Filming vortex lines reconnecting in a turbulent superfluid

Tiny tracer particles of hydrogen ice attach themselves to quantized vortex lines in superfluid helium-4, making it possible to study quantum turbulence with unprecedented resolution.

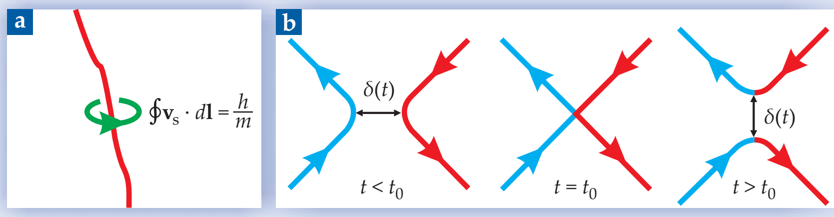

Figure 1. Quantized vortex lines in superfluids. **(a)** Circulation of superfluid around the vortex line is quantized so that the contour integral of its velocity around any loop enclosing one line is h/m, where m is the relevant boson mass for the particular superfluid. **(b)** Reconnection of two vortex lines at time t_0. The reconnection dynamics can be described by $\delta(t)$, the time dependence of their smallest separation. Colored arrowheads indicate the sense of the superfluid circulation around the vortex lines. (Adapted from ref. 1.)

The classical physics of turbulence in ordinary liquids is difficult enough. But turbulence in superfluids piles on its own special problems. Being manifestations of Bose–Einstein condensation, superfluids are subject to quantum constraints that might seem to thwart turbulence. (See the article by Joe Vinen and Russell Donnelly in PHYSICS TODAY, April 2007, page 43.)

Turbulence is largely characterized by the random formation of rotating eddies. So when Richard Feynman in 1955 suggested superfluid turbulence as a theoretical possibility, he had to address the constraint that the curl ($\nabla \times \mathbf{v}_{sf}$) of a superfluid's velocity field $\mathbf{v}_{sf}$ must vanish everywhere. Laszlo Tisza and Lev Landau had already explained that superfluids can be regarded as two coexisting and interpenetrating components: a normal viscous component and the inviscid (zero viscosity) superfluid component, each with its own independent velocity field. The superfluid fraction grows from zero at the transition temperature T_c to 100% at absolute zero.

Feynman argued that the superfluid component could exhibit rotational flow despite the required vanishing of curl $\mathbf{v}_{sf}$ if the turbulence created quantized vortex lines—linear topological defects along which the superfluid component vanishes. He added that reconnection of such vortex lines, when two of them encounter each other, might solve a related problem: Once turbulence has begun in the superfluid component, how can it dissipate in the absence of viscosity?

It wouldn't be long before Joe Vinen experimentally demonstrated turbulence in superfluid helium-4 and found that the contour integral $\oint \mathbf{v}_{sf} \cdot d\mathbf{l}$ of the rotational flow is quantized, as Feynman had predicted, in units of h/m, where m is the ^{4}He atom's mass. Turbulence in superfluids has now been studied in the laboratory for half a century, mostly in ^{4}He and more recently in ^{3}He. But until 2006 no one had directly observed a quantized vortex line freely moving in a superfluid, let alone a reconnection event.

In that year Gregory Bewley, a Yale graduate student working in Daniel Lathrop's lab at the University of Maryland, College Park, developed a technique for rendering quantized vortex lines in He II visible and recording their movement. (Liquid ^{4}He cooled below its T_c of 2.17 K is commonly called He II.)

"When Greg and I actually saw the first reconnection event," recalls Lathrop's student Matthew Paoletti, "it was a Eureka moment." Now Paoletti, Lathrop, and theorist Michael Fisher have reported their analysis of some 40 000 vortex reconnection events "filmed" in turbulent He II with Bewley's technique.[1]

The observed reconnection dynamics agree well with what one expects if the principal determining parameter is the "quantum of circulation," $\kappa \equiv h/m$. And the experiment has yielded the first clear evidence that a turbulent superfluid's velocity distribution differs markedly from what one finds in classical turbulence.

Recording reconnections

In He II, the effective thickness of a quantized vortex line—the distance over which the superfluid component falls to zero—is just a few angstroms. The circulation $\oint \mathbf{v}_{sf} \cdot d\mathbf{l}$ around a loop that encloses a single vortex line is κ, irrespective of the loop's size (see figure 1a). So the circulating superfluid velocity falls off linearly with distance from the line. For He II, κ is close to 0.1 mm^2/s.

