just affect reaction yield. Ejected ions can also ablate material from protective tiles on the walls, which pollutes the plasma. More catastrophically, but also more rarely, the ions may damage the vacuum vessel enough to cause an air leak.¹

Recent experiments and simulations of such wave-induced ion transport have supplied new details on the process. And scientists are gaining a deeper understanding of the sensitive phase dependencies that drive the wave-particle interactions. Indeed, although the interactions degrade the magnetic confinement, their observed behavior is a helpful diagnostic for researchers trying to ameliorate their effects in future fusion reactors. More fundamentally, the studies beautifully illustrate basic processes that occur in plasmas throughout the universe.

Alfvén waves and modes

To appreciate the complexity of particle motion in a tokamak, consider figure 1, which shows the orbit of a single ion through the machine. The ion gyrates rapidly around the local magnetic field at the same time it drifts more slowly in the toroidal direction in response to the field's spatial gradient and curvature. The slower drift follows a "trapped" orbit that traverses back and forth, tracing out a banana-shaped path in the cylindrical (Rz) plane. The motion originates from the fact that the magnitude B of the magnetic field from the toroidal solenoid varies inversely with radius in the torus.

The magnetic moment $\mu = \frac{1}{2}m_i v_{\perp}^2/B$ of an ion with mass m_i is invariant when the confining field changes on time and length scales long compared with the ion's gyrating motion. As the ion experiences an increasing magnetic field B on moving from an outer to an inner region of the plasma, its velocity perpendicular to B , given by $v_{\perp}$, must increase accordingly to keep μ constant. But because the magnetic field does no work on the ion, the total ion energy remains constant, which requires that the velocity parallel to the magnetic field must decrease. Eventually that parallel motion, around the axis of the torus, comes to a stop and turns around.

The other main type of path an ion can follow is a so-called passing orbit, which more closely follows the magnetic field lines and traces out a contour in the Rz -plane that is representative of the plasma shape or, more precisely, the contours of constant magnetic flux. An ion or electron will follow a trapped or passing orbit depending on whether it has enough velocity parallel to the magnetic field to avoid mirroring (that is, stopping and reversing direction) into a trapped trajectory.

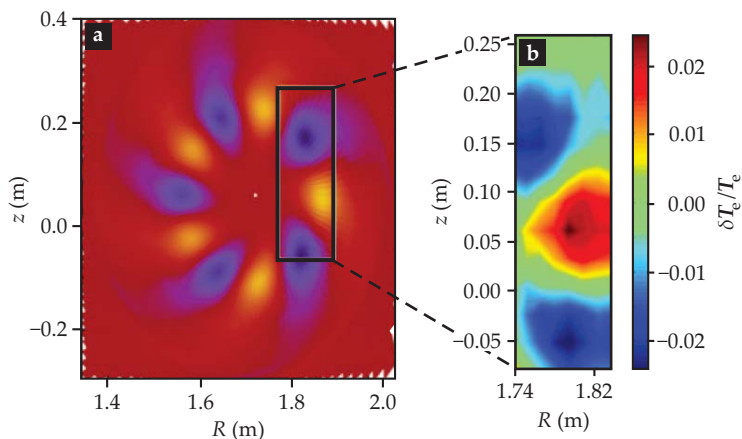


Figure 2. Simulation and measurement of an Alfvén eigenmode in an experiment conducted at the ASDEX Upgrade tokamak in Garching, Germany. (a) The simulated mode structure across the Rz -plane of the tokamak for a fixed toroidal position. The black rectangular outline marks the region over which electron temperature fluctuations δT_e are measured by an imaging system that monitors the subtly varying intensity of cyclotron radiation emitted by orbiting electrons. **(b)** The imaged mode structure demonstrates the accuracy with which simulation can reproduce an eigenmode. (Adapted from ref. 8.)

