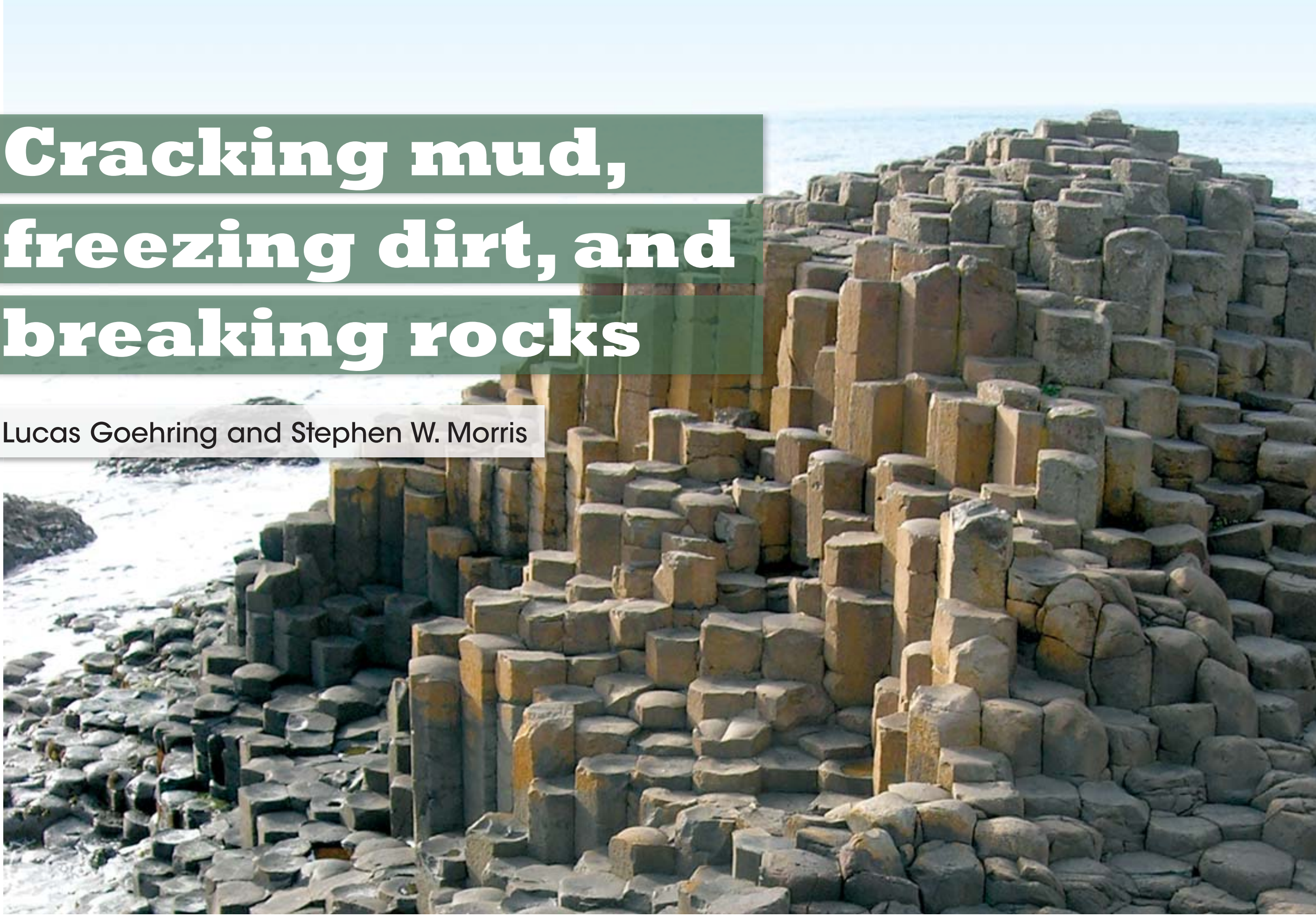
Cracking mud, freezing dirt, and breaking rocks



Lucas Goehring and Stephen W. Morris

The crack patterns in dried mud, permafrost, and lava columns exhibit subtle variations on simple physics.

Ordered crack patterns are so common in nature that they are often overlooked. From tile-like formations in ordinary mud, as shown in figure 1, to the vast polygonal networks that stretch across the polar deserts of Earth and Mars, as seen in figures 2a and 2b, they are typical features in geomorphology.[1,2] On smaller scales, crack networks add an artistic flourish to Japanese *raku* pottery and are found on the paintings of the old masters.[3] Cracking even determines the pattern of scales on the snouts of Nile crocodiles,[4] such as the one in figure 2c.

