letters

Time-symmetric quantum mechanics questioned and defended

In their feature article "A Time-Symmetric Formulation of Quantum Mechanics" (PHYSICS TODAY, November 2010, page 27), Yakir Aharonov, Sandu Popescu, and Jeff Tollaksen state that there exists a "freedom to impose independent initial *and* final conditions on the evolution of a quantum system" without having to modify quantum mechanics "by an iota." The supporting illustrations they give, however, are based on an inadequate analysis of the measurement process in quantum mechanics.

Consider their *gedanken* experiment in which measurements are made at two successive times, t and t_1, after the system has been prepared in a state Ψ at $t_0 < t < t_1$. Now suppose that the experiment is repeated, but without any measurements made at t_1. Then the standard statistical prediction of quantum mechanics for the outcome at the intermediate time t is identical in both experiments, contradicting the authors' claim that "the results at [the intermediary time] t depend not only on what happened earlier at t_0, but also on what happens later at t_1."

To illustrate their arguments, the authors describe some measurements of polarization with spin-½ particles as follows: "We could, for example, start at t_0 with an ensemble of spin-½ particles, each one polarized 'up' in the z-direction. Then at t_1 we measure each spin in the x-direction and select only the particles for which the spin turned out to be up again, but in the new direction. Thus, at any intermediate time t, the spin components in both the z and the x directions—two noncommuting observables—would seem to be completely determined." The rationale given for that strange conclusion is that "if at t we measure the spin along x, we must also find it up [along x], because otherwise the measurement at t_1 wouldn't find it up [along x]." But that claim is incorrect. Selecting, after a measurement, a subset of particles with spin up along a given axis does not imply that before the measurement such particles had spin up along that axis. On the contrary, if some particles at t_1 also emerge with spin down along x, then, according to quantum mechanics, the state Ψ at $t < t_1$ does not represent particles polarized along the x-direction. To find that state, the axis of the measurement device—for example, a Stern–Gerlach magnet—must be rotated until all particles emerge with the same direction of polarization. For the example under consideration, that would be the direction of the z-axis, which corresponds to the polarization at t_0 and t. Hence, a measurement at any later time t_1 is not lost information; instead, it is redundant information about the outcome of a measurement at an intermediate time t when at the initial time t_0 the particles are in a state Ψ.

The claim by Aharonov and coauthors that at various stages of the measurement process ensembles can be separated into subensembles that can be associated with quantum states leads to contradictions with the principles of quantum mechanics, and gives rise to the paradoxes of "impossible ensembles" discussed in the article. Their unphysical description of the measurement process leads them to the false conclusion that "quantum mechanics offers a place to specify both an initial *and* an independent final state," and to such outlandish statements as the idea "that quantum mechanics lets one impose . . . a putative final state of the universe."

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"A Time-Symmetric Formulation of Quantum Mechanics" by Yakir Aharonov, Sandu Popescu, and Jeff Tollaksen is riddled with errors. That is not surprising, given that their starting point is an erroneous "postselection" process. Postselection ignores the measurement postulate of standard quantum physics, according to which a system's quantum state abruptly switches, upon measurement, into an eigenstate of the measured observable.

With that postulate abandoned, it's to be expected that Aharonov and coauthors obtain results that persistently violate the standard quantum uncertainty principle. For example, in their discussion on page 28 of a spin-½ system, they incorrectly state that "the measurement of $S_{\pi/4}$ [the spin component along the diagonal between the x-axis and z-axis] is effectively a simultaneous measurement of S_x and S_z." If that were true, it would violate the uncertainty principle. But the statement is not true. A measurement of $S_{\pi/4}$ could be made by positioning a Stern–Gerlach apparatus with its inhomogeneous magnetic field in the $\pi/4$ direction, without measuring either S_z or S_x. In fact, the measurement would put the system into an eigenstate of $S_{\pi/4}$, leaving both S_z and S_x indeterminate, in agreement with the uncertainty principle.

Aharonov and coauthors continue: "The idea that they [S_z and S_x] are both well defined stems from the fact that measuring *either one* yields +½ with certainty." But that supposed violation of the uncertainty principle is wrong. Using an obvious notation, in their example the system is claimed to be in the eigenstate $|+S_z\rangle$ at time t. This eigenstate can also be written as $(|+S_x\rangle + |-S_x\rangle)/\sqrt{2}$, showing that the spin component S_x is indeterminate whenever the system is in the state $|+S_z\rangle$, in agreement with the uncertainty principle.

