QUANTUM black holes

Georgi Dvali

Living in a middle ground between the light fundamental entities of particle physics and the massive black holes of astrophysics are objects that trap light but don't obey classical rules.

BLACK CIRCLE BY KASIMIR MALEVICH

Black holes were first conceived by English polymath John Michell in a paper completed in May 1783. Michell thought that the speed of light corpuscles emitted from a star was affected by the star's gravitational attraction and, based on a then-reasonable estimate for the speed of light, he concluded, "If the semi-diameter of a sphere of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1 . . . all light emitted from such a body would be made to return towards it, by its own proper gravity."[1] Thirteen years later Pierre Simon Laplace offered an independent proof for the existence of "invisible bodies" that could trap light.

According to the modern perspective of general relativity, a sphere of mass M will become a light-trapping black hole when its radius shrinks to the so-called Schwarzschild radius $R = 2GM/c^2$; here G is Newton's gravitational constant and c is the speed of light, which I will henceforth set equal to one. The special radius is named in honor of Karl Schwarzschild, the first to find the black hole solution to the Einstein equations. Interestingly, the Newtonian and relativistic calculations give the same result for the radius corresponding to light trapping.

Classical (that is, in a nonquantum world) black holes exhibit many fascinating properties, some of which I will briefly review. But my main focus will be on the quantum world, and I will address several key questions: What are quantum mechanical black holes? How small and light can they be? Can they be produced in laboratory experiments? To begin, let's consider how force is conveyed in quantum mechanics.

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Gravitating gravitons

In classical physics, one way of introducing the notion of force is via the concept of a field. The field endows a test particle with a location-dependent potential energy—for example, a test mass m located a distance r from a source mass M has a gravitational potential energy $V(r)$ equal to $-GMm/r$. The test particle tends to move to regions of lower potential energy—in other words, it feels a force attracting it toward the source mass. To define the field itself, one abstracts out the test particle. Thus the gravitational field is given by $V/m = -GM/r$. Notice that in units for which the speed of light is one, the gravitational field is dimensionless.

Quantum mechanics paints a very different picture. A force does not arise from a field, but rather from the exchange of a messenger particle. (Particle physicists call such a mediating particle an intermediate boson.) For instance, the mediator of the electric force is the photon. Gravity is mediated by a massless spin-2 particle called the graviton. In quantum theory, even large objects such as Earth and the Moon attract each other due to the exchange of gravitons.

When a system can be described by a static gravitational field, the gravitons mediating the gravitational force are necessarily virtual. That is, the messenger gravitons cannot be detected as real particles. As a consequence, the processes of emission and absorption are not happening in well-defined moments of time. But in other contexts—for example, when gravitational waves are produced by a binary pulsar—real, detectable gravitons are created.

General relativity teaches us that any source of nonzero energy creates a gravitational field. In quantum theory, any such source emits and absorbs gravitons. That statement applies to all sources, including the energy-carrying gravitons themselves. To put it succinctly, gravitons gravitate.

The quantum phenomenon of gravitons emitting and absorbing other gravitons leads to countless possibilities. Two gravitons can, for example, exchange a virtual graviton; as a result of that exchange, the momenta and energies of the two original gravitons will change. Or two gravitons can temporarily merge into a single graviton that then decays back into a pair of gravitons. Such scattering processes are nicely visualized by Feynman diagrams like the ones shown in figures 1 and 2. Among the possibilities are some with incredibly complicated Feynman diagrams representing hard-to-calculate physics. Fortunately, in many situations of interest, the more complicated processes are less probable and thus can be ignored.

Strong interactions, small black holes

Classical gravity is a universal force whose strength is proportional to the energy of the source. Likewise, in quantum mechanics higher-energy gravitons interact more strongly. In quantum theory, the strength of the interaction between elementary particles (not just gravitons) can be parameterized by a dimensionless number, usually denoted by α, that gives the probability amplitude for the particles to change their energies and momenta by scattering off each other. The actual scattering probability is given by α^2, so strong interactions mean a high likelihood of scattering. For electromagnetic interactions, the probability amplitude is the famous fine-structure constant, approximately equal to 1/137.