The physics behind all those patterns can be captured by the requirement that cracks obey a simple elastic energy balance as they grow. Unlike many other physical problems, however, crack growth involves only a local energy minimization at the point and time that a crack is growing, rather than a global minimization of some free energy functional. That subtle distinction means cracking can exhibit some interesting and surprising features. In this article we discuss some modern insights into crack patterns in geomorphology, including their formation and dynamics, the role of energy, and the mechanisms of scale and pattern selection.

Crack networks form sequentially, crack by crack, rather than all at once. Before considering the larger pattern, therefore, we need to understand something about single cracks. That is the domain of the practical discipline of fracture mechanics—the physics of failure. Engineers obviously want to avoid cracks in bridges, ships, and airplanes. Or, if a crack were to appear, they might reasonably want to know how big it can be before something catastrophic happens. The history of fracture mechanics is, perhaps not surprisingly, tied to the history of warfare and disaster.[5] The basic theory is given by two different but equivalent formulations, which sprung from studies begun during the two world wars. Alan Griffith, who worked alongside G. I. Taylor for the UK's Royal Aircraft Establishment during World War I, studied metal fatigue and the effects of scratches on the strength of aircraft parts. In Griffith's theory, the threshold for cracking occurs when the energy consumed to create new surfaces by extending a crack just balances the strain energy released by such an extension.[6] That threshold is called the Griffith criterion.

Although Griffith's theory is both elegant and intuitive, applying it to engineering problems can often be a challenge. Of more practical use is the stress-based theory of George Irwin,[7] who worked for the US Naval Research Laboratory during World

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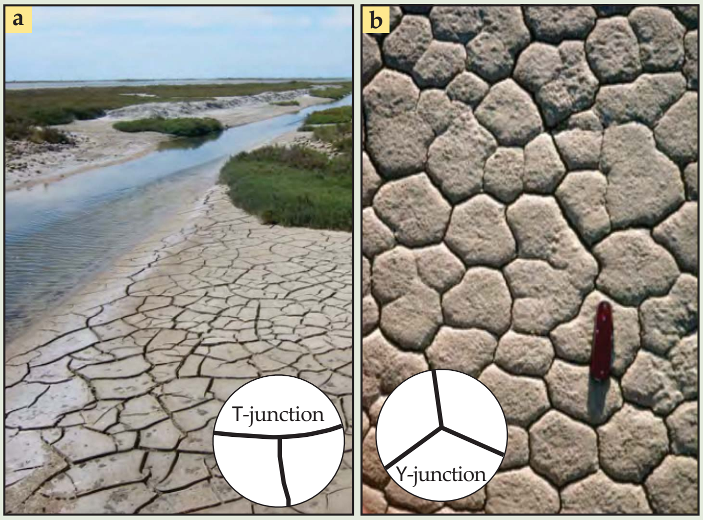


Figure 1. The shapes of mud crack patterns. (a) In dried river clay the crack pattern is built up from individual, sequential cracking events. The T shape in which cracks intersect reflects the order in which the cracks appeared, as later cracks curve to intersect earlier ones at right angles. Here the crack pattern is dominated by four-sided polygons.[8] **(b)** Cracks in mud near Death Valley, California, show a mainly hexagonal pattern. In this case, the cracks meet at Y-junctions. The hexagonal pattern results from a process similar to annealing; a network of cracks like that in panel a is healed by rewetting and re-forms many times. With each iteration, the pattern evolves toward greater hexagonality as the order of cracking changes from iteration to iteration. Figure 3 further elaborates on the process. (Panel b courtesy of Bernard Hallet.)

War II. The navy was having problems with sudden catastrophic fracture: Cargo-carrying Liberty ships were abruptly and spontaneously breaking in two, even before they saw action. Irwin's theory identified the universal way in which stress concentrates around the tip of an existing crack, a result that can be used to estimate the critical stress at which the crack will advance. The physics of stress concentration near the end of a crack is similar to that describing how lightning rods attract lightning by concentrating electric field gradients near their tips (see the article by Philip Krider, PHYSICS TODAY, January 2006, page 42). Often the existing cracks in either Griffith's or Irwin's theory are simply tiny flaws in the material. The box on page 41 describes a couple of simple paper-tearing experiments that illustrate some of the ideas behind the two theories (see also the article by Michael Marder and Jay Fineberg, PHYSICS TODAY, September 1996, page 24).

