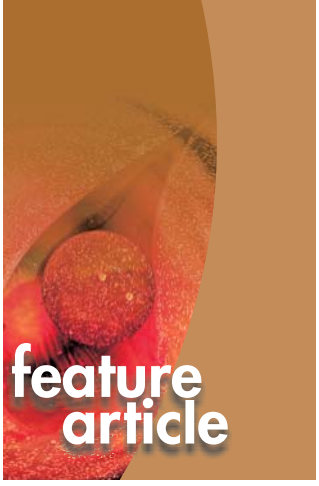

What black holes teach about strongly coupled particles

Clifford V. Johnson and Peter Steinberg

Classical systems with strongly interacting particles can, in principle, have zero viscosity. But string theory and experiment suggest that in the quantum world, the viscosity can be only so low.

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The notion of a perfect fluid arises in many fields of physics. The term can be applied to any system that is in local equilibrium and has negligible shear viscosity η. In everyday life, viscosity is a familiar property associated with the tendency of a substance to resist flow. From a microscopic perspective, it is a diagnostic of the strength of the interactions between a fluid's constituents. The shear viscosity measures how disturbances in the system are transmitted to the rest of the system through interactions. If those interactions are strong, neighboring parts of the fluid more readily transmit the disturbances through the system (see figure 1). Thus low shear viscosities indicate significant interaction strength. The ideal gas represents the opposite extreme—it is a system with no interactions and infinite shear viscosity.

Perfect fluids are easy to describe, but few substances on Earth actually behave like them. Although often cited as a low-viscosity liquid, water in fact has a substantial viscosity, as evidenced by its tendency to form eddies and whorls when faced with an obstacle, rather than to flow smoothly as in ideal hydrodynamics. Even the famous helium-3, which can flow out of a container via capillary forces, does not count as a perfect fluid.

Two remarkable fluids

Considering how useful superfluids have proved to be as test beds for fundamental quantum mechanics, one might expect the study of truly perfect fluids to be of great value. It is thus fortuitous that over the course of the past five years, investigators have discovered systems that seem to approach perfect-fluid behavior. Experimenters at Brookhaven National Laboratory's Relativistic Heavy Ion Collider (RHIC) reported in 2005 that collisions of gold nuclei produced a liquid state of strongly interacting matter with a shear viscosity so low that it was effectively negligible.[1] At about the same time, a group at Duke University reported that a system of strongly coupled lithium atoms behaved nearly perfectly.[2] (New work suggests that other experimental systems—for example, graphene[3]—are also displaying perfect-fluid behavior.)

The apparent near perfection of the RHIC and Duke fluids—one with a temperature exceeding 2 trillion kelvin, the other a few microkelvin above absolute zero—means that the usual intuitions based on perturbation theory don't apply. The perturbative approach proceeds from well-defined particles that have an associated mean free path. However, the typical mean free paths required to obtain the observed low viscosities are so short that they are of the same order as the thermal and quantum fluctuations of the system. Thus many kinetic theory estimates and perturbative computational techniques are doomed to fail (see figure 1). Consider, for example, quantum chromodynamics, the fundamental theory of the strong nuclear interactions of quarks and gluons. Perturbative QCD has demonstrated successes in explicating a wealth of collider data, but it has not been as successful at describing the physics at RHIC.

As with the theories describing the electromagnetic and weak nuclear forces, QCD has a local internal symmetry group—in its case, $SU(3)$—and is commonly referred to as a gauge theory. (For a quick tutorial and definitions of terms, see the box on page 31.) Gauge theories are the cornerstones of the standard model of particle physics, but most of their remarkable quantitative success has been in the perturbative regime. QCD is perturbative when the interacting particles transfer a lot of momentum, a property called asymptotic freedom. But otherwise, the quarks and gluons described by QCD are strongly coupled. As a result, quarks are bound together into hadrons such as protons and neutrons. Systems of strongly coupled quarks and gluons are hard to describe in detail using QCD, and the strong-coupling behavior seen in the RHIC experiments is a profound example of a new phenomenon that needs to be understood. The powerful numerical techniques that enable physicists to extract some nonperturbative physics from QCD can access only a small subset of the observed phenomena. A more detailed understanding requires new tools.

