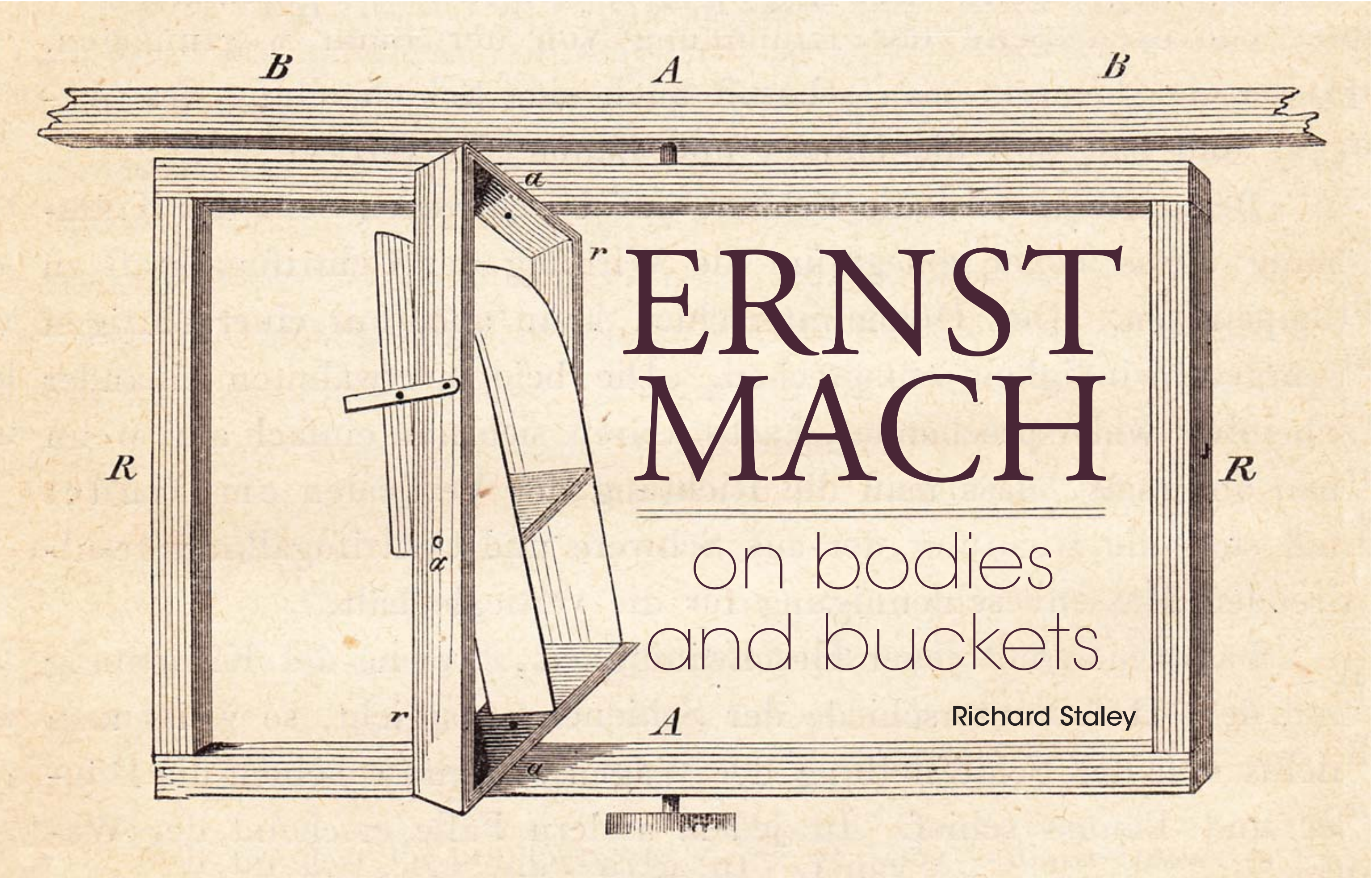
ERNST MACH

on bodies and buckets

Richard Staley



The scientist-philosopher's critique of Newtonian mechanics was informed, in part, by a startling phenomenon he experienced as the train he was riding negotiated a sharp bend.

