

Rogue waves

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Rich and challenging physics lies behind the gigantic ocean waves that seem to appear without warning to damage ships or sweep people off rocky shores.

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Media accounts often portray rogue, or freak, waves in dramatic terms. *New Scientist* proclaimed on 30 June 2001, “It came from nowhere, snapping giant ships in two.” Tales of such monsters of the deep give the impression that the waves are huge and unpredictable. In this Quick Study, we address two basic questions. First, what causes rough seas with large ocean waves? Second, for a given sea state (a general term used to describe the surface roughness), do extra-large waves occur more frequently than random superposition of different wave trains would predict?

Roughing it

Ocean waves are initially generated by random pressure fluctuations in the turbulent wind, then reinforced in a feedback process that involves the airflow over the wavy surface. The crests and troughs of a single sinusoidal wave travel at the “phase” speed, which is related to the wavelength by a so-called dispersion relation. The first waves to be created are short waves that are easily outrun by the wind. Weak interactions between different waves then transfer energy to longer, faster waves.

The stronger the wind and the greater the distance over which it blows, the longer and larger are the dominant waves. Nonetheless, a spectrum, or mix, of waves of different wavelengths is always present, and that mix is responsible for the rough appearance of a stormy sea. In a large and long-lived storm—say, 1000 km across and a couple of days’ duration—the peak of the spectrum eventually corresponds to waves whose phase speed is close to or just above the wind speed. If, for example, the wind blows at 20 m/s, the corresponding wavelength is 256 m, the period is 12.8 s, and the average trough-to-crest wave height is approximately 8 m. That is large, but observers have reported waves as high as 34 m from trough to crest in the open ocean. After the storm dies down, the longer waves outrun the shorter ones; the result is the familiar and regular swell that can break on beaches thousands of kilometers away.

When discussing waves and speed, one needs to be aware of an important subtlety. The energy in a wave travels at the “group” speed, which is different from the phase speed. For ocean waves in deep water, the group speed is half the phase speed for all but the shortest waves, which are influenced by surface tension as well as gravity. The difference in the two speeds is readily observed in the spreading circle of waves generated by a stone dropped in a pond. The circle as a whole travels more slowly than individual waves, so that

a wave appears on the inside of the ring of waves, travels through the ring, and disappears at the front.

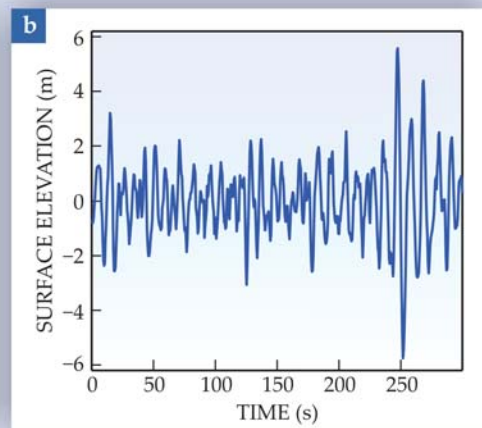
The unequal phase and group speeds make waves more sensitive to currents than one might expect. If waves whose phase speed in still water is c propagate into an opposing current whose strength increases in the direction in which the waves are moving, the waves can be stopped completely by a current of just $c/4$. One factor of 2 comes from the difference between phase and group speeds. The other is an effect of the waves being compressed; as the wavelength decreases, the phase and group speeds are reduced. Moreover, as the wave train is compressed, the wave amplitude increases, partly because the energy is squeezed into a shorter distance and partly because work is done against a “radiation pressure” exerted by the waves.

Because of the sensitivity of wave amplitude to opposing currents, particularly rough seas can occur in regions such as that off the east coast of South Africa, where swell from the Southern Ocean runs into the Agulhas Current. There and elsewhere, waves can also be focused and amplified as they are refracted in current jets and eddies, much as light waves can be concentrated by a lens.

An interesting feature of the interaction with currents is the constancy of the energy of a group of waves divided by the waves’ changing frequency with respect to the water. Called the wave action, the conserved quantity is akin to a well-known adiabatic invariant of classical mechanics: A pendulum made up of a mass on the end of a string is a simple example; as the string is shortened, the energy and frequency both increase but their ratio remains the same.