Cracking is, of course, a highly nonequilibrium and irreversible dynamic process and cannot be entirely explained by equilibrium energy or stress-concentration arguments. For example, a dynamic crack tip may be unstable to meandering or branching, or it may exhibit other effects not easily explained by Griffith's or Irwin's models. Fracture in solids, like turbulence in fluids, is an incompletely solved problem. Nevertheless, local energy considerations can guide physical intuition about the motion of cracks. Below we present some of the more complex phenomena pertinent to geomorphological crack networks. But first we consider the deceptively mundane question, How do mud cracks form?

Cracking mud

If you look at garden-variety mud after it has dried, you will usually see a network of cracks like the one shown in figure 1a. The pattern results from the sequential growth of many individual cracks, and it provides a record of how each one grew.

Mud is a mixture of soil and water. As it dries, water retreats from the pores between the soil grains, leaving countless tiny menisci at the new water–air interfaces. The capillary forces generated by those highly curved surfaces compress the mud into a smaller shape. However, if the mud layer cannot move laterally—for example, because it is stuck to something on its underside—it will build up an internal stress. At some point, perhaps around a defect like a bubble or other inhomogeneity, a single crack will start. Nucleating the crack is the hard part (the box explains why); once the crack has been established, it will rapidly grow across the mud layer in a roughly straight line until it hits something or runs out of mud.

Sometime later a second crack will start. As it grows, it can be influenced by the first crack, which has already released some of the stress energy around itself. Stress is a tensor quantity, and a crack releases more of the stress normal to its edges than along the direction of its growth. If the second crack approaches the first, it will be guided toward the direction in which it tends to release the most remaining energy. Therefore, the second crack will turn to hit the first crack at a right angle. The history of the two cracks—that is, which came first—is recorded in the asymmetric shape of the resulting T-junction.[8]

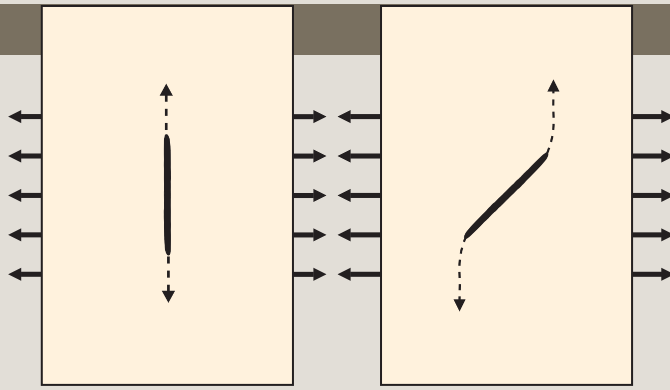
To make a dense network like the one shown in figure 1a, the junction-forming process is repeated by means of many single cracking events. Cracking continues until the polygons reach a characteristic size proportional to the depth of the mud layer. At that time, cracking saturates and higher stress simply widens existing cracks rather than triggering new ones.[9]

When the cracking process ends, it leaves behind a network with distinct statistical features. The angles between cracks at their junctions are mostly near 90° and 180°, as one would expect for T-junctions. The number of sides of the peds (as the broken bits of mud are called) is mostly four, and the peds are roughly square or rectangular. The T-junction network is the most familiar sort of crack network, and it is typically seen in the craquelure on pottery glaze and old paintings. We now consider how it could possibly be any different.

Freezing dirt

Crack networks form in frozen dirt as well as in dried mud. Permafrost often cracks during the deep winter months due to thermal contraction; indeed, the cracks may extend a few meters into the frozen ground. The size of the polygons in the permafrost crack network is proportional to the depth of fracture, just as it is for networks in dried mud—and the

Two simple paper-tearing experiments



Take a piece of paper with a small vertical slice cut in the middle of it and pull its parallel sides as indicated in the left-hand panel of the figure. For a gentle pull, the paper will not tear and the process is reversible. The ability of the paper to withstand your action illustrates the physics of the Griffith criterion—there is a threshold stress below which fracture cannot proceed. Now tug harder, so the paper begins to tear. You will find that the "crack" then advances, even with less force than was needed to initiate tearing. The strain energy you supply by pulling contributes to the widening of the crack all along its length and, for a given pulling force, is larger for longer cracks. But to a good approximation, the strain energy you supply is consumed only at the growing tip. That is because the stress concentrates at the tip. Thus longer cracks are more dangerous because they require less applied stress to grow, and new cracks activated from microscopic flaws require the highest stresses.