Austrian physicist Ernst Mach (1838–1916) is best known for his 1883 book *Die Mechanik in ihrer Entwickelung* (*The Science of Mechanics*), a work widely credited with helping instigate a reconsideration of the Newtonian worldview that led to the formulation of special and general relativity in the early 20th century. Mach's first critiques of mechanics had been published as early as 1868 and 1872 in studies of mass and inertia. But the section in *The Science of Mechanics* in which Mach attacks Isaac Newton's concept of absolute space develops a line of thought begun in 1879. There, Mach responds to Newton's primary argument for absolute space, the rotating-bucket experiment.

If a bucket of water is set spinning, the walls of the bucket at first rotate relative to the stationary water, and the surface of the water remains level. Then the water starts to rotate also, and it rises toward the walls. When the spinning bucket and water are at rest relative to each other, the water surface is concave. Interactions between water and walls can't explain the deformation, and Newton argued that the surface concavity demonstrated the bucket's absolute rotation in space. For Newton, absolute space remained similar and immovable, without regard to anything external, but the experiment showed that the absolute rotation of water could be measured.

In 1883 Mach wrote that Newton's experiment merely shows that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces and that such forces are instead produced by its relative rotation with respect to Earth and the other celestial bodies. Referring to a situation he seems to have first sketched in

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a notebook entry in 1879, Mach added, "No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick."[1] He went on to argue that when we say that a body retains its direction and velocity in space, that expression is an abbreviated reference to the entire universe.

Over time, Mach's attack on absolutes and his discussion of the bucket experiment became the most celebrated features of his account of mechanics. Indeed, for many years Albert Einstein saw the demand to provide an account of inertial mass that depends on the masses of the universe as one of the central criteria for his emerging theory of gravitation. Yet whether and how to satisfy such a demand—what Einstein christened Mach's principle—have remained open questions, and there have been many different formulations of the principle. The collected essays in Julian Barbour and Herbert Pfister's book *Mach's Principle: From Newton's Bucket to Quantum Gravity* are particularly insightful on the subject.[2]

For all its subsequent importance, Mach's section on absolute time and space in *The Science of Mechanics* is interesting for what it leaves out. In his teaching, notebooks, and early publications, Mach consistently discussed mechanics in conjunction with a flotilla of other thoughts—about economics, the physiology of sense perception, psychology and the ego, and politics—which seem at different times to have formed and been stimulated by his perspective on mechanics. This article highlights a few of those diverse elements to help place in context the origins and character of Mach's contributions to relativity; Mach's perspective may also help us better appreciate what his work meant to others—Einstein in particular.

Indeed, Einstein twice commented on the extraordinary nature of Mach's influence: Even Mach's opponents were hardly aware of how much of his way of thinking they had absorbed "so to speak, with their mother's milk,"[3] as Einstein put it in his obituary of Mach. But Einstein also viewed Mach's epistemology critically and wrote to a friend, "It cannot give birth to anything living, it can only exterminate harmful vermin."[4] Nonetheless, as we'll see, Einstein himself was unaware of the debt he owed Mach for the happiest thought of his life, which came in 1907.

"History can change everything"

After completing his doctorate on electrical discharge and induction in 1860, Mach (shown in the roughly contemporaneous photograph in figure 1) began his career with lectures on physics for medical students and with research that related the senses and physical phenomena—what German philosopher Gustav Fechner called psychophysics. Mach conducted research on the Doppler effect in acoustics and optics and, like his contemporary Hermann von Helmholtz, studied the anatomy of the ear. Mach's critical responses to atomism and the basic concepts of mechanics at the time are both likely to have had their origins in his teaching.

In 1867, aiming to clarify the relations between *a priori*, empirical, and hypothetical elements in the foundations of mechanics, Mach offered a groundbreaking definition of mass solely in terms of the mutual acceleration of different bodies. The paper's rejection by the prestigious *Annalen der Physik* reinforced his caution about discussing such ideas with physicist colleagues. (The paper eventually appeared the following year in Carl's *Repertorium*.[5]) Four years later he took up Newton's first law in the notes on a lecture he gave on the history of the conservation of work to the Royal Bohemian Society of Sciences in 1871.[6]

Like German mathematician Carl Neumann, Mach believed that any formulation of the law of inertia that did not specify the motion of a body in relation to other bodies would be meaningless. But whereas Neumann proposed that all motion should be determined relative to some hypothetical body in absolute space, Mach believed it critical to specify the motion in relation to real, concrete bodies. To show why, he posed the case of the relative motion of Earth with respect to the stars. The two possibilities—with Earth rotating on its axis, or at rest with the stars spinning around it—are equivalent geometrically. Yet not only is the first case simpler