In view of asymptotic freedom, the properties of the material created at RHIC came as something of a surprise. Asymptotic freedom suggests that a new phase of quark–gluon matter should form when temperatures are high enough that the coupling between quarks and gluons decreases and the particles are liberated from being bound inside hadrons. That new phase of matter is known as the quark–gluon plasma (QGP). To estimate the phase-transition temperature, theorists turn to lattice QCD, a technique in which the quark and gluon fields are placed on a discrete spacetime grid and powerful computers are then used to calculate transition matrix elements (see the article by Carleton DeTar and Steven Gottlieb in PHYSICS TODAY, February 2004, page 45). Lattice QCD is a natural complement to perturbative QCD and allows for

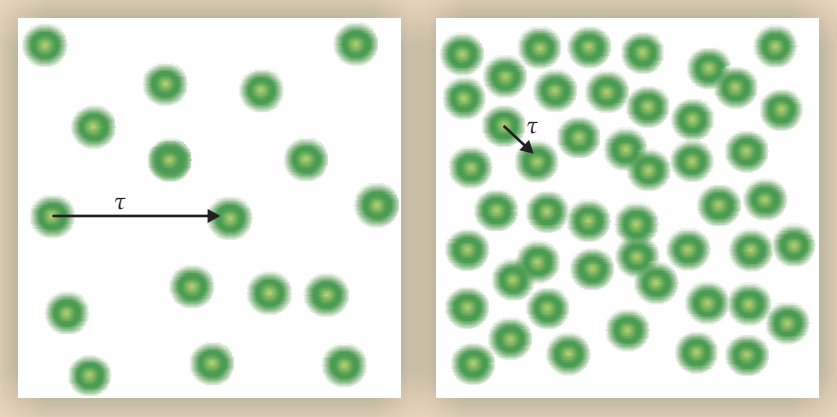

Figure 1. Strongly interacting particles such as those depicted in the right-hand panel typically have a short mean free path and mean free travel time τ before they interact with others. The system of weakly interacting particles illustrated in the left panel is amenable to traditional perturbative calculational techniques, but as τ decreases, particles begin to lose their individual identity and those methods begin to break down.

the computation of some physical quantities—for example, the energy density at nonzero temperature—even for strongly coupled particles. It predicts a phase transition from hadrons to the QGP at about 2×10^{12} K.

The primary goal of RHIC, which collides heavy nuclei together, was to find experimental evidence of the QGP and study its properties. RHIC scientists expected to turn up evidence of a weakly coupled gas of quarks. However, their scrutiny of the first several years of data revealed that the azimuthal distribution of emitted particles has an elliptical shape; that is, it suggests a collective push in the direction of the largest matter-density gradients. An ideal gas boils away nearly isotropically, with particles emitted equally in all directions. The RHIC system was better modeled as a liquid with an extremely low shear viscosity. That discovery, coupled with the observation of a phenomenon called jet quenching (a consequence of the opacity of the system to fast quarks), led the heavy-ion community to announce in 2005 that whatever RHIC produced was a strongly coupled system, something for which theoretical tools did not yet exist.

Behavior suggestive of a perfect fluid was observed at about the same time by a group at Duke studying droplets of ultracold ^{6}Li atoms. There, the system was tuned to the strongly coupled crossover region between two characteristically different regimes, one with a Bose–Einstein condensate of molecular pairs and the other characterized by the Bardeen-Cooper-Schrieffer behavior associated with the formation of Cooper pairs of atoms. A later explicit measurement of the shear viscosity of the system yielded an extremely low value, just as for RHIC.

The following two articles, on pages 34 and 39, discuss the Duke and RHIC experiments in greater detail and present the evidence that the systems observed are nearly perfect fluids. In this piece, we focus on what we regard as the true surprise in the strange empirical convergence of the ultrahot and ultracold: the apparent ability of string theory to describe the emergent strong-interaction physics in terms of properties of a black hole living in a higher-dimensional spacetime. A number of key physical properties of the RHIC and Duke systems seem to be best captured in terms of those unexpected new variables, which may herald an entirely new class of techniques for studying certain types of quantum systems at strong coupling.

The holographic principle

Over the past 12 years, the string theory community has introduced an unexpected component into the study of gauge theories: quantum gravity. At the core of the new technique is the holographic principle, a conjecture formulated by Gerard 't Hooft[4] and further developed by Leonard Susskind.[5] The principle asserts that any theory of quantum gravity in a $(d + 1)$-dimensional spacetime has an equivalent description in terms of a theory that can roughly be thought of as living on the spacetime's d-dimensional boundary.

The holographic principle was motivated by results obtained when physicists applied the rules of quantum mechanics to black holes, which are described by general relativity. That work began in the early 1970s with semiclassical efforts by Stephen Hawking, Jacob Bekenstein, and others, and culminated in the 1990s with work in a full theory of quantum gravity—string theory.