Now repeat the experiment with a piece of paper having an oblique slice as depicted in the right-hand panel of the figure. You'll find that you need a harder pull to initiate crack growth, assuming the initial oblique and vertical slices are the same length. That is because a crack is most efficient at relieving stress perpendicular to its length. When the oblique crack does advance, it will tend to curve toward the vertical direction to most efficiently release the strain energy.

The experiments described here illustrate the importance of local energy considerations in the growth of a crack. The Griffith criterion involves only the point and time of crack advance and compares the energy before and after an infinitesimal advance of a crack. When the Griffith criterion is exceeded, a crack will tend to grow along the path that maximizes the energy difference.

physics leading to the proportionality is the same too. The resulting landform is called polygonal terrain. Processes similar to terrestrial permafrost cracking occur on Mars, and picturesque polygonal terrain covers a significant fraction of the polar land area of both Earth and the red planet; figure 2 shows examples. A key difference between the frozen-terrain networks and the dried-mud crack patterns is that frozen-terrain networks consist of cracks that open and close seasonally. During each cycle, the network is partially erased and updated. The polygons observed today usually represent many thousands of years of iterated evolution.

The network's telltale feature that betrays their iterated origin is their mostly hexagonal polygons bounded by cracks that meet at nearly 120° junctions. Those Y-junctions do not form all at once; rather, they appear to evolve from T-junctions in the following way. During every summer the cracks close and partially heal. In the subsequent winter, new cracks appear that tend to follow the existing weak places established by the previous year's cracks, but they can do so in a different order. In an existing T-junction, a new crack may enter by the arm of the T and exit by the stem, for example. Figure 3 shows how the process plays out in a simple analogue experiment during which cracks in a mud layer are healed by repeated wetting.[10] Over time the cracks in each cycle will approach each junction in an essentially random order, so a symmetric Y-junction emerges. The precise location of the vertex of the junction moves slightly to accommodate the evolving junction shape.

In a hexagonal crack network, the centers of the polygons are often raised above their surroundings, so each polygon has a mounded shape. That feature can be the result of external material, such as ice, filling the cracks while they are open. The subsequent closing of the cracks develops stresses that push up the polygon centers. Over many cycles, the soil inside each polygon may flow in a manner reminiscent of fluid in a convection cell with an upflow in the center.

Of course, freezing and the transport of groundwater and material in a heterogeneous soil can be much more complex than in the simple laboratory analogue featured in figure 3. Furthermore, in arctic soils, other types of ordering mechanisms can produce striking patterns—sorted circles of stones, for

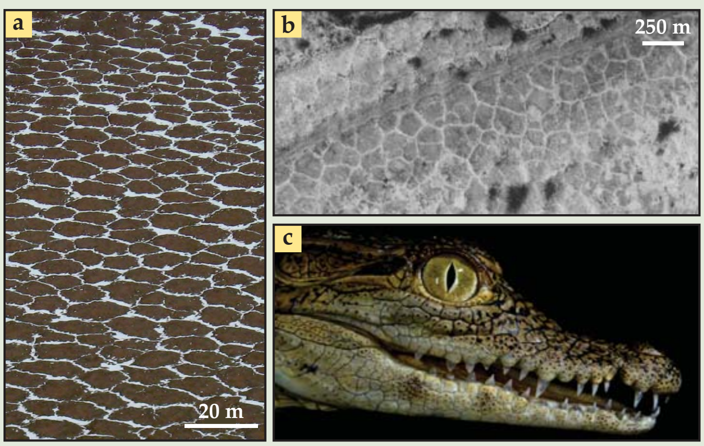


Figure 2. Polygonal terrain on Earth and Mars . . . and a crocodile snout. **(a)** Polygonal terrain is decorated by snow in the McMurdo Dry Valleys, Antarctica, as seen from a helicopter. **(b)** Polygonal terrain on Mars is decorated by seasonal frost. Such patterns are reasonably common in the high latitudes of both Martian hemispheres. (Courtesy of NASA/JPL/Malin Space Science Systems.) **(c)** A mechanism similar to the one producing the ordered fracture in panels a and b forms the scales on the snouts of the Nile crocodile.[4] (Courtesy of Michel Milinkovitch.)