Ions interact with many different waves in a plasma. Alfvén waves—a ubiquitous type of plasma wave named after Hannes Alfvén, recipient of the 1970 Nobel Prize in Physics for his foundational work developing magnetohydrodynamics in plasmas—are transverse electromagnetic waves that can be found in the presence of a magnetic field, including those in the Sun’s corona, Earth’s ionosphere, and various astrophysical sources. Think of a magnetic field line as the electromagnetic equivalent of a guitar string. Plucking the string—that is, perturbing the field line in a transverse direction—causes an Alfvén wave to propagate along the field line in a way that resembles propagation on the string, with the magnetic field amplitude being analogous

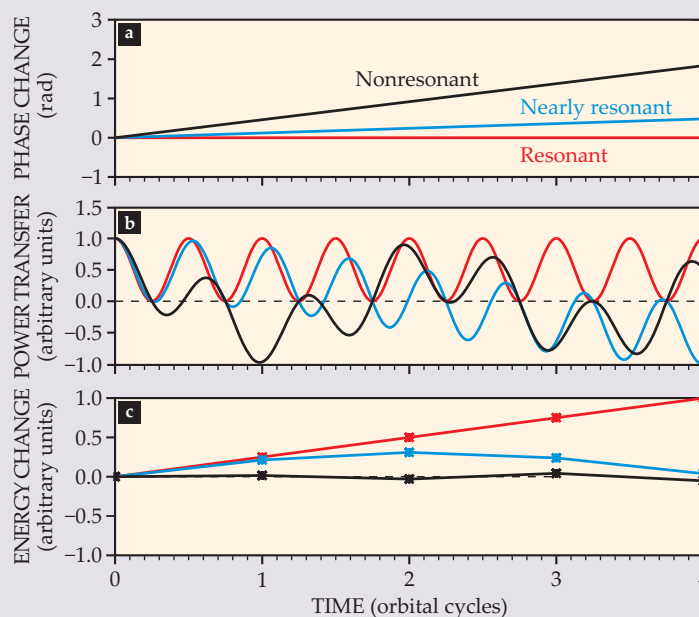
to the tension in the string and the plasma’s mass density being analogous to the string’s mass density.

A special class of Alfvén wave, the Alfvén eigenmode, exists in tokamaks thanks to the periodic boundary conditions imposed by the machine’s toroidal geometry and the magnetic field profile. That geometry causes the plasma’s index of refraction to vary periodically. As Lord Rayleigh recognized in 1887, a natural consequence of the periodic variation is Bragg reflection,² which prevents wave propagation in certain frequency bands. Just as variations in the refractive index give rise to forbidden transmission frequency bands in photonic crystals and optical fibers, they do so in plasmas as well.

Box 1. The wave-particle interaction

The phase relationship—resonant, nearly resonant, or nonresonant—between a charged particle and a plasma wave determines the power and energy transferred between them. **(a)** Whereas a resonant particle experiences the same phase of the wave as it orbits a tokamak, nonresonant particles do not. The

relationship is quantified by the phase change $\Delta\theta = (\omega_p - \omega_w)t$, where ω represents the frequency of the particle p or the wave w and t is the time normalized to the particle’s oscillation period. **(b)** The power $P(t)$ transferred from a wave to a particle resonant with it always



(in this simplified example) has the same sign and a magnitude given by $|\mathbf{v}_p \cdot \mathbf{E}_w|/(v_p E_w)$, where $\mathbf{v}_p$ is the particle velocity and $\mathbf{E}_w$ is the electric field of the wave. By contrast, the sign of the power transferred to either a nonresonant or nearly resonant particle continuously changes, though a nearly resonant particle has a greater likelihood of experiencing long periods at a fixed sign. **(c)** Unlike a resonant particle, which experiences a constantly increasing exchange of energy, a nonresonant particle exchanges very little energy with the wave, and a nearly resonant particle eventually returns its received energy back to the wave, as $\int_0^t (P(t)/t)dt$, where the power transfer is normalized to the orbital period of the particle. In this example, $\mathbf{v} \cdot \mathbf{E}_w \geq 0$ at all times for the resonant particle. More generally, resonant energy transfer occurs whenever $\oint (\mathbf{v} \cdot \mathbf{E}_w)dt \neq 0$ over an integer number of orbital cycles.