Although classical black holes have a surface of closest approach from within which nothing can escape—the event horizon—Hawking discovered that when quantum mechanics is put in play, a black hole emits radiation at a definite temperature set by its mass–energy.[6] Bekenstein, realizing that a black hole with temperature should obey the laws of thermodynamics, argued that it should have a well-defined entropy.[7] The resulting Bekenstein–Hawking entropy formula relates a black hole's entropy S to the area A of its event horizon, $S = A/4 \cdot (k_B c^3/G\hbar)$. Here k_B, G, and $\hbar$ are the Boltzmann, gravitational, and Planck constants, respectively. That striking result—the entropy is proportional to the area of the horizon bounding a black hole, and not to the volume encompassed by the horizon—suggests that the number of degrees of freedom needed to describe a quantum black hole likewise scales with its area, not its volume. In other words, a black hole creates a "hologram" of the physics of the spacetime region inside it, which is projected onto the black hole's boundary, the horizon.

Since a black hole generically forms if the mass–energy density is high enough, it might be expected that any theory of quantum gravity would exhibit a holographic nature when one tries to determine its degrees of freedom by probing short distances with high energies. From that expectation emerged the idea that a quantum gravitational theory has an alternative description in terms of a nongravitational theory—a "holographic dual"—in one fewer dimension. Since, peturbatively, quantum gravity is fundamentally different from nongravitational physics, the dual relationship must be subtle. It is, in fact, a strong–weak coupling duality, meaning that when one theory involves weak coupling and thus has a perturbative description, the dual description involves strong coupling, and vice versa.

That quantum gravity arises in the context of strongly coupled systems such as high-temperature quarks and gluons and cold ^{6}Li atoms might at first be confusing. After all, many of us were taught that gravity is much weaker than all

Here comes the *SU*(*N*)

Gauge theories are invariant under gauge transformations, symmetry operations that can be implemented independently at each point in space and time. The most familiar example of a gauge transformation comes from classical electrodynamics, in which the scalar (V) and vector ($\mathbf{A}$) potentials can be changed via $V \rightarrow V - (1/c)\,\partial\Lambda/\partial t$, $\mathbf{A} \rightarrow \mathbf{A} + \nabla\Lambda$ without altering the physical electric and magnetic fields. In the transformation, Λ is some arbitrary function, and c is the speed of light; for simplicity, we don't explicitly indicate the dependence of Λ on space and time.

The gauge transformation, with its derivatives, may look unusual. Indeed, its form arises because the scalar and vector potentials are parts of a geometric object called a connection. To understand the significance of the connection, consider gauge transformations in the context of quantum electrodynamics. There, the electron–positron field ψ transforms according to the more normal-looking $\psi \rightarrow \exp(i\Lambda)\psi$. The equations of QED, however, include derivatives of ψ, and as a result the phase multiplication introduces derivatives of Λ that threaten to ruin the invariance. The vector and scalar potentials save the day: The derivative terms in their transformations cancel the potentially ruinous terms that come from the phase multiplication.

Gauge transformations can be applied sequentially. In QED, that would lead to the transformation $\psi \rightarrow \exp(i\Theta)\exp(i\Lambda)\psi$, with two distinct gauge functions Λ and Θ. Clearly, the order in which the phase multiplications are applied does not matter, either for the field ψ or for the scalar and vector potentials; in the jargon of the field, the QED gauge transformation is said to be abelian.

In general particle-physics scenarios, ψ represents a vector-like collection of fields—a triplet of quarks or an octet of gluons, for example—and the gauge transformation is realized as a matrix multiplication $\psi \rightarrow \mathbf{M}\psi$. Symmetry again requires the introduction of fields, whose transformations involve derivatives of $\mathbf{M}$. Matrix gauge transformations can be compounded, too, but in general, the order does matter; the transformations are nonabelian. Gauge theories built on nonabelian groups are often called Yang–Mills theories.

In many examples from particle physics, $\mathbf{M}$ is an element of the group $SU(N)$. That is, it is an $N \times N$ matrix that is special (meaning the determinant is one) and unitary (the matrix's complex-conjugate transpose is also its inverse). In the simpler QED case, the phase multiplication may be viewed as a unitary 1×1 matrix, so the gauge group appropriate to QED is $U(1)$, which is abelian.

the other forces and that the quantum gravity scale, the Planck scale, is well away from the regime that experiments probe. The simple resolution of the puzzle is that it is not the gravity of our universe that is being employed in dual constructions but one that is essentially an auxiliary theory. We live in a flat universe with, possibly, a small positive cosmological constant, but the holographic constructions relevant to the RHIC and Duke results involve quantum gravity with at least one extra spatial direction and a negative cosmological constant. Quantum gravity in such spacetimes seems to capture key physics of the experimental nongravitational systems of interest.

