requires that our Nows must coincide at each of two consecutive meetings. That is just what we find whenever any of us move apart and then come back together. But at the slow relative speeds at which we move, the possibly complicating effect of relativistic time dilation on the advances of our individual Nows is utterly negligible compared with the psychological width (many milliseconds) of each of our private Now experiences. Can our Nows coincide when we come back together no matter how rapidly we move back and forth?

The twin "paradox"—the relativistic requirement that personal time keep pace with proper time—assures us that according to physics, the Nows of the traveling and the stay-at-home twin will indeed coincide at their reunion if they have coincided at their separation, even when the departure and return involve speeds comparable to the speed of light. Far from having nothing to say about the Now, physics actually describes it in a way that makes psychological sense, even in a world of many people, all moving about at relativistic speeds.

Erwin Schrödinger had it almost exactly right when he wrote to Arnold Sommerfeld about an "emergency decree" that quantum mechanics "deal only with the object-subject relation. Although this holds, after all, for any description of nature, it evidently holds in a much more radical and far-reaching sense in quantum mechanics."3 My only reservation is that although quantum mechanics has indeed forced uswell, at this point only some of usto recognize that physics is about the object-subject relation, this holds in just as radical and far-reaching a sense in classical physics too.

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Letters

Early chaos theory

he article "Chaos at fifty" by Adilson Motter and David Campbell (PHYSICS TODAY, May 2013, page 27) pays a well-deserved homage to Edward Lorenz for his contribution to chaos theory and meteorology. However, I take exception to the statement at the beginning of the article that "in 1963 an MIT meteorologist revealed deterministic predictability to be an illusion and gave birth to a field that still thrives." On the contrary, the exponential growth of errors in some deterministic systems and the practical consequences for predictability have been appreciated by scientists for more than a century. I am particularly fond of what Henri Poincaré says in "Le hasard," chapter 4 of his book Science et Méthode (Ernest Flammarion, 1908; my translation):

A small cause, that escapes us, determines a considerable effect that we cannot ignore, and we then say that this effect is due to chance. If we knew exactly the laws of nature and the situation of the universe at the initial instant, we could predict exactly the situation of this same universe at a later instant.

But Poincaré remarks that we know the initial situation only approximately, and that it may happen that small differences in initial conditions generate large differences later, so that prediction becomes impossible. Poincaré offers meteorology as an example:

Why do meteorologists have such difficulty in predicting the weather with any certainty?... [They see] that a cyclone will appear, but they are unable to say where; a tenth of a degree added or subtracted at some arbitrary place, the cyclone appears here and not there, and causes destructions in countries which it would have spared. If one had been aware of this tenth of a degree, one could have known it in advance, but the observations were not spaced closely enough, and were not precise enough, and this is why everything seems due to chance. Here again we find the same contrast between a tiny cause, that the observer cannot measure, and considerable effects, which may be appalling disasters.

So, Poincaré knew about the butterfly effect 50 years before Lorenz. Lorenz's contribution is not so much the



knowledge of the Lorenz attractor (a somewhat abstract model for atmospheric convection) as it is the demonstration that with computers, meteorologists can progressively improve their modeling of the dynamics of the atmosphere.

Instead of a serendipitous discovery giving birth to a new field of science out of the blue, I see the blooming of chaos theory as a consequence of the progress in mathematical, experimental, and computational techniques, which over several decades have given rise to a formidable self-organized multidisciplinary effort.

Using the mathematical theory of dynamical systems developed after Poincaré and Jacques Hadamard, and based on their work, Floris Takens and I, for instance, showed that Landau's quasi-periodic theory of turbulence was unstable and led to hyperbolic dynamics and "strange attractors." That was an early contribution to what was not yet called chaos theory.

A fundamental problem Poincaré explicitly left open was that of the stability of the solar system. The problem was not solved by the discovery of homoclinic tangles, because they may involve only sets of measure zero. The Kolmogorov-Arnold-Moser theory gave the hope that one could prove the stability of the solar system. But delicate computational work by Jack Wisdom and Jacques Laskar in the 1980s finally proved instability and thus solved the important classical problems of stability² (or long-term predictability). Laskar's contributions in particular are chaos theory at its best:3 They provide new views on the history of climates and other important geological questions.

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■ The feature article "Chaos at Fifty" by Adilson Motter and David Campbell highlights Edward Lorenz's discovery¹ in 1963, which, the authors say, "gave birth to a field that still thrives." Without a doubt, Lorenz's contribution was outstanding, but the real history of the scientific research of chaos starts with Boris Chirikov a few years earlier. Work done by Chirikov in 1959 established a resonance overlap criterion for the onset

of chaotic motion of plasma confined in a mirror magnetic trap.² The criterion was later shown to also apply to a number of other deterministic Hamiltonian systems, and it is now known as the Chirikov criterion.

Over the ensuing decades, Chirikov made a great many seminal contributions to what became known as the field of chaos.³ (See also his obituary in PHYSICS TODAY, June 2008, page 67.) It would be a shame if readers of the magazine forgot about this pioneer of chaos.

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■ Motter and Campbell reply: We thank Professors David Ruelle and Dima Shepelyansky for their clarifying comments, which expand on some important aspects of the rich history of chaos that the stringent length and number of reference limits of PHYSICS TODAY did not allow us to include in our article. We chose to focus our article on the contributions of Edward Lorenz and the role of computation in the development of the modern theory of chaos.

We are well aware of, and in our article we explicitly quoted from, Henri Poincaré's insights into "sensitive dependence on initial conditions." Indeed, almost precisely the same paragraphs that Ruelle quotes in his letter appeared in an article by one of us published more than 25 years ago. 1 Had space permitted, we would also have included quotes from James Maxwell,2,3 who, decades before Poincaré, clearly recognized that sensitive dependence on initial conditions implies loss of predictability. As noted by Richard Kautz,4 "it is perhaps fairest to say that chaos was discovered many times, although most discoverers did not understand their discovery as fully as Lorenz."

Our focus on Lorenz's work was also motivated by its central role in bringing the quantitative aspects of chaos to the awareness of the scientific community. This is reflected in the paper that named the field,⁵ in which the first four references were to publications by Lorenz.

We are pleased that Ruelle's final comments on the importance of Jack

Wisdom and Jacques Laskar's "delicate computational work" reinforce our point about the essential role played by computation—both the numerical results and the visualizations—in the full development of chaos theory and its applications. That point is discussed in detail in reference 12 of our PHYSICS TODAY article.

Shepelyansky's remarks about the significance of the work of his mentor and close collaborator Boris Chirikov in developing an approximate theoretical approach—the Chirikov overlap criterion—to the study of chaos in Hamiltonian systems are pertinent. We chose to focus our brief discussion of Hamiltonian chaos on the more general and prior Kolmogorov-Arnold-Moser theory,6 mentioned in Ruelle's letter. Interested readers are encouraged to consult Chirikov's papers. As noted at the end of our article, "There have been many other important developments in chaos that could not be discussed in this brief, nontechnical article."

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