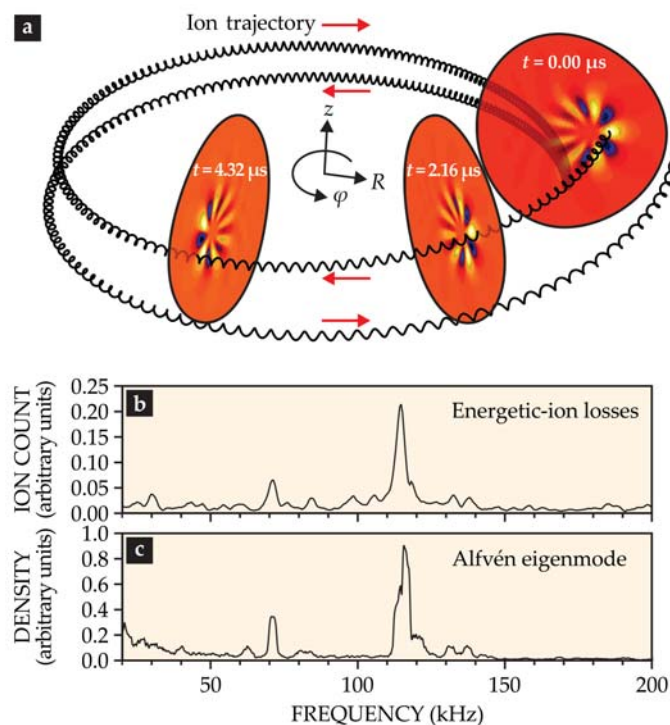


Figure 3. The spatiotemporal evolution of an ion interacting with an Alfvén eigenmode. **(a)** An ion and eigenmode are in resonance with each other when the ion orbit in the confinement vessel stays in phase with the eigenmode—that is, when it passes through the same lobe of the mode while traversing the plasma. The interaction leads to a large displacement of the ion from the orbit a nonresonant ion would follow. The mode structure the ion passes through at different times and locations is illustrated in three simulated Rz cross sections of the plasma. The ion begins its trapped orbit at time $t = 0$, reverses its toroidal direction after nearly a complete orbit, and eventually ends at a location marked x , where a sensor detects it outside the plasma. Power spectra from fluctuations of **(b)** energetic-ion losses and **(c)** plasma density measured during experiments at the DIII-D tokamak reveal peaks at a mode frequency of 115 kHz for orbits that stay in phase with the eigenmode.

phase of an Alfvén wave over many cycles around the tokamak—or more precisely, experiences the same phase at the end of an integral number of those cycles—energy is efficiently transferred from the ion to the wave or vice versa. Box 1 outlines the influence of the phase relationship on the energy transfer.

And Alfvén eigenmodes can exist as weakly damped standing waves within those bandgaps.¹

The eigenmodes are apparent as measurable fluctuations in magnetic field, electron density, electron temperature, and other quantities. The opening figure on page 34 illustrates a single eigenmode superimposed along with ion trajectories on a photograph of the confinement vessel in the DIII-D tokamak located in San Diego, California, whose center post has a diameter of 2 meters. Various diagnostic systems can detect the fluctuations. Figure 2, for instance, presents results from a particularly powerful temperature-imaging system installed on the ASDEX Upgrade tokamak in Garching, Germany. That imaging system measures the electrons' cyclotron emission, which is a temperature-dependent quantity, across a wide two-dimensional cross section of the plasma. Importantly, the 2D image can be rigorously compared with the theoretically expected wave structures. That makes it an exciting advance in plasma physics over prior work, in which fluctuation measurements came from either 1D profiles or single-point time-series measurements of electron temperature, density, or magnetic field.

Simulations are advancing to keep pace with the increasing fidelity of experimental results. The eigenmode simulation shown in figure 2 is based on experimentally determined plasma properties, such as its magnetic-field profile, which allows scientists to determine the diverse range of bandgaps in which eigenmodes may be excited. Increasingly, details of the ions' velocity distribution—and thus the waves' influence on their orbits—are included as inputs to the simulations.