The AdS/CFT correspondence

The best understood example of holography is the so-called AdS/CFT (anti–de Sitter/conformal field theory) correspondence, formulated by three groups in the late 1990s (see reference 8 and the article by Igor Klebanov and Juan Maldacena, PHYSICS TODAY, January 2009, page 28). Anti–de Sitter spacetimes are the negative-cosmological-constant analogue of the more familiar Minkowski spacetime, the cosmological-constant-free flat spacetime of special relativity. One convenient feature of the $(d+1)$-dimensional AdS spacetime is that its volume (the "bulk") has radial slices that are d-dimensional Minkowski spacetimes. The holographic duality works in part precisely because of that feature: Fields that propagate in the bulk gravitational system have well-defined expectation values at asymptotic infinity in the radial direction, often referred to as the boundary. The asymptotic values behave like fields and couplings in the d-dimensional Minkowski boundary spacetime, defining a self-contained theory in its own right—the holographic dual theory. Amazingly, those dual theories are sometimes gauge theories, familiar to nuclear and particle physicists.

In perhaps the most celebrated case, the quantum gravity theory lives in five dimensions, and the 4D holographic dual is a Yang–Mills theory with a local scale invariance known as conformal invariance (whence the "CFT" of the correspondence). The Yang–Mills theory, in addition, is invariant with respect to supersymmetry (which pairs integer-spin particles with half-integer-spin ones). The gauge symmetry of the theory is $SU(N_c)$; as we will soon show, if the duality is to be useful, N_c must be large. A combination of N_c and the dimensionless Yang–Mills coupling g_{YM} sets the so-called 't Hooft coupling of the theory, $\lambda = g_{YM}^2 N_c$, which determines the effective strength of interactions. Recall that QCD is also a Yang–Mills theory, but in that case $N_c = 3$, corresponding to the three colors of quarks. Moreover, the matter content of QCD—the quarks—is different from the matter content of the supersymmetric Yang–Mills theory. Nonetheless, calculations show that the differences between the two theories are not so stark in the high-temperature regime. Thus the supersymmetric Yang–Mills theory is potentially relevant to the RHIC results.

To motivate the strong–weak coupling duality property of AdS/CFT, we begin by asking what determines whether quantum gravity calculations can be carried out perturbatively. The local strength of gravity in the bulk spacetime is given by the spacetime curvature, which can be characterized by a length scale l_c. As with the radius of a circle, smaller l_c implies greater curvature. String theory, the relevant quantum gravity theory, also has a natural length scale l_s, which is inversely related to the string tension. If $l_s \ll l_c$, then weakly coupled string theory is an appropriate tool for quantum gravity calculations: Theorists compute the relevant quantities as a perturbative expansion in the small dimensionless ratio l_s/l_c.

The remarkable feature of the duality is that the 't Hooft coupling of the boundary Yang–Mills theory is inversely related to that ratio: $\lambda \propto (l_c/l_s)^4$. Thus the perturbative regime on the gravity side corresponds to the strongly coupled regime for the dual Yang–Mills theory. Conversely, if the 't Hooft coupling is small, then the dual gravitational theory requires a fully nonperturbative string theory treatment, since strong-curvature and string-coupling effects cannot be ignored. Moreover, the string coupling and the Yang–Mills coupling are proportional to each other. Given that the string coupling is small in the perturbative regime, the 't Hooft coupling can be strong only if the N_c of the $SU(N_c)$ gauge group is large.