One sentence later, the authors state, "If we first measure S_z and then S_x, . . . then, given the pre- and postselection, both measurements yield +½ with certainty." That supposed violation of the uncertainty principle is wrong. If we first measure S_z, we'll get +½ because the system was previously prepared in that state. But if we then measure S_x,

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we'll find the result +½ with 0.5 probability and −½ with 0.5 probability, consistent with the uncertainty principle. To use the misleading "ensemble" language of Aharonov and coauthors, every member of the ensemble is in the state $|+S_z\rangle = (|+S_x\rangle + |-S_x\rangle)/\sqrt{2}$. Contrary to the authors' postselection process, it's not true that 50% of the ensemble is in the state $|+S_x\rangle$ (which would violate the uncertainty principle), and 50% is in $|-S_x\rangle$. To misinterpret quantum superpositions such as $(|+S_x\rangle + |-S_x\rangle)/\sqrt{2}$ in this manner is an elementary misconception. It also directly contradicts the experimental facts about measurements of the Stern–Gerlach type.

Contrary to the assertion of Aharonov and coauthors on page 32 that they "have not modified quantum mechanics by one iota," their postselection process would change the foundations of quantum mechanics. The fallacy in that process was pointed out by Asher Peres 16 years ago.[1]

Reference

1. A. Peres, *Phys. Lett. A* **203**, 150 (1995).

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The article by Yakir Aharonov, Sandu Popescu, and Jeff Tollaksen is stimulating and raises some interesting and profound issues regarding the foundations of quantum mechanics and quantum measurements. One point the authors make is that a measurement of spin component $\sqrt{2}/2$ of a spin-½ particle is unphysical and can be attributed only to errors in weak measurements performed on a collection of N spins. I offer a more mundane interpretation that does not require the introduction of any error or postselection concepts. The physical, textbook picture of spin-½ is a vector of length $\sqrt{S(S+1)} = \sqrt{3}/2$ with some distribution of orientations (that is, polar coordinates θ and φ). A conventional single measurement of S_z can yield only one of the eigenvalues ±½. This may be interpreted in the classical vector model by envisioning that the spin lies on a cone with θ defined by $\cos\theta = \sqrt{3}/3$. The expectation value of S_x then vanishes due to the uniform distribution of φ. Having some control over φ can yield a finite value of S_x. Thus I would argue that only an expectation value greater than $\sqrt{3}/2$ is unphysical; a value of $\sqrt{2}/2$ is quite physical and can even be interpreted classically in terms of some distribution of φ and θ.

I would phrase the important observation of Aharonov and coauthors in a different way. While spin-½ has a magnitude of $\sqrt{3}/2$, a fundamental limitation of conventional *single-point* quantum measurements is that they can yield only the values +½ and −½ and any expectation value must therefore lie between those two extremes. Classically, therefore, the spin may not be fully aligned along the z-axis with $\theta = 0$. Such alignment is impossible since it would yield well-defined values of the three noncommuting variables $S_z = \sqrt{3}/2$ and $S_x = S_y = 0$. However, once multiple time measurements are performed, one can define a reduced distribution of some of the measurements that is conditional on the outcome in the other measurements; such a distribution can be interpreted in terms of S_z greater than ½ but less than or equal to $\sqrt{3}/2$. That interpretation does not violate any of quantum mechanics' fundamental rules, which do not usually consider such quantities.

Rather than a new, time-symmetric formulation of quantum mechanics that involves pre- and postselection, one can simply treat the scheme of the authors' figure 1b as a three-point correlation function, whereas figures 2a and 2b show a four-point correlation function, each panel having its own set of conditional probabilities. Error, time reversal, and postselection need not be invoked. Instead, one may think in multiple dimensions and develop the right language for the interpretation of the observables.

The combination of pre- and postselection is an attempt to create an artificial ensemble that reproduces the results of multiple-point measurements in a single one-point measurement. As shown by Aharonov and coauthors, that is possible by introducing errors. An alternative physical picture is obtained by retaining the multipoint analysis. Multidimensional thinking is well developed in coherent nonlinear spectroscopy, where a system of spins or optical chromophores are subjected to sequences of short impulsive pulses.[1,2] Similar ideas may be applied for the interpretation of multiple measurements. One can think of various types of n-point observables obtained by combining $n - m$ perturbations and m measurements. The nonlinear n-point response functions in spectroscopy represent $n - 1$ impulsive perturbations followed by a single measurement. The objects Aharonov and colleagues considered correspond to $n = m$. Proper multiple-distribution functions could then be naturally used for the interpretation of such generalized measurements.

continued on page 62