A tale of loss

Alfvén eigenmodes are excited through wave-particle resonances. If an ion experiences the same

The study of wave-particle resonance in plasmas has an illustrious history.³ As first shown nearly 70 years ago by Lev Landau,⁴ an important quantity is the slope of the particle distribution function at the velocity corresponding to the phase velocity of the wave. If the slope, or velocity gradient, is negative, the number of particles slower than the wave's phase velocity exceeds the number of those that are faster. As a result, more particles gain energy from the wave than lose energy to it, and the wave damps. Conversely, if the slope is positive—with more particles having higher velocities relative to the wave's phase velocity—the wave grows.

Resonant wave-particle interactions alter the particle orbit. Advances in the detection and measurement of Alfvén eigenmodes are now allowing physicists to observe the transport of energetic ions outward from the center of a plasma. Figure 3 illustrates one example from an experiment conducted at DIII-D. An ion source launches particles—just one of which is pictured in the diagram—on banana-shaped orbits that traverse an Alfvén eigenmode before approaching an ion detector mounted near the outer wall of the vacuum vessel. Ions that stay in phase with the wave as they traverse the mode are effectively pushed into or away from the detector, which registers the loss. The result is that the ion flux measured by the detector oscillates at the frequency of the Alfvén eigenmode.

The loss of energetic ions is problematic in a fusion reactor. If fusion products reach the plasma-facing wall at nearly full energy, two undesirable effects occur: The products may damage the wall, and they can no longer contribute to heating the background plasma. Fortunately, the orbits that energetic ions will follow in future large reactors such as ITER—whose tokamak is, by far, the largest ever built—will be small relative to the devices' size.

Because the ions will have much farther to travel to escape the plasma, scientists don't anticipate losses through a single wave-particle interaction. Nonetheless, concerns remain about a single particle that is resonant with multiple waves and about multiple particles that are resonant with a single wave.

Resonance islands

Because of the complexity of its orbit, an energetic ion often experiences multiple resonances with a single Alfvén eigenmode, which makes significant transport possible from a single mode.¹ The situation is a familiar one in nonlinear dynamics.⁵ If the mode amplitude is small, each resonance acts independently to perturb the equilibrium orbit only slightly, as shown in box 2, and subsequent orbits show some periodic displacements that are manifest as islands, or gaps, in phase space. As the mode

amplitude increases, ion displacements become larger, the resonance-induced islands widen, and particles can begin to interact with more than one resonance.⁶ Eventually, if the amplitude is large enough, the islands overlap and the particle motion becomes chaotic, which allows ions to diffuse rapidly through the plasma.

A simulation of results from DIII-D outlined in box 2 illustrates the evolution to chaotic motion as the mode amplitude is varied from low to high. The rapid transport flattens the spatial profile of the energetic ions and redistributes them from the plasma center outward. Experimentally, the redistribution reduces the central fast-ion pressure or density, relative to that predicted in a plasma with no eigenmodes (see box 2, panel c). Even if ions are not ejected from the plasma, their loss from the central, core region leads to reduced temperatures and fusion yield.

Box 2. The onset of chaotic ion orbits

The greater the strength of an Alfvén eigenmode (AE), the greater the fluctuations in plasma magnetic and electric field experienced by an ion traversing the AE, resulting in a more perturbed ion orbit. Three separate simulations—with the AE's amplitude scaled by factors of 0.1, 10, and 30—bear out the extent of the perturbation. **(a)** Imag-