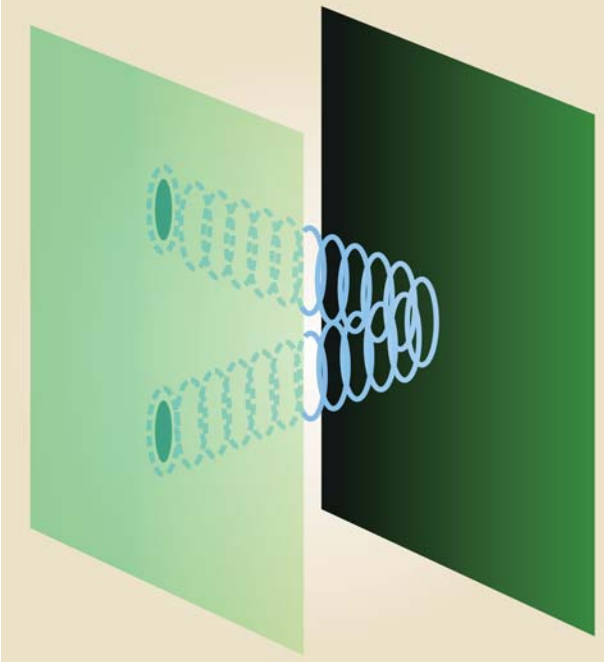

Figure 2. Holographic duality enables hydrodynamic calculations in a gauge theory to be recast as scattering calculations in a higher-dimensional spacetime. The plane to the left represents 3D space, a slice of the 4D spacetime in which the dual gauge theory resides. Motion perpendicular to that plane is in an extra fifth dimension; gravity operates in the bulk 5D spacetime. The plane on the right is a 3D black hole horizon. The hard computation of how disturbances propagate in the gauge theory's 4D spacetime is equivalent to a simpler-to-compute process in which a graviton (blue) scatters off a black hole in the bulk spacetime.

There is a wonderful serendipity here. Holographic dualities were originally discovered in an effort to understand the physics of strongly coupled gravity. Dual weakly interacting gauge theories enabled string theorists to prove the Bekenstein–Hawking entropy formula for a large class of black holes, to give just one example. The dualities are now being turned around so that perturbative-gravity techniques can be employed in the service of studying nongravitational, strongly coupled physics.

The limit to perfection

Lattice QCD has enabled the calculation of certain thermal properties such as the equation of state, but the technique has had great difficulty extracting transport properties. That is because lattice QCD calculations employ a so-called Euclidean time formalism. In that approach, a fixed time corresponds to a definite temperature, and so explicit time evolution cannot be studied. Holographic approaches are not limited in that way. For a thermal system, the dual gravity theory has a black hole in the extra dimension, away from the boundary. The black hole's Hawking radiation is in thermal equilibrium with the entire spacetime and hence with the dual gauge theory defined at the boundary at infinity. Because of the nonzero temperature, the dual theory is neither supersymmetric nor conformally invariant, which allows it to better represent experimental systems of interest.

As discussed earlier, the shear viscosity η reflects interaction strength. Dividing η by the entropy density s of a system gives a measure of the interaction per constituent that better allows comparison across different systems at widely different scales. The viscosity is proportional to both the mean free time τ between collisions of a constituent particle and the energy density ε of the system. The entropy density is proportional to the particle number density n, so the ratio η/s should be something like $\tau E/k_B$, where E is the energy per particle. The product τE, by Heisenberg's uncertainty principle, is of order $\hbar$ or greater, and so the ratio η/s is set by $\hbar/k_B$, with a dimensionless multiplier determined by the details of the substance in question. Strong coupling tends to reduce τ, so strongly coupled fluids will have smaller shear viscosity than weakly coupled ones.

In 2005 Pavel Kovtun, Dam Son, and Andrei Starinets (KSS) used the above dimensional argument, combined with Heisenberg's relation, to motivate the existence of a universal lower bound on η/s that would be approached for strongly coupled systems (see reference 9 and PHYSICS TODAY, May 2005, page 23). The idea grew from their observation (subsequently strengthened by studies from several groups) that, to leading order, a variety of strongly coupled gauge theories with known holographic duals yielded a strikingly small universal value for η/s. The key quantum-gravity computation had been carried out earlier by Giuseppe Policastro, Son, and Starinets:[10] The calculation of how a disturbance propagates from one place to another in the gauge theory may be recast as a calculation in which gravitons from a point on the boundary propagate in the higher-dimensional bulk theory, scatter off a black hole, and return to another point on the boundary (see figure 2). The scattering cross section is proportional to the area of the black hole horizon, and, as Policastro and colleagues showed, the viscosity is proportional to the scattering cross section. According to the Bekenstein–Hawking formula, the entropy is also proportional to the horizon area, so in the ratio η/s, the horizon area cancels to yield $\eta/s = 1/4\pi \cdot \hbar/k_B$. That value holds for specific classes of systems, but KSS conjectured it was a universal lower bound.

Ordinary substances like water, and even most specially prepared laboratory fluids—liquid helium, for example—have η/s ratios that lie well above the KSS value. But the RHIC and Duke systems seem to have an η/s at most only a few times higher than the KSS value and thus far below that of all previously known laboratory systems.

