

Gaping holes in a gorgeous web of theorems

Perspectives in Computation

Robert Geroch

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Reviewed by William H. Press

True or false:

1. There exist real numbers between 0 and 1 that cannot be computed—or even approximated—by any finite algorithm.
2. An “ideal” quantum computer can compute some problems that are not calculable by an “ideal” classical computer.
3. Among problems that are computable both classically and quantum mechanically, some can be solved much faster with quantum computation (asymptotically, with respect to problem size).

The answers (acknowledging some imprecision in the questions) are

1. True.
2. False.
3. Nobody knows.

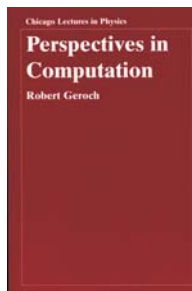
Did you pass? If not, you need to read mathematical physicist Robert Geroch's *Perspectives in Computation*, a short, beautiful set of seminar lecture notes for physics graduate students on the theory of computing with an emphasis on the flowering field of quantum computing. *Perspectives in Computation* is not an encyclopedic treatment; the book's references to the literature are sparse. Rather, it is a carefully constructed single story line presented with outstanding clarity. It contains few equations but many carefully conceived logical arguments.

The first half of the book discusses material covered in upper-level undergraduate texts on computer science theory, including Turing machines, formal

languages, and difficulty functions. However, few computer scientists will approve of Geroch's style: He presents some key proofs as plausibility arguments, then shows their flaws; only after that does he reveal his true intent, which is to prove the negative of the original argument! That kind of playfulness in print may be appealing to physicists, who love intuition-building exercises, but it may be less so to others. I'm reminded of Richard Feynman's self-reported ability to best mathematicians at guessing whether their conjectures were true or false.

Occasionally, Geroch truly goes rogue, as when he makes a remark in passing about the notion “X never happens.” That notion is central to the “halting problem” (the question of whether a computer program ever stops) and to computability theory (what can and cannot be computed) in general; Geroch proposes that “X never happens” could be formally replaced in any theorem by the nonce word “swerm” without any loss of formal rigor. Here, and in other places, he is the physicist reflecting on whether we might be overstating the applicability of formal truths to realizable systems. In another eccentric sidebar, Geroch muses on the possibility that a physical theory, perhaps quantum gravity, might predict for the result of an experiment a noncomputable number—a value that might be measured to arbitrary precision but never confirmed theoretically.

After the first half's classical introduction, Geroch turns his story line to quantum computing. The essence of quantum mechanics—Hilbert spaces, unitary evolution operators, Hermitian measurement operators, and probabilistic measurements—is economically summarized in two pages. The Grover construction to find a needle in a haystack of size N is taken as a canonical quantum algorithm—indeed, it is the only specific quantum algorithm discussed at any length—and it is beautifully explained. The importance of actually constructing the initial state and operators by manipulations whose



complexity is no larger than claimed for the algorithm in toto is emphasized. One glaring shortcoming is the lack of a rigorous discussion of the quantum error correction problem, which cries out for the same kind of lucid treatment that Geroch brings to other areas.

Perhaps a central theme of the book is that despite the hard work of several generations of computer scientists who have produced a gorgeous web of theorems, both classical computation theory and quantum computation theory contain gaping holes. According to Geroch, computability theory is “in excellent shape,” but not so for knowing a problem's *difficulty* in any useful way. Indeed, not a single problem of interest—classical or quantum—has a good, known lower bound on its difficulty, and we don't even know for sure whether quantum entanglement is essential to quantum computing. If a quantum algorithm ever proves to be faster than its fastest possible classical counterpart, it would remain to prove that there is no equally fast probabilistic algorithm, with quantum properties used only as the random number generator. And not much is known about how one might hope to construct such a proof.

If the intent of *Perspectives in Computation* is to leave experimental physics graduate students enthusiastic, energized, and ready to get to work and to leave theoreticians, whether physicist or computer scientist, dejected at the refractory difficulty of the field, it achieves its goal. The book is an eccentric and rewarding tour de force.

Scientific Computation

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Computational science—the third branch of science, joining experimental

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