A rigorous calculation of the interaction strength α_g between gravitons must use quantum mechanics. After all, the gravitons are massless and they can change their number by merging and multiplying. That said, imagining a Newtonian interaction between two gravitons will take you a long way toward estimating α_g. The only place where you need quantum mechanics is to relate the wavelength L of a graviton to its energy E through the well-known relation $E = \hbar/L$, where $\hbar$ denotes Planck's constant.

The fictional Newtonian interaction would be described by the Newtonian gravitational field $-GM/r$, whose magnitude will serve to estimate α_g. In evaluating the field, one should replace the source mass M with $\hbar/L$ and set the distance r equal to L, the minimum distance within which a graviton of wavelength L can be localized. The result is $\alpha_g = \hbar G/L^2$. To make the dimensionless nature of α_g manifest, let $\hbar G \equiv L_P^2$ so that $\alpha_g = L_P^2/L^2$. The scale L_P is called the Planck length, and it is the most fundamental scale of quantum gravity and probably all of nature. Its numerical value is 10^{-35} m. The corresponding mass or energy scale $M_P \equiv \hbar/L_P$ is called the Planck mass. Its numerical value is approximately 10^{-8} kg, the mass of a biological cell.

Because the Planck length is so tiny, the interaction of macroscopic-scale gravitons is incredibly small. To give an example, two 1-cm-wavelength gravitons will scatter off each other once every 10^{114} years! The corresponding coupling α_g is on the order of 10^{-66}. With such a weak coupling, all the higher-order processes, such as the one shown in figure 2, are enormously suppressed and can be ignored. As the wavelength of the graviton decreases, the perturbation approach that generates Feynman diagrams becomes less and less reliable, and it simply breaks down when the wavelength is of order L_P.

At the breakdown point, the interaction of gravitons becomes strong enough that colliding gravitons can, with a probability close to one, capture each other and form a bound state. Those microscopic states are called quantum micro black holes, or quantum black holes for short. Their properties lie between the realms of quantum elementary particles and macroscopic black holes, but they belong to neither of the two worlds. On one hand, they are strongly gravitating objects, but on the other hand, quantum fluctuations are so strong that one can't describe them in terms of classical geometry.

To understand the middle ground where quantum black holes live, let's approach it from the realm of elementary particles and from the realm of classical black holes. We start the journey from the world of the elementary particles. On the way, we'll learn why L_P and M_P describe the smallest, lightest possible quantum black holes.

From quantum particles to black holes

It might seem that a massive elementary particle such as the electron must necessarily be a black hole. After all, if the particle is elementary, it seems logical to suppose that the particle is point-like and has zero size. But if it is massive, it has a nonzero Schwarzschild radius. The conclusion is that the particle is localized within its Schwarzschild radius and is thus a black hole.

The problem with the above reasoning is that

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in quantum theory, elementary means "having no substructure." But lack of substructure absolutely does not mean zero size. Indeed, in quantum mechanics, any form of energy, mass included, has an associated length scale, and any finite length has an associated energy. Specifically, a particle with energy m has a strict lower bound on the length within which it can be localized. That length, called the Compton wavelength, is given by $L_C = \hbar/m$. Roughly speaking, L_C is the length at which the energy of quantum fluctuations becomes as important as the particle mass.

To get a feel for why the Compton wavelength is a limiting length, imagine you want to resolve an electron under a microscope with a resolution scale r. According to the Heisenberg uncertainty principle, the minimum momentum you have to transfer to the electron is $\hbar/r$. Thus measuring the electron's location more precisely than L_C would impart to the electron an energy comparable to or larger than its rest mass.

The Schwarzschild radius R of an electron is much smaller than L_C, and therefore any attempt to localize the electron within R would require such a distortion of the system that it could hardly be called an electron at all. To perform such a measurement would require a transfer of something like 10^{40} GeV (equivalently 10^{13} kg) of momentum to the electron. The result would be a particle with an energy something like the mass of Halley's comet. The lesson from the microscope thought experiments is that it's impossible to probe the Schwarzschild radius of the electron by means of any experiment; the electron is not a black hole.

For an elementary particle to be a black hole, it must be heavy enough that L_C and R meet. Not surprisingly, that happens just about at M_P, essentially the smallest mass for which R starts to have a physical meaning. However, since L_C and L_P are comparable, quantum gravitational effects are important for those lightest of black holes. For example, one cannot talk about geometric properties of spacetime near the surface of a quantum black hole. And it would be a mistake to apply to those objects intuitions derived from large black holes.