The reconnections Feynman envisioned, illustrated in figure 1b, are not unlike what happens when field lines reconnect in magnetized astrophysical and fusion plasmas (see the article by Forrest Mozer and Philip Pritchett, PHYSICS TODAY, June 2010, page 34). To investigate the dynamics of reconnection in turbulent He II, Lathrop and company primarily sought to record how the separation between two reconnecting vortex lines changes in the moments before and after the reconnection.

To create and visualize quantized vortex lines, the Maryland experimenters inject a room-temperature gas of ^{4}He and a little hydrogen into quiescent liquid ^{4}He just above its T_c in a cryostatic vessel with portholes for laser illumination and filming. The injected hydrogen promptly freezes into a snow of reflective, micron-sized tracer particles. At that moment the team drops the temperature below T_c and begins alternately agitating the He II to turbulence and then digitally filming the tracer particles at about 80 frames per second as the superfluid relaxes back to quiescence.

As Lathrop and company reported in 2006, the tracer particles attach themselves firmly to the quantum vortex lines generated by the turbulence.[2] The pressure gradient that drives them there is a Bernoulli force. It reflects the minimization of kinetic energy when a particle displaces the fastest-flowing superfluid nearest the vortex lines. The particles themselves don't participate in the circulation, because the superfluid, by its nature, exerts no shear force on them. Their micron size is a double bonus: It renders the lines visible with excellent resolution, and it does so with minimal effect on their dynamics.

In the newly reported experiment, Paoletti filmed the relaxation of turbulence in He II at various temperatures down to 1.7 K. In each cycle he generated turbulence by passing current through a heating coil at the bottom of the cryostat for 5 seconds. Because only the normal component absorbs heat, it alone is driven upward. But that convective flow displaces the superfluid downward in a counterflow that induces turbulence and generates a profusion of quantized vortex lines. Each time the heating current stopped, 10 seconds of filming began.[3]

The reconnection analysis is based on about a thousand such minimovies. Figure 2 shows a close-up of one reconnection event, atypical only in that, for display purposes, this cycle was given an unusually high dose of tracer particles. Because the particles do have some small effect on the dynamics, one wants the minimum number that will do the trick.

With almost a million digitized frames in hand, Paoletti needed an efficient search algorithm for finding event candidates. Reconnection events take about half a second, and they exhibit very fast approach and separation near the crossover point. So the algorithm sought out pairs of tracer particles whose separation either shrank or grew at least fourfold within 0.25 s. A hundred thousand such pairs yielded the 39 000 reconnection events on which the analysis is based.

Analyzing the dynamics

The principal record abstracted from each event was $\delta(t)$, the time dependence of the minimum separation between the two reconnecting vortex lines (see figure 1b). If κ is the only relevant physical parameter in reconnection dynamics, dimensional analysis leads one to expect that

$$\delta(t) = A(\kappa|t - t_0|)^{1/2}, \qquad (1)$$

where t_0 is the instant of reconnection and A is a dimensionless coefficient of order 1. The equation's unphysical ve-