Figure 1. Ernst Mach (1838–1916) is now well known for his research on sound and the eponymous number representing the ratio of the speed of an object moving through a fluid and the local speed of sound. He is known, too, for the principle Albert Einstein named after him, which relates the law of inertia—or in Einstein's formulation, the inertial mass of any given body—to the large-scale distribution of masses in the universe. (Courtesy of the AIP Emilio Segrè Visual Archives.)

astronomically, it also has real inertial effects: the flattening of Earth and the circular motion of a Foucault pendulum. For Mach, that asymmetry meant that a proper expression for the law of inertia had to include the interactions of a body in question with the masses of numerous other bodies—including, at least in principle, the masses of the entire universe.

Mach's criticism is remarkable for its cosmic scale. Often, the corner of a room is enough to determine a frame of reference, and for astronomical observations the fixed stars will do. But because Mach took the causal interaction between bodies as central in his philosophy, he asked what one would make of the law of inertia if, for instance, an earthquake struck or if the stars suddenly swam in confusion. What would become of the law? How would it be expressed and applied?

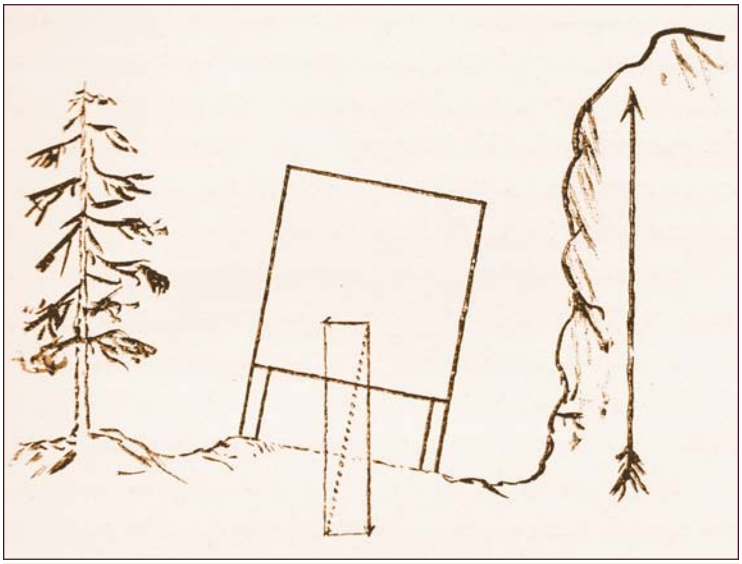

Figure 2. Ernst Mach sketched his analysis of the forces responsible for his perception of trees appearing to tilt outward as the train he was riding rounded a curve. The diagram, from Mach's 1897 account of the experience, parses the gravitational and centrifugal forces his body felt into vertical and horizontal components. His rotation experiments showed that the resultant of those vector forces is perceived as vertical; Mach's bodily sense of the vertical coincided with the compensating pitch of his train, and the trees appeared at an angle. (Adapted from ref. 16.)

Since his earliest teaching, Mach argued that science was chiefly concerned with economy of thought; that is, the goal of science was to discern the simplest, most economical descriptions of experience. But he also understood the necessity of concept formation by abstraction, analogy, and generalization. And for him, the process was similar across different fields of study, even if their methods should be quite different. For instance, Mach underscored his argument for the causal role played by surrounding bodies in the law of inertia by drawing an analogy to the role that the entire currency in circulation in an economy plays in funding any given paper note.

In another instance, Mach argued that matter is an abstraction of the same kind as the soul, and he believed that science could say as much about the soul as it did about matter.[7] His notes for "On Some Principal Questions of Physics," an 1872 course he taught, show how seriously he took the issue. He began by noting that we experience both an inner and an outer side of nature, and that given such experience we may attribute, by analogy, an inner life to others. He then compared an extension of gravity to all matter with Fechner's generalization of a soul inherent in all matter. But the value of such generalizing was, for Mach, practical.

Indeed, Mach concluded his lecture notes with methodological advice: "We can hope to come to a better understanding of the world if we measure ourselves with the standards of the outer world and the outer world with our own standards, considering it as physical process, but attributing sensation to matter."[8] According the inner world equal status with the outer world and simply accepting that matter has the capacity to sense was for Mach a pragmatic stance that allowed him to investigate the relation between mind and matter.