Statistics

For a given sea state, waves vary in size. But the question remains open whether extra-large waves, such as the one shown in panel a of the figure, occur more frequently than predicted by statistical models that assume random superposition of the different wave trains present. The observational evidence from ships, instrumented buoys and oil platforms, and spaceborne synthetic aperture radar is still hotly debated by oceanographers. To have confidence in the results, one needs long records that capture enough rare events, but the basic sea state can change rather quickly. One can allow for those changes by choosing short sections of a record and appropriately scaling the data in the various sections, but basic statistical questions remain.



Stormy weather. (a) Large ocean waves, such as this one photographed in the Bay of Biscay, can threaten merchant ships. The photo first appeared in the Fall 1993 issue of *Mariners Weather Log*. (Courtesy of the NOAA Photo Library.) (b) Large waves can appear suddenly, after a period of relative calm. The measurements shown here were taken off the west coast of Vancouver Island, Canada, on 16 June 1985. The wave at 250 seconds is much larger than any in the preceding 4 minutes. (Courtesy of Integrated Science Data Management, Canada.)

Treating a sea state as statistically stationary when in fact it varies gives the false impression of extra-large waves occurring more frequently than random superposition implies they should. That misinterpretation can happen not only at a fixed location but also from the deck of a ship as it steams through sea states that, because of interactions with currents, vary on a scale of a few kilometers or less.

One factor that often leads to a wave being called a rogue is that it appears without warning after a period of considerably smaller waves (see panel b of the figure). In our own research, we have found that the frequency of occurrence of such “unexpected” events is the same in the data we have examined as it is in simulations based on random superposition of waves of different frequencies. A wave with a crest twice as high as any of the preceding 30 occurs approximately every two days in the open sea and more frequently in shallow water near shore. The simulations allow for the standard result that a wave crest is sharper and a wave trough more rounded than for a pure sinusoid, but they do not invoke the more complicated physics of the Benjamin–Feir instability, to which we now turn.

Benjamin–Feir instability

Extra-large and unexpected waves may be mostly a consequence of random superposition. On the other hand, a physical mechanism does exist that can lead to large waves popping up out of a regular series of waves. Known as the Benjamin–Feir instability, it was discovered in the 1960s after vain attempts to make a perfectly regular series of waves in a wave tank. An initially regular series of waves always became irregular, with some waves higher than others.

A simple explanation of the BF instability is that the basic equations involve not just linear terms that permit a solution for the surface elevation η of the form $\eta = a \cos \omega t$, a sinusoidal wave of amplitude a and frequency ω . The equations also involve small nonlinear terms that include η^3 —the nonlinear Schrödinger equation is a commonly invoked model. Suppose one starts with a wave and a small sideband, $\eta = a \cos \omega t + \epsilon a \cos(\omega - \delta\omega)t$, with $\epsilon \ll 1$ and $\delta\omega \ll \omega$. The cubic term η^3 can be expanded into harmonics, one of which has frequency $2\omega - (\omega - \delta\omega) = \omega + \delta\omega$. A new wave is there-

fore forced at a second sideband frequency, and it, in a similar manner, reinforces the first sideband at $\omega - \delta\omega$. Both sidebands grow and, however small they are initially, eventually become large enough to disrupt the regular nature of the wave train. The larger the initial amplitude a , the greater the instability’s growth rate.

In a wave tank, where the BF instability has been observed, all waves propagate in the same direction. In the open ocean, however, waves travel in a spread of directions, and it is unusual for a group of waves to hang together long enough for the BF instability to cause the emergence of one or two large waves from a smooth group. Recent research suggests, though, that locally driven wind waves in a storm can feed energy into a preexisting unidirectional swell and make it large enough to undergo BF instability and kick up extra-large waves.

In brief, it seems to us that most large and unexpected waves are a consequence of strong wind, weak interactions that move energy to longer waves, refraction by currents, and random superposition. Occasional rogues, though, may come from the strong interactions of the BF instability.

Several questions concerning extreme waves remain to be explored. For example, are extreme waves transient events lasting for a few seconds as crossing waves combine, or can circumstances cause them to persist? Also, the breaking of large waves is poorly understood. Breaking can limit the height of waves, but it also packs great destructive force. The physics of ocean waves, like many other topics in physical oceanography, is fascinating and important.

Further reading

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