Information in classical black holes

In the previous section, I argued for the inevitability of quantum black holes as a consequence of increasing the mass of an elementary particle. I now develop the idea of the quantum black hole starting from the realm of macroscopic black holes—that is, black holes with $R \gg L_P$. Such black holes are well described by classical physics, but just how well they are described is a subtle issue.

To begin to address the question, consider the classical world in which $\hbar = 0$ and thus $L_P = 0$ as well. Rigorous proofs establish that classical black holes are fully characterized by their mass, angular momentum, and electric charge. Physicists summarize that result by saying, "a black hole has no hair," meaning that it is featureless. And because black holes lack features, they cannot encode information, which is necessarily carried by elements that can be arranged and modified for that encoding.

A classical black hole can be compared to Kasimir Malevich's famous painting *Black Square*, shown in figure 3. As opposed to, for example, the artist's earlier self-portrait (figure 4), the black square carries almost no features. Only one artist can get famous by creating a painting such as *Black Square*, and producing new black squares won't add anything to the original breakthrough because the square provides no features that can be rearranged. In contrast, creating a human portrait offers unlimited possibilities.

The no-hair property of a classical black hole becomes puzzling when classical physics is regarded as the limit of quantum mechanics in which $\hbar \to 0$. The reason has to do with information storage. During a phone conversation, for example, you and your friend send messages that are encoded in electromagnetic waves. Quantum mechanically, those waves represent bunches of photons carrying a rich variety of features that can be decoded. How compact can the message be? Quantum mechanics says there is no free lunch: Each photon of wavelength L costs an energy on the order of $\hbar/L$. Thus increasing the quantity of information with more photons or making the message more compact costs energy.

In the classical limit, however, the energy cost per photon goes to zero. Hence, in a classical world, an arbitrarily short electromagnetic pulse of arbitrarily low energy can encode an arbitrarily large message. In other words, classical fields have an infinite capacity for no-cost information storage.

Infinite information capacity is also what one would expect from classical black holes. Yet the no-hair property says that black holes are featureless. That's not an inconsistency, but it is a strange dichotomy—one that was highlighted by remark-

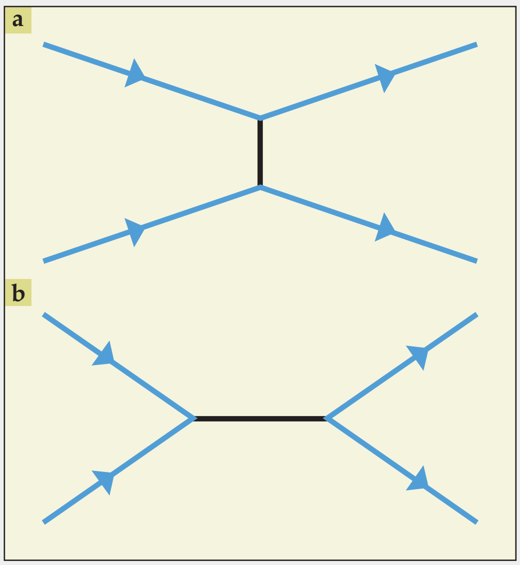

Figure 1. Gravitons, the conveyors of the gravitational force, interact by exchanging virtual gravitons—that is, gravitons that cannot actually be measured in the lab. The various interactions can be depicted with Feynman diagrams such as the two shown here; virtual gravitons are represented by black lines. **(a)** Two gravitons scatter off one another via virtual graviton exchange. **(b)** Two gravitons merge into a single virtual graviton that then decays into two gravitons.