In the early 1870s Mach thus moved between inner and outer worlds in the classroom, and between physics and economics in his lecture on the conservation of work. But he wanted his audiences to recognize his politics too. For that reason, he began his 1871 lecture at the Royal Bohemian Society of Sciences with a manifesto for history. As a young child, Mach said, he could never understand why people should suffer the reign of a king for even a minute, and why there should be such inequalities between rich and poor. Over time one either simply gets used to such facts, he continued, or one learns to understand them through history and can thus approach them without hatred. But understanding wasn't enough. He told his audience, "We should not let go of the guiding hand of history. History has made everything, history can change everything."[9] He meant that statement as true not only for politics but also for the ongoing work of the sciences. Those political dimensions were largely invisible and unstated in Mach's *The Science of Mechanics*, written a few years later. But they help explain why he would be an important and attractive figure for socialists, including many in the circles Einstein got to know as a student in Zürich, Switzerland.

Railway phenomena

Shortly after Mach considered the interrelation of bodies in the universe, he turned his attention to his own body—more specifically, to the connection between movement and his physiological sense of it. In the autumn of 1873, while aboard a train rounding a sharp curve, Mach suddenly saw the trees and buildings along the side of the track angle outwards—something he termed "Eisenbahn Phänomene" (railway phenomena). That unusual event led him to recognize the need to explain how we sense the vertical. Even as he began research, however, he already believed that a suitable sense organ lay inside our heads—specifically in the semicircular canals of the inner ear. And his notebook entries contained calculations of the motion of heaving ships and sketches of Atwood's machines—systems typically involving two masses connected by string over a pulley—that would allow Mach to investigate the sensations of rising and falling.

To experimentally settle the issue, Mach built a frame 4 meters long that could be spun on a central axis; that frame carried a second frame 2 meters high which itself carried a chair,[10] as illustrated on page 42. The chair, sometimes described as an ancestor of modern devices used to train astronauts, could be set in position anywhere inside the outer frame, be rotated on its own vertical axis, or even be set in a tilted position if desired. Ensconced in a paper box to block all visual cues and set spinning about the outer frame's central axis, a seated subject could thus observe his sensations firsthand. The results were unambiguous: Subjects immediately sensed the direction and magnitude of their rotation. However, if they continued to rotate at a constant speed, the sensation gradually waned until they believed themselves stationary. And if the device was slowed to a standstill, they believed themselves to rotate in the opposite direction at a rate proportional to the deceleration. The sensation persisted for some seconds.

When the inner frame is positioned outside the central axis—approximating the train's turn—the subjects immediately perceived their direction of rotation but also believed themselves and the interior of the paper box to be inclined outward, tipping away from the central axis, even though they actually remain vertical. (To experience the effect, simply close your eyes while riding a playground merry-go-round.) Mach concluded that the resultant of the outward-directed centrifugal force and the downward-directed gravitational force is perceived as vertical. His train must have traveled at the appropriate speed for the pitch of the track on its curve to fully compensate for centrifugal effects of the turn. So rather than feeling himself pushed to the edge of the train, Mach realized that his bodily perception of vertical had coincided with the frame of the train compartment and the trees had appeared inclined, as sketched in figure 2.

But Mach also sought to explain the mechanism behind that bodily perception. Moving his head while rotating helped confirm that the primary sense lay there. Indeed, as he had earlier surmised, he found the behavior of fluids in the semicircular canals of the inner ear to be critical: The three semicircular canals, oriented roughly perpendicular to each other, provide a mechanical system that responds to motion in each direction, and the changing pressure of the fluids against membranes in the ampullae, located at the ends of the canals, as shown in figure 3, provides the basis for our sensation and for our distinction between angular acceleration and angular velocity.

Mach was investigating our inner sense of motion with the measures of the outer world, and he showed that mechanics could bring insight to physiology. His paper giving the first full account of the role of the inner ear in the perception of motion was received by the Austrian Academy of Sciences on 6 November 1873. Mach experimented with pigeons and rabbits; studied the brain, skin, and muscles; and examined the relations between motion and vision. He used a seesaw-like device and a trolley rolling down an inclined plane to investigate the sensation of falling, and he gathered information on elevators in mine shafts.[11] Years before Mach wrote extensively on Galileo Galilei and Newton, he had rolled down Galileo's inclined plane and had experienced firsthand the forces exerted by a spinning bucket—indeed, he had been the bucket! So what contribution did Mach's physiological studies make to his conceptual critique of mechanics?[12]

