quick study

A new twist on the Doppler shift

Miles Padgett

Light with orbital angular momentum will undergo a frequency shift when it bounces off a rotating object.

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The well-known Doppler shift is typified by the audible rise and fall in the pitch of a siren as a fire engine races past. That Doppler shift applies to light too, and it is perhaps best exemplified by the redshift of the spectral emission lines from distant stars. Indeed, it was in the context of the color of light from binary stars that Christian Doppler first proposed the phenomenon in 1842; three years later Christoph Buys Ballot verified the effect for sound waves.

For light, the angular-frequency shift $\Delta\omega = \omega_0 v/c$, where ω_0 is the unshifted frequency and v, the relative speed between source and observer, is presumed to be much less than the speed of light c. If the velocity is not parallel to the line of sight between source and observer, then the frequency shift picks up a factor of $\cos\alpha$, where α is the angle between the line of sight and the velocity vector. One then speaks of a reduced Doppler shift $\Delta\omega = (\omega_0 \cos\alpha)\, v/c$.

A police radar gun applies the Doppler effect to measure longitudinal velocity and catch those of us who drive too fast. But the Doppler effect is encountered in other applications too. In laser surface velocimetry, also called Doppler velocimetry, the idea is to measure the speed of transverse fluid flow—for example, of blood. In that technique, summarized in figure 1, the scattered light is measured in a different direction from that by which the fluid is illuminated. Typically, two laser beams illuminate the fluid at angles $\pm\alpha$ with respect to the surface normal. When light scattered from particles in the fluid is detected in the direction of the surface normal, it is subject to a reduced Doppler shift. The Doppler shifts from the two illuminating beams are equal in magnitude but opposite in sign. As a result, the components in the reflected beam interfere to give an intensity modulation in the scattered light with an angular frequency of $(4\pi \sin\alpha)\, v/\lambda$, where λ is the wavelength of the light. From α, λ, and the measured modulation, one can readily obtain the fluid speed.

Doppler shifts can also arise from rotation, even when source and receiver are not moving farther apart or closer together. As demonstrated in the late 1970s by Bruce Garetz, when a circularly polarized light beam is rotated about its propagation axis with rotational velocity Ω, its frequency shifts by $\Delta\omega = \sigma\Omega$. Here $\sigma = \pm 1$ for right- and left-handed circularly polarized light respectively; the spin angular momentum per photon in such light is $\sigma\hbar$. The frequency shift can be understood through a simple physical picture. Think of the second hand on a stopwatch, which is analogous to the electric field vector in a circularly polarized light beam. If the stopwatch is placed face up on the center of a turntable, the revolution of the turntable causes an observer in the rest frame to see the second hand rotating at a different speed.

Fusilli light waves

In the 1930s Charles Darwin (grandson of the Darwin of evolution fame) reasoned that if angular momentum was to be conserved when light is emitted from a high-order atomic transition, the spin component of light's angular momentum had to be supplemented with an additional angular momentum. Called orbital angular momentum, that addition was originally associated with rare, "forbidden" transitions. However, in 1992 Les Allen and colleagues realized that one could actually produce laser beams for which every photon is in a well-defined orbital-angular-momentum state. The electric field is not constant in the plane perpendicular to the propagation of such beams. Rather, it picks up a phase of $l\theta$, where θ is the azimuthal angle and each photon has an orbital angular momentum $l\hbar$ (see the article I wrote with Johannes Courtial and Les Allen in PHYSICS TODAY, May 2004, page 35). As with the spin parameter σ, l can be positive or negative.