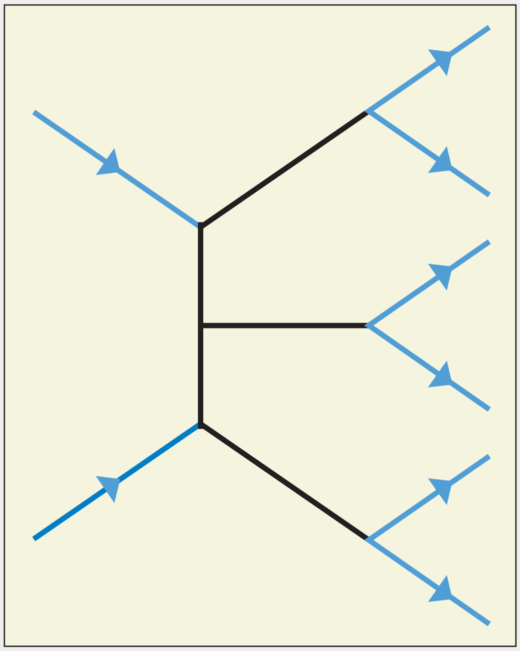

Figure 2. Processes with many interactions are called higher-order processes. In the Feynman diagram shown here, two incoming gravitons produce six outgoing ones. Each interaction vertex contributes to the probability for the process. At low energies, that contribution is small and higher-order processes are suppressed. But at high energies, such processes are significant.

able discoveries by Jacob Bekenstein and Stephen Hawking.

Infinite information, zero retrieval

In his 1973 work,[2] Bekenstein showed that a black hole has an entropy S proportional to $(R/L_P)^2$. Since entropy is the measure of information-storage capacity, Bekenstein also showed that a black hole stores information; in fact, he showed that it stores the maximal possible information for a given energy or, equivalently, size. But as $L_P \to 0$, Bekenstein's formula says that the information storage capacity becomes infinite. How can that be reconciled with the no-hair property of classical black holes?

The resolution of the puzzle is that the information in a classical black hole cannot be accessed. Bekenstein's formula says only that the classical black hole stores infinite information; it does not say how easy it is to retrieve or decode that info. In fact, in the classical limit, the information retrieval time becomes infinite.

Thus in classical theory, a black hole is indeed an infinite reservoir of information, but it is also eternal: No amount of information can be retrieved in a finite time. That is why, from the point of view of an outside observer, the black hole is featureless.

Hawking's discovery followed from a calculation in a regime midway between the classical and quantum worlds.[3] In the "semi-classical" arena where he worked, quantum particles live in a black hole geometry that is treated as an external classical field—in other words, particles are quantized but spacetime geometry is not. You can think of working in the semiclassical world as analogous to ignoring the influence of ballet dancers on the stage on which they perform.

Semiclassical calculations are exactly doable and mathematically rigorous only if one takes an infinitely massive black hole and simultaneously sets Newton's constant equal to zero in such a way that R remains finite. Working in that limit, Hawking showed that black holes radiate. Moreover, the Hawking radiation is ideally thermal, with a temperature T proportional to $\hbar/R$. The black hole radiates, on average, on the order of one quantum per time R, and the spectrum of emitted quanta is as if they were coming out of an ideal oven set at T. But since the black hole is infinitely massive, its mass never decreases, even though it radiates at a constant rate. And because the radiation spectrum is ideally thermal, an observer detecting the black hole radiation will be unable to decode any information. The conclusion is that information remains eternally stored inside a semiclassical black hole.

Our understanding of black holes in both classical and semiclassical worlds is fully consistent. In both cases black holes are eternal and featureless. In both descriptions, $L_P = 0$, so black holes never reach the quantum threshold, no matter how small they may be. In the classical and semiclassical worlds, particles and black holes are strictly segregated; no entity can be regarded as a black hole particle. Indeed, in classical theory, $\hbar = 0$, so elementary particles are massless and carry zero energy, in contrast to black holes, which necessarily have nonzero mass. In semiclassical theory, elementary particles can have finite mass, but black holes are infinitely massive; again, the two notions never meet.

The real world of finite constants

The merging of particles and black holes can only happen in a fully quantum world, such as ours, in which both $\hbar$ and G are finite, and therefore so is L_P. In the real world, finite-sized black holes have finite mass. The mass decreases as the black hole radiates, and the black hole temperature increases. The changing temperature means not only that the spectrum of radiation coming from a real black hole is not exactly thermal but also that features of the black hole are evolving. The information released by a black hole must somehow be encoded in the departures from the exact thermality.

Real black holes evidently differ in profound ways from their semiclassical idealizations, but one can still address the question of how well the semiclassical approximation does for a finite-mass black hole. Great care is needed, because the half-life of a black hole is on the order of $N = (R/L_P)^2$ emission times, and N is an enormous number. To get a feel for just how large it is, note that the half-life of an Earth-mass black hole is roughly 10^{38} times the age of the universe.