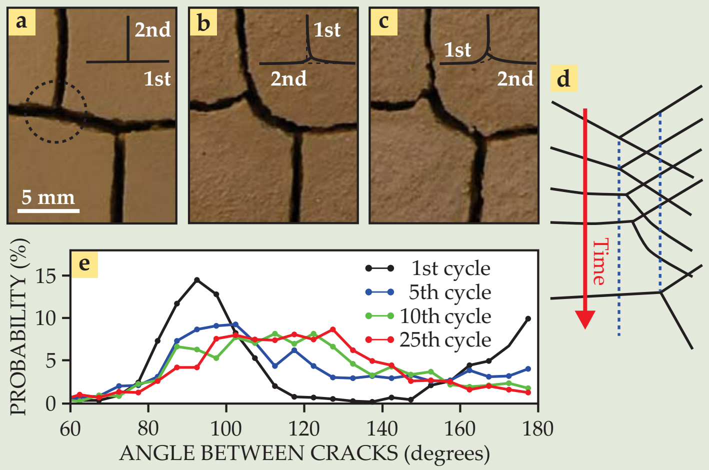


Figure 3. The evolution from T-junctions to Y-junctions. A laboratory experiment in which a mud layer is cracked, wetted, and cracked again many times illustrates some of the essential features of the process that forms polygonal terrain in permafrost and the Martian polar regions. In that process, as in the mud experiment, the crack pattern re-forms in each cycle; later incarnations are guided to some extent by earlier ones. **(a–c)** An initial T-junction (circled) evolves toward a Y-junction because cracks from subsequent generations approach the junction in different sequences. The insets show the changing crack order. **(d)** The location of a junction evolves slightly to accommodate the junction's changing shape. **(e)** The distribution of angles between cracks initially peaks at 90° and 180° but evolves toward 120° over time. (Adapted from ref. 10.)

example—that do not necessarily involve fracture (see reference 11 and PHYSICS TODAY, April 2003, page 23). Those mechanisms are likely to be different in detail on Earth and Mars, and in general, disentangling their effects can be a challenge.

Occasionally, an undisturbed mud puddle will mimic the lab experiment and form a mostly hexagonal crack network as a result of repeated wetting and drying cycles, as shown in figure 1b. Those patterns also typically show polygons that are raised in the center, which indicates that the mud has been lifted by a ratcheting process similar to that described above.

Something analogous to the dynamic process that forms polygonal terrain is also, remarkably, responsible for the ordering of the scales on the snouts of Nile crocodiles (see figure 2c). During embryonic development, the snout scales (but, interestingly, not body scales) emerge and evolve due to the local failure of a brittle skin rather than being directly organized by gene expression.[4]

Breaking rocks

A particularly spectacular kind of ordered cracking is columnar jointing, a three-dimensional organization of cracks found in many lava flows, that results in massive outcrops of nearly hexagonal, often meter-scale pillars. Famous examples include the Giant's Causeway in Northern Ireland, shown on the cover and on page 39; Fingal's Cave on the island of Staffa in Scotland, shown in figure 4a; the Devils Postpile in California; and the Devils Tower in Wyoming. The evocative names of all those places are indicative of their uncanny geometry.

Columns are formed by a traveling version of the ordering process described above for polygonal terrain. After it is deposited, a thick lava flow cools from the outside in. That cooling produces a network of shrinkage cracks at the surface. As the cracks deepen, the extraction of heat eventually becomes dominated by the boiling and reflux of groundwater in the cracks, as in a heat pipe. That is a much more efficient heat-transfer mechanism than pure thermal diffusion, but even so, complete cooling of a large flow can take decades.

As always, crack growth is dictated by local conditions at the crack "tips," which in 3D are actually lines at the ends of 2D cracks. The network of tips is confined to a thin, downward-moving, roughly planar front that carves out columns with polygonal cross sections as it proceeds in step-like increments. The water-reflux mechanism sets an overall constant speed for the front.

Initially, the crack pattern is dominated by T-junctions, but within a few meters of the surface it evolves into a Y-junction-dominated network and the polygonal column width evolves toward a common scale L. As in polygonal terrain, successive cracks adjust their positions as the network develops step by step.[12] In the columnar case, adjustment occurs as the cracks advance deeper into the flow, rather than by the cyclic opening and closing of cracks. The cracks deepen and lengthen the columns but approach the emerging Y-junctions in a random sequence in each step. Extended lines of such Y-junctions eventually form the corners of a hexagonal column. That evolving ordering and incremental advance of the columns can be deduced from features left behind on the crack faces. In many locales, however, those delicate surface markings are weathered away as the rock is eroded.