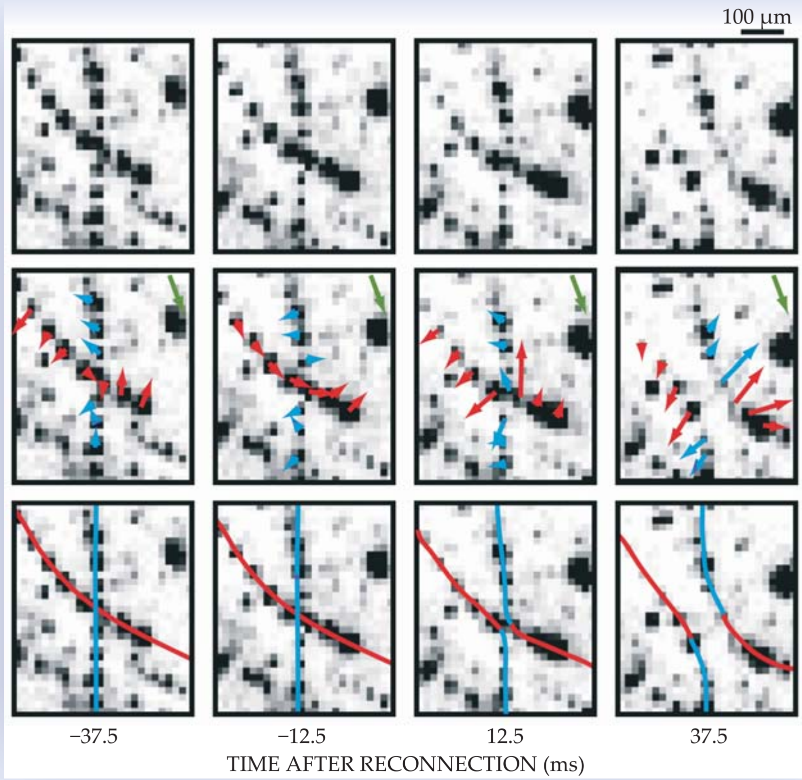

Figure 2. Close-ups of a reconnection event in superfluid helium-4 digitally filmed in an experiment at the University of Maryland. Reflective tracer particles adhering to the reconnecting vortex lines make them visible in these negative images. In the middle row, the green arrow indicates the local background flow velocity, and the red and blue arrows give background-subtracted velocities for the two vortex lines whose reconnection is delineated by the colored curves in the bottom row. (Adapted from ref. 1.)

locity singularity at t_0 makes it, at best, an asymptotic approximation. But it was deemed suitable for analyzing the Maryland experiment with its 0.01-second intervals between frames. And, indeed, the equation describes the data quite well. "That was our major finding," says Lathrop. "But a rather broad event-to-event fluctuation in the value of A required further attention."

The approximate validity of equation 1 for measurement times relative to t_0 ranging from about 0.01 to 0.25 s implies that the quantum of circulation is, in fact, the dominant parameter in that range. At shorter times one might expect the vortex diameter, its surface tension, or the limiting sound velocity to come into play. And at longer times, the typical spacing between vortex lines, which in this experiment was a fraction of a millimeter, could become important.

Tentatively attributing the observed event-to-event fluctuation to environmental variables like the proximity of the nearest nonparticipating vortex line, the team fitted the data with a one-parameter correction factor appended to the idealized form of equation 1. Fitting each reconnection event separately with the corrected form

$$\delta(t) = A(\kappa|t - t_0|)^{1/2}(1 + c|t - t_0|) \qquad (2)$$

yields the distributions of the free parameters A and c shown in figure 3.

The rather broad A distribution peaks near 1, as expected. The c distribution peaks at zero, where the correction vanishes. But the mean-square value of c can be interpreted as reflecting a characteristic length, which turns out to be about 0.4 mm. That's a typical spacing between vortex lines in the Maryland experiment, strengthening the idea that spectator vortices nearby are major sources of fluctuation. Temperature variations from run to run, on the other hand, appear to have little effect on the reconnection dynamics.

Dissipation and time reversal

For most events, the digitized data were not equally good before and after t_0. So the analysis divided the 39 000 reconnection events into two groups of roughly equal population: "forward" events, whose reconstruction was based primarily on movie frames

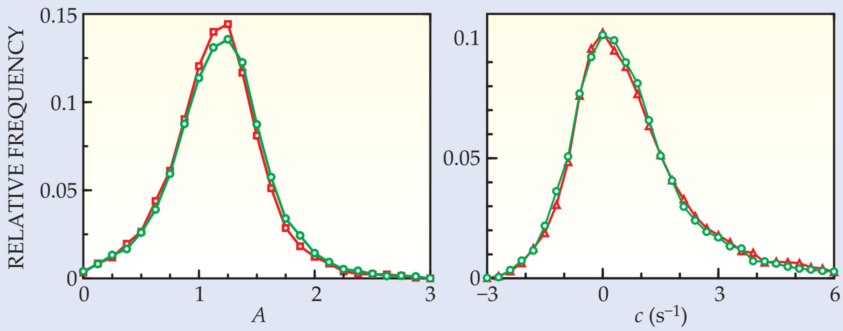

Figure 3. Separately fitting each of the Maryland team's 39 000 reconnection events with the free parameters of equation 2 yields the distributions shown here. The distribution for the dimensionless amplitude parameter *A* peaks near 1 as expected. But its width bespeaks considerable fluctuation from event to event. The width for the correction parameter *c* suggests that much of that fluctuation is due to spectator vortex lines near the reconnection site. The forward (green) and reverse (red) event distributions, as defined in the text, are almost identical, which argues for time-reversal invariance in the dynamics of reconnection. (Adapted from ref. 1.)

before t_0, and "reverse" events based on post-t_0 frames. The separate curves in figure 3 for the two classes show essentially no distinction. This apparent time-reversal symmetry implies that—unlike magnetic reconnections—the reconnection of quantized vortex lines in superfluids generates little or no energy dissipation. The widely used Gross–Pitaevskii theoretical approximation of vortex reconnection in Bose fluids is explicitly time-reversal invariant.