Quantum corrections are derived from specific microscopic quantum models of a black hole. At the moment, no commonly accepted model exists, though theorists are exploring a few promising avenues. For example, in certain classes of string theory, black hole microstructure can be understood in terms of the number of string states.[4] A more recent idea,[5] applicable in different theoretical regimes, is that a black hole is a composite of something like N gravitons with wavelengths of roughly R.

I won't discuss specific models further, except to relate an important point about any model positing that a black hole has a corpuscular substructure whose constituents have a wavelength of order R: All such models imply that relative corrections to ideal thermal behavior scale with $1/N$. The individual corrections are minuscule. But the corrections, accumulated over the life span of a black hole, completely change the semiclassical picture. Indeed, the size of the accumulated corrections illustrates just how important quantum mechanics is for black holes. For most macroscopic objects, quantum corrections are far less important; they are totally negligible, for example, for Earth's gravitational field.

I now come to the punch line. The smaller and lighter the black hole is, the smaller N is and the more important relative quantum corrections are. If the radius is sufficiently small, quantum corrections are so great that the tiny object is a new kind of entity: a quantum black hole, now discovered via an approach from the realm of macroscopic black holes. When the black hole has a radius of L_P (and thus is basically at the Planck mass), the quantum corrections become order one. For radii smaller than L_P, black holes no longer exist; we are in the realm of elementary particles.

Thus well-established physics implies the existence of entities that serve as a transitional point between the world of particles and the world of black holes. They share properties of both, but are quite different from either. Neither the rules of weakly interacting elementary particles nor semiclassical black hole physics accurately predicts their physical properties.

Physicists can only guess at the qualitative behavior of quantum black holes by extrapolating known laws of physics from above and below. The reason quantitative calculations are so difficult is that for quantum black holes, the interaction-strength parameter α_g is of order one, so it's not OK to apply conventional calculational methods based on a perturbation series in the interaction strength. For particles just a couple of orders of magnitude heavier than quantum black holes, one could apply semiclassical black hole methods, and for particles similarly lighter, perturbative methods would be fine. Quantum black holes, alas, sit right at the point where both of those approaches break down: We'll need to develop new and powerful theoretical methods if we are to gain a satisfactory understanding of those entities.

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If decades of experience with the nuclear interaction are any guide, even the most powerful theoretical methods won't be enough without help from experiments. The nuclear interaction earns its moniker "strong"—its α becomes of order one—at energies of around 1 GeV. In crude terms, that energy, called the quantum chromodynamics scale, plays the same role for the nuclear force that the Planck mass plays for gravity; in particular, it corresponds to the transition point between the low-energy theory with mesons as fundamental entities and the high-energy theory of quarks and gluons. Physicists have excellent tools for the regimes away from the transition: chiral perturbation theory for low energies and quantum chromodynamics for high energies. But near the transition, those techniques are essentially powerless. As a result, despite a wealth of experimental data, our theoretical understanding of the strongly coupled nuclear interaction is still far from being satisfactory.

Experimental prospects

How realistic is the possibility of experimentally studying quantum black holes—or even observing them? If physicists get incredibly lucky, we may detect the last stages of evaporating black holes that were produced early in the history of the universe and that have by now diminished to Planck size. Such black holes could wander into a particle detector—for example, one searching for decays of dark-matter particles—and explode there in a final burst. The probability of such an event depends strongly on features of specific models and on unknown aspects of the universe's early history.

Direct production of a quantum black hole requires particle collisions at energies greater than the

Figure 3. ***Black Square***, a 1915 painting by Russian artist Kasimir Malevich, is analogous to a classical black hole in that it encodes little information.

Planck mass. The only known possibility for realizing such high energies is in the collisions of cosmic rays. However, even for cosmic rays, collisions with energies above M_P are extremely rare. To date, no such collisions have ever been detected in Earth's atmosphere.

The energies achieved by humankind's most impressive particle accelerators are well below M_P, so the prospects for producing a quantum black hole at an accelerator may seem hopeless. Theorists recognized some time ago, however, that the experimental prospects for manufacturing quantum black holes may not be so bleak.[6] The key insight is that with certain types of new physics—in particular, in theories with large extra dimensions—the fundamental Planck length can be much longer than the L_P dictated by the usual four-dimensional theory.