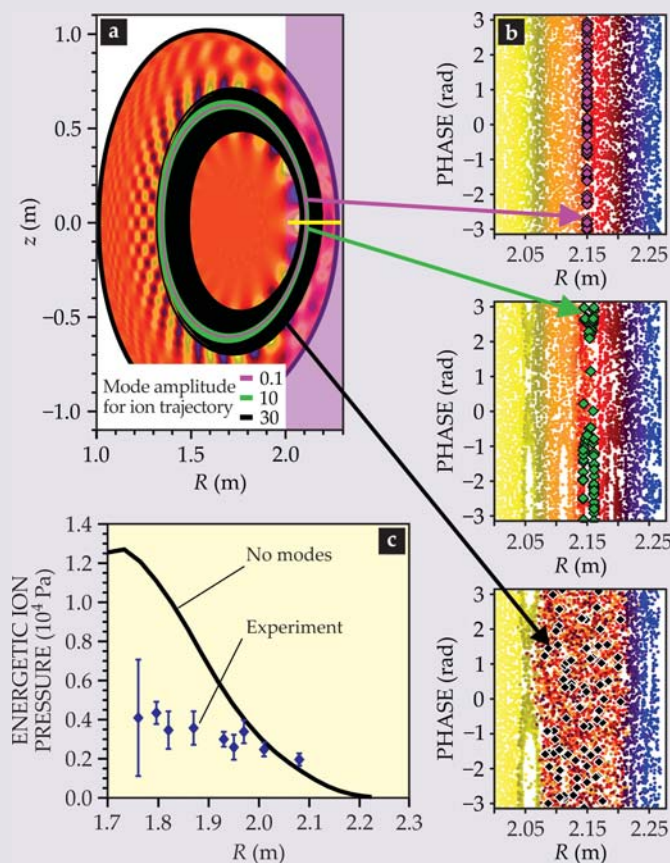
ine the ion circulating through the plasma that hosts an AE with a checkerboard mode structure. This plot maps the cross-sectional projection of ion positions in the Rz -plane after dozens of ion orbits. Colors signify the amplitude of the simulated mode the ion feels.

To appreciate the progressively greater spatial range of the orbit,

consider the ion's behavior in a narrow slice of phase space centered at the midplane (the yellow line at $z = 0$) for $R \geq 2$ m (purple region).

(b) Each map highlights the radial position of a single ion—started with an initial orbit at $R = 2.15$ m—and the phase of the AE mode it feels for each of the dozens of times the ion crosses the midplane. In the background of each map are points that represent dozens of other ions, whose colors indicate their initial radius.

For the 0.1-amplitude case (top), the highlighted ion (pink) orbits as if no AE were present, so the ion's radius at $z = 0$ remains unperturbed. When the amplitude is increased by two orders of magnitude (middle, which is close to experimental conditions), gaps known as phase-space islands open vertically and horizontally between data points (green) due to wave-ion resonances. The ion's trajectory extends across a range of radial positions. Finally, when the amplitude is increased to a value of 30 (bottom), the ion (black) experiences so much perturbation that it orbits at very disparate radii, and still wider gaps open. **(c)** The radial displacement of energetic ions at large amplitudes produces a flattened profile of the ion pressure, compared with the theoretical expectation for the pressure profile in a plasma without AEs. The data points shown here came from a DIII-D tokamak experiment. (The AE in this box is analyzed in ref. 9.)



Toward ITER

The effects of energetic ions are studied in today's tokamaks because a better understanding of their behavior is needed to predict and produce the operating parameters required for a reactor.⁷ The next step in magnetic confinement fusion is currently taking shape in the form of ITER, a tokamak designed to produce many times more fusion power than it consumes. Although the project has experienced a turbulent management history (see, for example, *PHYSICS TODAY*, April 2010, page 20, and May 2015, page 21), scientists are optimistic that ITER, whose components are being assembled in a region of southern France that is home to France's Atomic Energy Commission, will succeed as a burning-plasma tokamak.

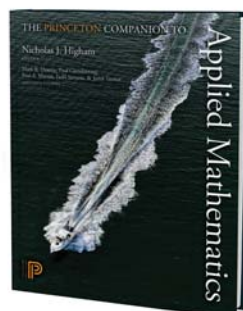
Once the reactor reaches fusion temperature, the 3.5-MeV alpha particles produced during the reactions are expected to maintain the plasma at that temperature. Further development of the wave-particle physics described in this article will arm researchers with the ability to predict, and then avoid or mitigate, scenarios at ITER in which alpha particles are transported out of their confined orbits in the plasma by Alfvén eigenmodes.