As discussed above, the scale of the polygons in mud cracks and polygonal terrain is proportional to the depth of the cracking layer. In columnar jointing, a subtler, rate-dependent, nonequilibrium mechanism sets the column scale. The crack network advances into a stressed region near the front whose thickness is determined by the speed v at which the whole planar network advances.[13] That speed, in turn, is proportional to the overall heat flux extracted by the water reflux to the surface.

Flows that are cooled slowly exhibit wider columns, and rapidly cooled flows show narrower columns, even in otherwise identical materials. The thickness of the active layer where thermal stresses are present ahead of the crack tips scales with D/v, where D is the diffusivity of heat. More formally, the configuration of stresses and temperatures in the active layer is determined by a quantity called the Péclet number, $Pe = vL/D$, the only dimensionless number that appears in the steady-state diffusion equation for temperature in a reference frame moving at speed v. The value of Pe is always reasonably close to 1, and thus all columnar joints are expected to be dynamically similar. That universality is evident in figure 5a, which shows data for columnar joints from various geological locales.

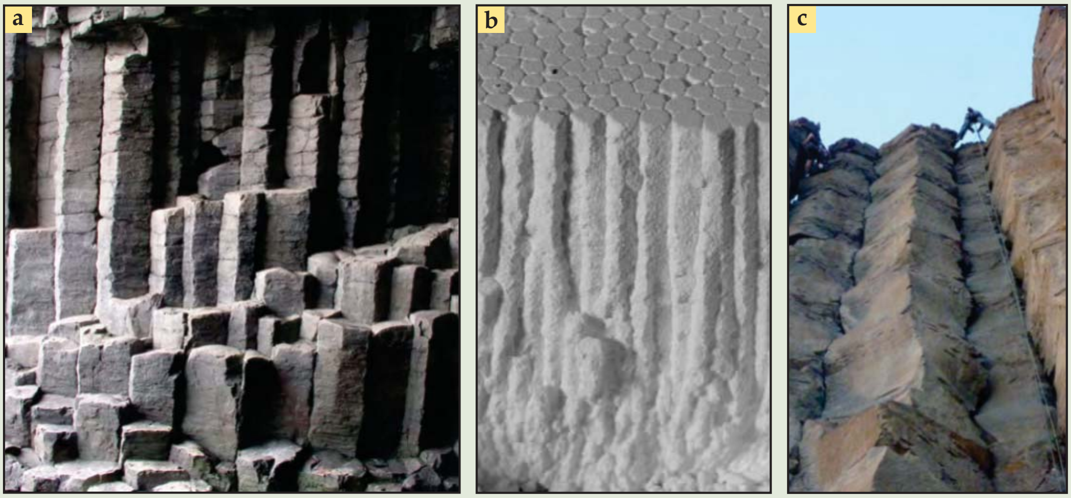


Figure 4. Pillars large and small. The meter-scale basalt columns seen in **(a)** were photographed in the interior of Fingal's Cave on the island of Staffa, Inner Hebrides, Scotland. (Courtesy of Robert Mehew.) **(b)** Processes similar to those forming the columns in panel a can be replicated in the laboratory with drying cornstarch. The columns here are a couple of millimeters across. **(c)** These wavy columns, a favorite of rock climbers, are located at Frenchman Coulee in Washington State. (Courtesy of Laurel Fan.)

Remarkably, columnar joints can also be produced in the laboratory by drying cornstarch, as shown in figure 4b. When a thick layer of wet starch is dried from above, the shrinkage stresses produce millimeter-wide columns that are analogous to the much larger lava columns. The peculiarities of how cornstarch dries have been studied in some detail, and we now know that the drying proceeds via the passage of a sharp front, with dry starch above and wet starch below. When the front is controlled to have a constant speed,[13] the column scale is again set by the Péclet number. Figure 5b shows the scaling for starch columns for a series of experiments that found $Pe \approx 0.1$. The behavior of cornstarch in the laboratory is not perfectly analogous to what happens with geological formations; nonetheless, the experiments have allowed investigations of various aspects of columnar jointing that would be impossible to explore in the geological context.