To see why the Planck length can change with extra dimensions, suppose that the universe includes an additional n compact space dimensions of radius l. At distances much shorter than l, the Newtonian gravitational field would be $V(r) = -G^{(n)}M/r^{n+1}$, consistent with Gauss's law in $3+n$ dimensions. Here, $G^{(n)}$ is the fundamental $(3+n)$-dimensional Newton's constant, which defines the fundamental Planck length L_* via $L_*^{2+n} = \hbar G^{(n)}$. Matching the 3D and $(3+n)$-D gravitational fields at l fixes the relation between the high-dimensional and conventional Planck lengths: $L_*^{2+n} = L_P^2 l^n$ (see also the article I wrote with Nima Arkani-Hamed and Savas Dimopoulos, PHYSICS TODAY, February 2002, page 35).

Thus larger extra dimensions mean a larger fundamental Planck length. Experimental bounds on L_* come from particle physics, astrophysics, and cosmology[7] and from a number of astrophysical[7] and tabletop[8] experiments specifically searching for deviations from Newtonian and Einsteinian gravity. Altogether, those results imply that the largest possible value of L_* is something like 10^{-19} m, just about what can be probed at CERN's Large Hadron Collider (LHC).

If the LHC observes quantum black holes, the lightest of them will look nothing like semiclassical black holes. In particular, they won't lose their mass slowly, and they won't decay into a large number of low-energy particles. Instead, they will decay extremely quickly into just a few highly energetic quanta. The decay products, of course, will behave according to their own quantum properties—for example, quarks and gluons will produce jets. So two-jet events would be the right place to look for evidence of the lightest quantum black holes.

With our current calculational abilities, we cannot predict the properties of quantum black holes with any certainty. For example, precise decay channels, branching ratios, black hole lifetimes, and the minimum mass for a quantum black hole can be predicted only up to coefficients of order one. On a more positive note, we can predict a surprising amount, given that we are dealing with quantum gravity.

An observation of quantum black holes would be one of the most exciting discoveries ever. However, no observation at a single energy can distinguish between a new strongly interacting particle and a quantum black hole. To tell the difference, experimenters will need to probe an interval of energies above the threshold for the quantum black hole (or new particle) production. Then, if the newly created entities are indeed black holes, as the energy increases they will be more likely to be produced, will live longer, and will decay into a greater number of products. Moreover, with increasing energy, black hole decay is increasingly democratic—that is, the probabilities for decay into the various possible product species tend to become equal.

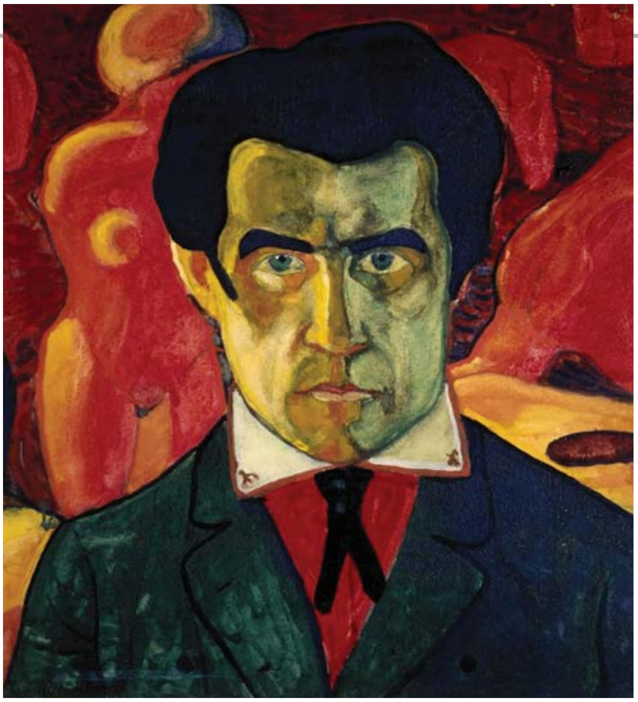

Figure 4. A self-portrait such as this circa 1910 work by Kasimir Malevich has much more information than a classical black hole.

Such behavior with increasing energy would nearly be a smoking gun for black hole discovery, though it is possible that some other entity could mimic the experimental signature of a quantum black hole—at least for a small energy range. To establish the observation of a quantum black hole will require an extremely careful analysis of a great deal of data. So we theorists studying quantum black holes can be counted among those physicists eagerly waiting to see what the LHC delivers in its new run.

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