The challenge of ITER, one of the greatest engineering endeavors of all time, mirrors that of another incredible engineering feat, the Apollo mission. Whereas the Apollo mission sought to send humanity out into the greater universe, ITER seeks to bring that universe, in the form of a star, to Earth. The peak thrust of the Saturn V rocket was 3.3×10^7 N; the structure containing the central magnet of ITER is designed to withstand nearly twice that force, about 6.0×10^7 N. The volume of plasma its magnetic fields will confine is about 20 times that of the next-largest tokamak, the Joint European Torus. Expected to reach temperatures above 2.3×10^8 K, ITER's plasma will be an order of magnitude hotter than the center of the Sun (1.5×10^7 K). Existing tokamaks are also capable of surpassing solar temperatures, but ITER will be the first to maintain them for minutes or hours.

We are grateful to Alexis Briesemeister, Auna Moser, Jacob West, and Homer Wolfe for reviews of the manuscript. Research supported by the US Department of Energy, Office of Science, Office of Fusion Energy Sciences, using the DIII-D National Fusion Facility, a DOE Office of Science user facility, under Award DE-FC02-04ER54698. DIII-D data shown in this paper can be obtained by following the links at https://fusion.gat.com/global/D3D_DMP.

References

1. W. W. Heidbrink, *Phys. Plasmas* **15**, 055501 (2008).
2. J. W. Strutt, *Philos. Mag.* **24**, 145 (1887).
3. R. Z. Sagdeev, A. A. Galeev, *Nonlinear Plasma Theory*, T. M. O'Neil, D. L. Book, eds., W. A. Benjamin (1969).
4. L. Landau, *J. Phys. (Moscow)* **10**, 25 (1946).
5. A. J. Lichtenberg, M. A. Lieberman, *Regular and Stochastic Motion*, Springer (1983).
6. B. V. Chirikov, *At. Energy* **6**, 630 (1959).
7. S. E. Sharapov et al., *Nucl. Fusion* **53**, 104022 (2013).
8. I. G. J. Classen et al., *Plasma Phys. Control. Fusion* **53**, 124018 (2011).
9. M. A. Van Zeeland et al., *Phys. Plasmas* **18**, 056114 (2011). ■



Cloth \$99.50

The Princeton Companion to Applied Mathematics

*Edited by Nicholas J. Higham
Mark R. Dennis, Paul Glendinning,
Paul A. Martin, Fadil Santosa &
Jared Tanner, associate editors*

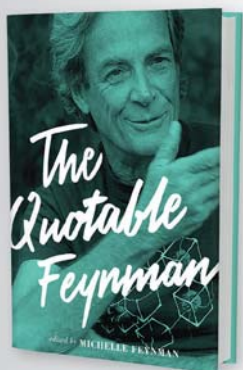
"Monumental and comprehensive, *The Princeton Companion to Applied Mathematics* does a breathtaking job of conveying the

richness, depth, and vitality of today's applied mathematics. . . . An instant classic."

—Steven Strogatz, Cornell University and author of *The Joy of x*

"The treasures [in the *Princeton Companion to Applied Mathematics*] go on and on. . . . PCAM shows us that applied mathematics is vast and it is confident. We see here a discipline engaged in every corner of the human enterprise, from cosmology to the spread of infectious diseases, from pattern formation to aircraft design, from financial portfolio optimization to the ranking of movie preferences."

—Lloyd N. Trefethen, *SIAM Review*



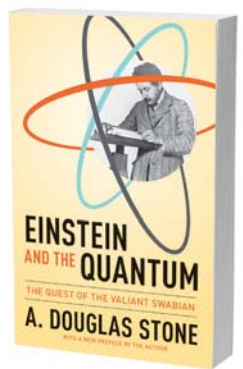
Cloth \$24.95

The Quotable Feynman

*Edited by Michelle Feynman
With a foreword by Brian Cox
and reflections on Richard Feynman
by Yo-Yo Ma*

"All evidence indicates that Richard Feynman was the most quotable physicist of all time. This collection is a vivid demonstration of his wit, wisdom, and unquenchable passion for finding things out."

—Sean Carroll, Caltech



Paper \$19.95

Einstein and the Quantum

*The Quest of the Valiant Swabian
A. Douglas Stone
With a new preface by the author*

"A wonderful reminder that Einstein's monumental role in the development of contemporary science is even more profound than history has allowed."

—*Kirkus Reviews*



PRINCETON UNIVERSITY PRESS

See our E-Books at
press.princeton.edu