One might imagine that the near perfection of hexagonal crack networks indicates that the fractures have followed some sort of optimization principle. Perhaps the regular pattern occurs because it maximizes the global strain energy that is released. The optimization explanation of columnar joints is an old one; it goes back to the work of Robert Mallet in 1875. In that superficially appealing scenario, the Y-junctions all appear simultaneously, organized into a globally "best possible" state with a single, optimal polygonal scale. Such is not the case, although that incorrect explanation is often given in tourist guidebooks. In fact, the scale of the pattern cannot be deduced from an equilibrium argument, and as discussed above, the Y-junctions do not form simultaneously. The hexagonal network emerges over time from local rules—no global optimum need exist. Indeed, the advance of the resulting hexagonal crack pattern may not even be steady.

Wavy cracks and other oddities

The nonequilibrium nature of fracture is made especially manifest by the existence of oscillatory and branching cracks. In a classic experiment, a hot, thin glass plate with a single notch at its base is dipped at a fixed speed into a bath of cooler water. Depending on the speed of dipping and the temperature difference between the plate and the water, the notch may nucleate a single straight crack, a wavy crack that follows a nearly sinusoidal path as it advances, or a complex branching pattern of sinuous cracks.[14] The unsteadiness of the crack-tip path and the strong rate dependence of the process clearly show that nonequilibrium effects are important.

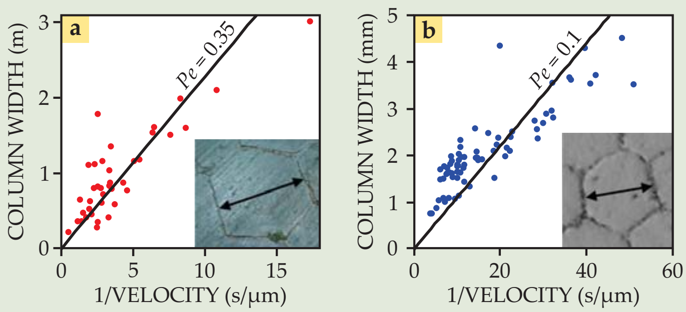


Figure 5. Scale selection in columnar jointing. The plots here give the widths of columns (as indicated in the insets) as a function of the inverse rate of crack advance. Theory predicts that for a constant heat diffusivity D, the data should fall on a straight line whose slope corresponds to the Péclet number Pe. **(a)** For these data, obtained from columns in various lava flows, the advance rate is deduced from detailed modeling of the patterns on the exposed faces of the columns.[15] The lava columns are seen to form with a Pe of 0.35, even though D is somewhat uncertain and varies a bit across the samples. **(b)** In drying cornstarch experiments, Pe is about 0.1. (Adapted from ref. 13.)

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An oscillatory crack traveling in a thin plate between two free edges or between two other cracks will release strain energy more efficiently by curving toward either boundary. That is the physics behind the appearance of T-junctions in mud cracks. However, the total strain energy density is higher near the middle of a plate than near the edges. Thus there is a competition between the energy release the crack achieves by changing its inclination and the energy release it sacrifices by moving away from the middle of the plate. Somehow, that competition is responsible for the instability and oscillation of the crack tip. Theories of fracture mechanics, however, are struggling to make the intuitive competition idea quantitative.

Perhaps surprisingly, columnar jointing can also display wavy cracks.[15] Figure 4c shows an example of the resulting wavy columns from Frenchman Coulee, a popular spot for rock climbing in Washington State. Wavy columns have not been observed in cornstarch analogue experiments so far, and their detailed formation mechanism is not understood, but it seems likely that they are 3D versions of the wavy cracks seen in thin plates.

The interpretation of crack networks is still something of an art. Like many far-from-equilibrium phenomena, they partially encode their whole history of formation, yet they also reflect subtle internal dynamics. Deceptively simple tabletop experiments can reveal new insights and sometimes even assist in the interpretation of large-scale patterns in geomorphology. At the most basic level, energy and symmetry considerations only get one so far. A fuller understanding requires stability considerations: Will a network evolve toward a dynamically steady configuration, or will it break into oscillation? Or will it become something more complex? The working out of local rules can have surprising global outcomes. From the snouts of Nile crocodiles to the surface of Mars, crack networks can tell us much about the hidden mechanisms at work during their evolution, if we can learn to read them. Ordinary mud is not a bad place to start.

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