How then can dissipationless reconnection contribute to the relaxation of turbulence in He II? In the two-component superfluid, a quantized vortex line can dissipate energy by friction as it moves through the normal component. But without reconnection, turbulence would produce a tangle of vortex lines so dense that no line could move. Feynman pointed out that reconnection would free up the tangle. In fact, he noted, vortex *loops* created in double-reconnection events could travel with particular ease.

At lower temperatures, other dissipation mechanisms must become important as the normal component dwindles. It's thought that the twang of reconnection excites the vortex lines to helical wave motion whose highest-frequency components would dissipate energy by generating phonons in the superfluid even in the absence of a normal component.

"We haven't tested how dissipation occurs near absolute zero," says Fisher. "But what we have found, by looking for the first time with resolution good enough to see quantized vortices in action, is that superfluid turbulence is a new beast, quite different from the classical turbulence it appears to resemble at lower resolution." In turbulent classical liquids, the distribution of velocities is essentially Gaussian. But for superfluids, the quantum theory predicts a much larger high-velocity population generated by reconnection.

Sure enough, Lathrop and company find that the velocity distribution—for all tracer particles, whether or not they're involved in reconnections—has the $1/v^3$ high-velocity tail implied by the reconnection dynamics.

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References

1. M. S. Paoletti, M. E. Fisher, D. P. Lathrop, *Physica D* (in press).
2. G. P. Bewley, D. P. Lathrop, K. R. Sreenivasan, *Nature* **441**, 588 (2006).
3. A movie clip with reconnection events is available at http://prl.aps.org/epaps/PRL/v101/i15/e154501/paoletti_movie1.mov.

Laboratory experiment shows that noise can be lessened for LISA

With a proposed spaceborne interferometer that dwarfs the Moon's orbit, researchers hope to detect gravitational waves. To do that, they need to eliminate the noise from laser frequency fluctuations.

Just as Maxwell's equations imply that an accelerating charge produces electromagnetic radiation, Einstein's theory of general relativity predicts that an accelerating mass produces gravitational radiation. As a gravitational wave propagates at the speed of light, it stretches space in one direction and compresses it in another. But because gravity is so weak, those distortions are minuscule, and extraordinary sensitivity is required to detect them.

The Laser Interferometer Space Antenna (LISA) is a proposed mission to look for gravitational waves through their effect on the distances between three spacecraft. The spacecraft would form, in essence, a Michelson interferometer 5 million kilometers on a side—more than 10 times the distance from Earth to the Moon. Researchers hope to be able to measure oscillations as small as 10 picometers in the interferometer's arm lengths.

Many technical challenges stand in LISA's way. One of the biggest has involved laser phase noise: Even with the laser frequencies stabilized as much as possible, they still exhibit fluctuations that are a billion times larger than the signal. In 1999 John Armstrong, Frank Estabrook, and Massimo Tinto of NASA's Jet Propulsion Laboratory (JPL) presented the theory for a method, called time-delay interferometry (TDI), of eliminating that noise through signal processing.[1] Now, a team of JPL experimentalists led by William Klipstein has shown in a laboratory demonstration that TDI can indeed reduce LISA's noise to the necessary extent.[2]

LISA and LIGO

Indirect evidence for gravitational waves is strong. Pulsars orbiting companion stars lose energy at exactly the rate predicted by general relativity. Researchers looking to detect gravitational radiation are less interested in proving the waves' existence than in the information they can provide: about merging black holes, core-collapse supernovae, galaxy formation—and the first 380 000 years after the Big Bang, a period when the universe was opaque and from which no electromagnetic information survives.

Different sources produce gravitational radiation of different frequencies. Ground-based interferometers, such as the Laser Interferometer Gravitational-Wave Observatory (LIGO), can be sensitive to frequencies from tens to thousands of hertz. (See the article by Barry Barish and Rainer Weiss in Physics Today, October 1999, page 44.) At lower