The possibility of a fundamental lower bound on η/s led to several fruitful discussions among physicists working in traditionally disconnected areas and also helped focus diverse experimental efforts. This convergence of experimental and theoretical work is particularly exciting and, notably, is the first dialog between string theorists and experimenters to produce actual experimental results. In a sense, the theory predicted the existence and properties of a class of fluids with extremely low η/s, and experimenters are now finding striking examples of such systems.

Questions of detail

Placing a black hole in a 5D spacetime certainly does not endow its dual 4D finite-temperature Yang–Mills theory with the specific content of finite-temperature QCD. The dual theory has no true quarks, and N_c must be large—not 3, as in QCD. Clearly, theorists would be thrilled to find a gravitational theory that is dual to QCD instead of to a Yang–Mills cousin. However, considerable research effort suggests that the goal will be extremely difficult to achieve. Even relaxing the requirement that N_c be large means that calculations on the string theory side are no longer tractable in general. So the question arises as to why the systems that have been an-

alyzed share some properties with QCD. Is it simply an accident, or is it an encouraging sign of a profound connection, despite differences in details?

The similarity of the experimentally and theoretically determined viscosity-to-entropy ratios suggests that the systems under study are primarily controlled by the temperature; the microscopic details of the physics are less important. In the strongly coupled hydrodynamic regime, the temperature determines the energy, pressure, speed of sound, and even the viscous deviations from perfect-fluid behavior. On the other hand, on the holographic dual side, the spacetime geometry is entirely determined by the temperature, which sets the size of the black hole. Further evidence that microscopic details may not be so important comes from successes in using the dual approach to compute qualitative features of other observed RHIC physics—for example, jet quenching and heavy-quark dynamics—and a qualitative agreement of RHIC physics with that predicted by the lattice QCD equation of state.

Theorists have studied dualities involving various gauge theories to better understand the sensitivity of the key results to differences in detail among those theories. They have learned, for example, that some theories have η/s smaller than the conjectured KSS bound of $1/4\pi \cdot \hbar/k_B$. That value, therefore, should not be thought of as an absolute limit but rather as a universal leading contribution to an expansion in inverse 't Hooft coupling and $1/N_c$. It may be a long while before theorists find holographic duals of exact experimental systems, so they are focusing on better understanding those physical quantities, computable with currently accessible models, that are not strongly sensitive to details. Perhaps that focus will reveal a kind of universality that explains why the computations work so well.

Black holes are rather universal objects in the sense that they are characterized by only a few parameters, such as mass, charge, and angular momentum. Therefore, all gravitational systems, and hence the various nongravitational theories they define via holographic duality—from the ultracold to the ultrahot—should have their thermal physics controlled by the same basic black hole dynamics. If further experimental and theoretical work strengthens the connection of strongly interacting systems to string theory, and if the universality proves robust, then physicists will have a powerful set of concepts and tools with which to explore a whole new class of experimentally accessible phenomena.

References

1. S. Aronson, T. Ludlam, *Hunting the Quark Gluon Plasma: Results from the First 3 Years at RHIC*, rep. no. BNL-73847-2005, Brookhaven National Laboratory, Upton, NY (April 2005), available at http://www.bnl.gov/npp/docs/Hunting the QGP.pdf.
2. A. Turlapov et al., *J. Low Temp. Phys.* **150**, 567 (2008).
3. M. Müller, J. Schmalian, L. Fritz, *Phys. Rev. Lett.* **103**, 025301 (2009).
4. G. 't Hooft, in *Salamfestschrift: A Collection of Talks from the Conference on Highlights of Particle and Condensed Matter Physics*, A. Ali, J. Ellis, S. Randjbar-Daemi, eds., World Scientific, River Edge, NJ (1994), p. 284.
5. L. Susskind, *J. Math. Phys.* **36**, 6377 (1995).
6. S. W. Hawking, *Nature* **248**, 30 (1974).
7. J. D. Bekenstein, *Phys. Rev. D* **7**, 2333 (1973).
8. J. M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998); E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998); S. S. Gubser, I. R. Klebanov, A. M. Polyakov, *Phys. Lett. B* **428**, 105 (1998).
9. P. Kovtun, D. T. Son, A. O. Starinets, *J. High Energy Phys.* **2003**(10), 064 (2003); *Phys. Rev. Lett.* **94**, 111601 (2005).
10. G. Policastro, D. T. Son, A. O. Starinets, *Phys. Rev. Lett.* **87**, 081601 (2001). ■