Newton's bucket, in abstract

For one thing, Mach's studies prompted him to turn to history again. Soon after he began research on the sensation of motion, Mach devoted a page and a half of notes to the history of mass—listing key players

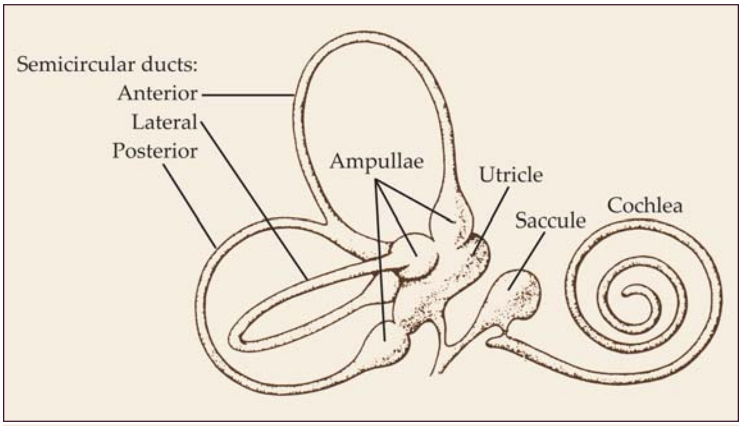

Figure 3. Inner-ear anatomy as Ernst Mach understood it. He discovered that the changing pressure of fluid in the semicircular canals was key to the sensation of rotation and found that structures in the utricle and saccule play a role in the perception of horizontal and vertical motion. (Adapted from ref. 17.)

such as Galileo, Christiaan Huygens, and Newton and highlighting the concept of work. Even so, there's little evidence he reexamined Newton's thought until much later—in the opening pages of a notebook begun in late October 1879. There Mach focused on acceleration and uniform motion and the relationship each indicates with different bodies; he noted that one cannot know whether motion is, in fact, uniform.

He then drew two diagrams, each depicting three concentric circles to delimit motion in two regions located within some outer boundary marked R, as shown in figure 4. On the left, the innermost circle, or ring, rotates clockwise, any material inside it subject to centrifugal forces marked by radially directed arrows. On the right, the middle ring turns counterclockwise, with material inside it also subject to centrifugal forces. The difference between the two cases and the existence of the outer ring, which symbolizes the influence of surrounding masses, were, for Mach, critical to his rejoinder to Newton's conception of absolute space and time. Our experience of centrifugal forces must be a result of relative motion, he reasoned.

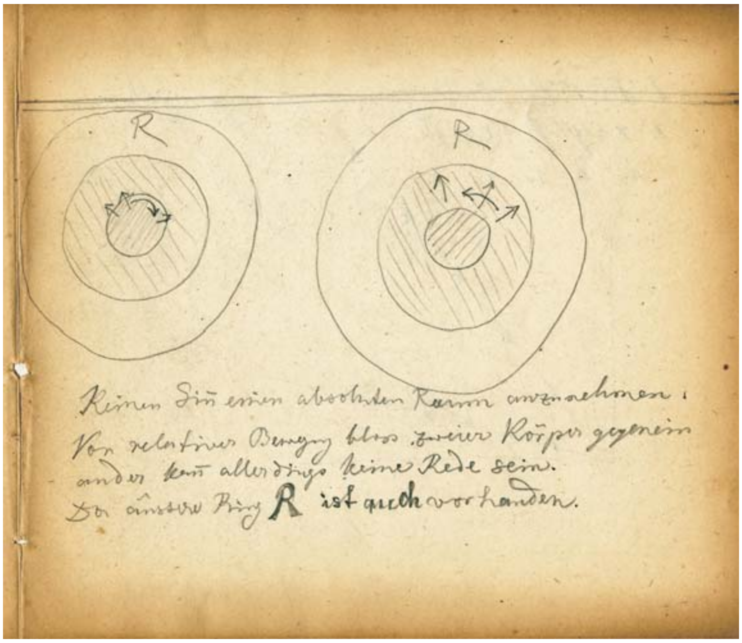

Figure 4. In 1879, Ernst Mach sketched what may be his first response to Newton's bucket experiment—Newton's argument for absolute space from the observation of changes to the water level in a spinning, water-filled bucket. The sketch depicts the relative motion of two concentric bodies (the hatched lines)—one rotating clockwise (left), the other counterclockwise (right), both some distance from an outer ring, R, presumably symbolizing the fixed stars. The radially outward arrows indicate centrifugal force. Below the diagram, Mach writes, "No sense assuming absolute space. In any case there can be no talk of relative motion between two bodies alone. The outer ring R is also present." (Adapted from ref. 13.)