As a consequence of that azimuthal phase structure, the surfaces of constant electric field are helical; the case $l = 1$ is shown in figure 2a. A second consequence is illustrated in figure 2b: The Poynting vector, which gives the local energy

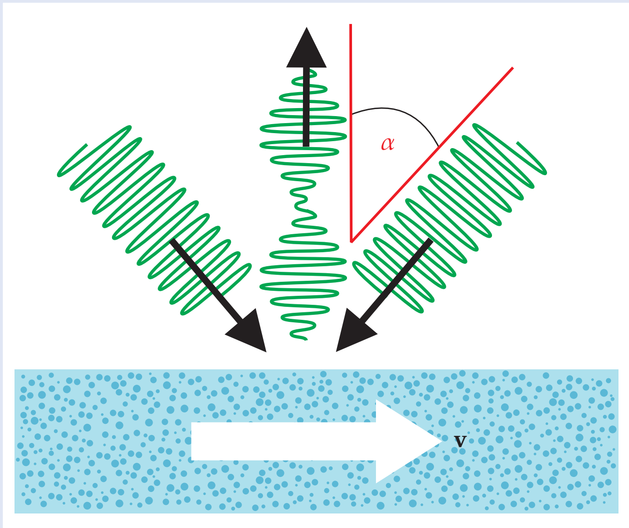

Figure 1. In laser surface velocimetry, light beams of wavelength λ striking a surface at incident angles $\pm\alpha$ scatter off particles moving with speed v and reflect off the surface in the normal direction. Due to the Doppler shift, the intensity of the scattered light modulates with an angular frequency of $(4\pi \sin\alpha)\, v/\lambda$. From the measured modulations, one can deduce the speed of the scattering particles.

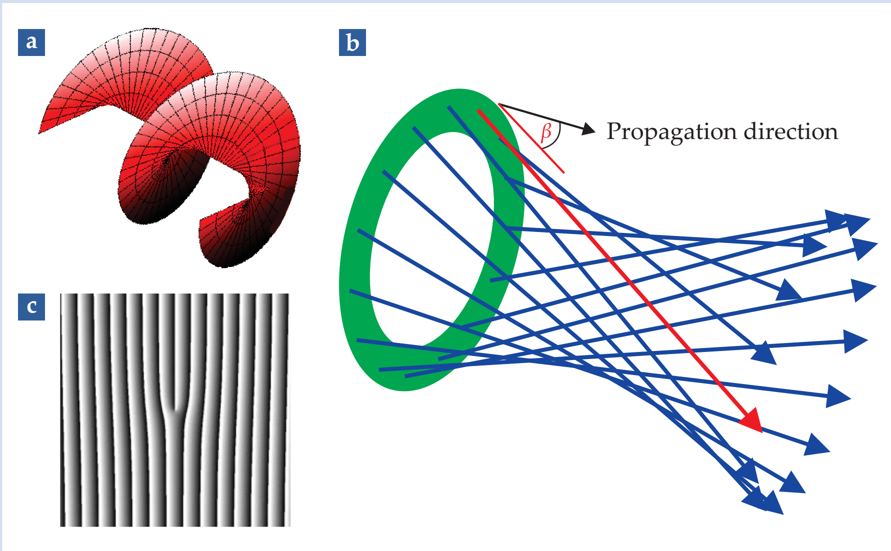

Figure 2. When light possesses orbital angular momentum, **(a)** surfaces of constant electric field have helical shapes. The surface shown here corresponds to light with one $\hbar$ unit of angular momentum per photon. (Adapted from PHYSICS TODAY, May 2004, page 35.) **(b)** Moreover, the propagation direction and the Poynting vectors giving energy flow (blue, but with one exemplar featured in red) are misaligned by an angle β, proportional to the orbital angular momentum. **(c)** Light with orbital angular momentum can be produced with diffraction gratings that have so-called fork dislocations, such as the one shown here.

flow, is misaligned with the propagation direction by an angle $\beta = l\lambda/2\pi r$, where r is the distance from the beam axis.

The helical surfaces corresponding to low l are familiar ones. For $l = 1$, the single-helix surface is typical of a screw thread. The two intertwined helices for $l = 2$ are similar to the double strands of DNA. You encounter the triple helix corresponding to $l = 3$ whenever you eat pasta fusilli.

