knowledge of the Lorenz attractor (a somewhat abstract model for atmospheric convection) as it is the demonstration that with computers, meteorologists can progressively improve their modeling of the dynamics of the atmosphere.

Instead of a serendipitous discovery giving birth to a new field of science out of the blue, I see the blooming of chaos theory as a consequence of the progress in mathematical, experimental, and computational techniques, which over several decades have given rise to a formidable self-organized multidisciplinary effort.

Using the mathematical theory of dynamical systems developed after Poincaré and Jacques Hadamard, and based on their work, Floris Takens and I, for instance, showed that Landau's quasi-periodic theory of turbulence was unstable and led to hyperbolic dynamics and "strange attractors."[1] That was an early contribution to what was not yet called chaos theory.

A fundamental problem Poincaré explicitly left open was that of the stability of the solar system. The problem was not solved by the discovery of homoclinic tangles, because they may involve only sets of measure zero. The Kolmogorov-Arnold-Moser theory gave the hope that one could prove the stability of the solar system. But delicate computational work by Jack Wisdom and Jacques Laskar in the 1980s finally proved instability and thus solved the important classical problems of stability[2] (or long-term predictability). Laskar's contributions in particular are chaos theory at its best:[3] They provide new views on the history of climates and other important geological questions.

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■ **The feature article** "Chaos at Fifty" by Adilson Motter and David Campbell highlights Edward Lorenz's discovery[1] in 1963, which, the authors say, "gave birth to a field that still thrives." Without a doubt, Lorenz's contribution was outstanding, but the real history of the scientific research of chaos starts with Boris Chirikov a few years earlier. Work done by Chirikov in 1959 established a resonance overlap criterion for the onset of chaotic motion of plasma confined in a mirror magnetic trap.[2] The criterion was later shown to also apply to a number of other deterministic Hamiltonian systems, and it is now known as the Chirikov criterion.

Over the ensuing decades, Chirikov made a great many seminal contributions to what became known as the field of chaos.[3] (See also his obituary in PHYSICS TODAY, June 2008, page 67.) It would be a shame if readers of the magazine forgot about this pioneer of chaos.

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■ **Motter and Campbell reply:** We thank Professors David Ruelle and Dima Shepelyansky for their clarifying comments, which expand on some important aspects of the rich history of chaos that the stringent length and number of reference limits of PHYSICS TODAY did not allow us to include in our article. We chose to focus our article on the contributions of Edward Lorenz and the role of computation in the development of the modern theory of chaos.

We are well aware of, and in our article we explicitly quoted from, Henri Poincaré's insights into "sensitive dependence on initial conditions." Indeed, almost precisely the same paragraphs that Ruelle quotes in his letter appeared in an article by one of us published more than 25 years ago.[1] Had space permitted, we would also have included quotes from James Maxwell,[2,3] who, decades before Poincaré, clearly recognized that sensitive dependence on initial conditions implies loss of predictability. As noted by Richard Kautz,[4] "it is perhaps fairest to say that chaos was discovered many times, although most discoverers did not understand their discovery as fully as Lorenz."

Our focus on Lorenz's work was also motivated by its central role in bringing the quantitative aspects of chaos to the awareness of the scientific community. This is reflected in the paper that named the field,[5] in which the first four references were to publications by Lorenz.

We are pleased that Ruelle's final comments on the importance of Jack Wisdom and Jacques Laskar's "delicate computational work" reinforce our point about the essential role played by computation—both the numerical results and the visualizations—in the full development of chaos theory and its applications. That point is discussed in detail in reference 12 of our PHYSICS TODAY article.

Shepelyansky's remarks about the significance of the work of his mentor and close collaborator Boris Chirikov in developing an approximate theoretical approach—the Chirikov overlap criterion—to the study of chaos in Hamiltonian systems are pertinent. We chose to focus our brief discussion of Hamiltonian chaos on the more general and prior Kolmogorov-Arnold-Moser theory,[6] mentioned in Ruelle's letter. Interested readers are encouraged to consult Chirikov's papers. As noted at the end of our article, "There have been many other important developments in chaos that could not be discussed in this brief, nontechnical article."

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