On subsequent pages, Mach's notes and diagrams reveal the difficulty of taking distant bodies into consideration in formulating the law of inertia. Mach wrote that Newton "affects logical rigor where it is not present," and "conceals assumptions."[13] Those 1879 notes have a starting point similar to Mach's 1872 discussion of Earth and the fixed stars, but they differ in character and scale in ways that point toward the strategy Mach followed four years later in *The Science of Mechanics*, in which he framed his argument as an attack on absolutes. The rings anticipate Mach's 1883 argument that no one can predict the outcome of the experiment using a bucket whose walls enlarge in size until they are several leagues thick.

Little of Mach's physiology, psychology, and politics made it into his 1883 account of mechanics. In a footnote he mentioned that his treatment would exclude physiological time and space, and he may have refrained from discussing the sloshing of fluids in the inner ear to avoid muddying the waters. Only later, in a popular lecture in 1897, did Mach comment that Newton would probably have welcomed his discovery that a person perceives rotation without any stationary reference points.[14]

Mach's 1879 notebooks on mechanics, however, are interrupted by passages on philosophy, psychology, and politics. One passage suggests that Mach observed the methodological principle he urged on his students: to explore the inner world with the measures of the outer world and the outer world with those of the inner. Between a series of notes first on action and reaction, inertia, pressure and counterpressure, and then on siphons, Foucault's pendulum, and gyroscopes, Mach wrote rather enigmatically, "Personal impersonal. The personal lies in strong connections. The person has only apparent unity. No boundaries. Renunciation of egotism. The person is an illusion. Ethical improvement.—What one now does unconsciously one will then do consciously."[13] With these comments, Mach quickly moves from physics to psychology. For him the transition was seamless because he believed that neither bodies nor the self could be correctly understood without recognizing their relations to the environment, which in each case played a greater role in their formation than was commonly appreciated.

While he discussed bodies rather abstractly in 1883, he kept his psychological arguments for his 1886 book *Beiträge zur Analyse der Empfindungen* (*Contributions to the Analysis of the Sensations*).

Origin of the equivalence principle

To appreciate the perspective Mach brought to his understanding of mechanics, we've focused largely on his writings on philosophy and physiology prior to *The Science of Mechanics* and arguments that later led Einstein to his formulation of "Mach's principle." But to appreciate the extent of Mach's influence on Einstein, let's consider another part of that 1883 work as well. In a section preceding his discussion of Newton's conception of space, Mach illustrates the principle of reaction by asking his readers to consider a thought experiment with a load on a table. The load presses the table only as much as the table, in turn, presses the load, preventing it from falling. If the table falls freely, all pressure on it ceases. Mach points out that no change in pressure is produced by ascent or descent at a constant velocity; it is the relative acceleration that determines pressure changes. Mach also discusses the behavior of a pendulum. Drawing without comment on his extensive physiological experiments, he refers to the sensations that we feel when we jump or fall and to those we would feel if suddenly transported to a different planet.[1] Clearly, his physiology had not completely disappeared.

Early in 1920 Einstein outlined the beginnings of his approach to gravitation in a manuscript reminiscent of Mach's study of the development of mechanics. In that essay, Einstein described the "happiest thought" of his life, which he says came in 1907 "in the following form":

> The gravitational field has only a relative existence to an observer, similar to the electric field produced by magnetoelectric induction. This is because for an

observer falling freely from the roof of a house there exists during his fall—at least in his immediate environment—no gravitational field. If the observer lets a body go, it will remain at rest relative to him or in uniform motion, independent of its chemical and physical nature. The observer is justified in thinking of his state as being "at rest."[15]

Mach's work provided central elements for Einstein's thought and helped set the form in which it arrived. Although Einstein never recalled his reading of Mach when describing that idea—the origins of the "equivalence principle," the equivalence of a gravitational field and a corresponding acceleration—his thought experiment is clearly a variation on Mach's, which was based on extensive empirical studies. So before pointing to the elevator and spacecraft to which Einstein and others allude in outlining the equivalence principle, we should recall the heaving ship, mineshaft elevator, and loaded table that Mach considered in the course of his physiological studies on motion.

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