Rotate a screw thread or pasta fusilli in your fingers and you'll see that the surfaces advance. Likewise, rotations to helically phased light beams carrying orbital angular momentum advance (or retard, if you rotate in the opposite sense) their surfaces of constant electric field and give a rotational Doppler shift $\Delta\omega = l\Omega$. That Doppler shift was postulated by Gerard Nienhuis and Iwo Białynicki-Birula in the 1990s and observed late in the decade by Johannes Courtial and coworkers. The experimental team used millimeter-wavelength beams so that the frequency shift could be seen via RF techniques. Subsequent to their work, other groups observed the frequency shift in optical beams and atomic spectra.

In 2012 my colleagues and I began to ponder the question, What happens to the angular momentum of light when the light is scattered from a rough surface? We thought that the surface might impart angular momentum to the light, but that turned out not to be the case. Eventually we decided to closely examine light with specific orbital-angular-momentum states, and we realized that one can draw parallels between the angle α characterizing the beams striking the surface in Doppler velocimetry and the angle β giving the offset of the Poynting vector from the propagation direction. It follows that when a beam of light carrying orbital angular momentum is normally incident on a surface rotating with angular frequency Ω, the normally reflected light experiences a frequency shift of the reduced-Doppler form $\Delta\omega = (\omega_0 \sin\alpha)\, v/c$; however, in our twisted context, the angle of incidence α is replaced by $\beta = l\lambda/2\pi r$ and the speed v is replaced by Ωr. The result of those substitutions, assuming small β, is a frequency shift $\Delta\omega = l\Omega$. Note that this shift scales with only the light's angular momentum and the scattering object's rotation rate. That behavior is quite different from the linear Doppler shift's proportionality to the initial light frequency.

As with the surface velocimetry, if the rotating surface is struck by two beams with opposite values of l, the frequency shifts are in opposite directions and the two reflected components interfere. The reflected light thus modulates in intensity with an angular frequency of $2|l|\Omega$.

Stick a fork in it

Helically phased light beams had actually been generated even before physicists recognized that such beams carry angular momentum. Marat Soskin and coworkers, for example, used simple diffraction gratings containing a fork dislocation (see figure 2c) to produce such beams. The same types of gratings can be used on light that includes many angular-momentum modes to select beams with a specific value or values of l and direct them to a single-mode optical fiber. Any resulting intensity modulations can then be observed by connecting the output of the fiber to a simple photodiode detector.

Last year Martin Lavery and I, along with coworkers Fiona Speirits and Stephen Barnett from the University of Strathclyde, used a forked grating to measure the intensity modulation resulting from the Doppler shifts of two beams with opposite values of l scattered from a spinning disk. Our experiments confirmed that the modulation frequency is $2|l|\Omega$. Note that due to the factor of l arising from light's orbital angular momentum, the modulation frequency can be many times greater than the angular rotation frequency of the surface Ω. Thus the rotational Doppler effect may allow for sensitive measurements of rotation, even when the conventional Doppler shift is absent. Such measurements may see applications to rotating astronomical bodies, atmospheric turbulence, or turbulence and vorticity in microscopic systems. In any case, more than 150 years after the identification of the Doppler effect, an understanding of classical wave physics can still spin a surprise.

Additional resources

▶ A. M. Yao, M. J. Padgett, "Orbital angular momentum: Origins, behavior and applications," *Adv. Opt. Photonics* **3**, 161 (2011).

▶ M. P. J. Lavery, F. C. Speirits, S. M. Barnett, M. J. Padgett, "Detection of a spinning object using light's orbital angular momentum," *Science* **341**, 537 (2013).

▶ C. Rosales-Guzmán, N. Hermosa, A. Belmonte, J. P. Torres, "Experimental detection of transverse particle movement with structured light," *Sci. Rep.* **3**, 2815 (2013).