Lie Groups, Physics, and Geometry

An Introduction for Physicists, Engineers, and Chemists

Robert Gilmore
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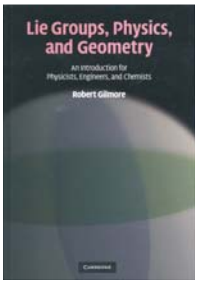
Robert Gilmore, author of *Lie Groups, Physics, and Geometry: An Introduction for Physicists, Engineers, and Chemists*, is a mathematical physicist who specializes in chaos theory and dynamical systems. His latest book, an update and expansion of his well-known *Lie Groups, Lie Algebras, and Some of Their Applications* (Wiley, 1974), is targeted to (mathematical) physicists. Indeed, according to its back cover, "Rather than concentrating on theorems and proofs, the book shows the relationship of Lie groups to many branches of mathematics and physics and illustrates these with concrete computations." Gilmore clearly conveys his enthusiasm for the subject and goes a long way toward fulfilling his stated goals.



Sophus Lie originally developed Lie groups as a tool for solving differential equations, drawing his inspiration from Évariste Galois' use of finite groups to solve polynomial equations. Motivated by that precedent, Gilmore begins his book in an unusual way, with the first chapter devoted to the basics of Galois theory. Although I find the approach intriguing, I have significant issues with its implementation. First, the treatment of Galois theory is both superficial and misleading. In particular, it implies that all polynomials of degree n have the same Galois group—the symmetric group on n letters. Although that result is true generically, individual polynomials can possess other Galois groups. Indeed, the power of Galois theory, which is used by such computer algebra systems as Maple and Mathematica, is that it ferrets out polynomials—of arbitrary degree—that have solvable Galois groups and hence are solvable by radicals. Second, the correct analogue of Galois theory for differential equations is the more refined and sophisticated Picard–Vessiot theory, which forms the foundation of modern differential algebra and underlies computer algebra algorithms for integrating and solving differential equations in elementary terms. However, unlike the Lie groups that have multifarious applications, Picard–Vessiot theory has had almost no impact in physics and so would not be an appropriate topic in a book for physicists. Third, the mathematics in Gilmore's introductory chapter has minimal connection with the rest of his book.

Once the author gets to the main subject, the book progresses through a lively presentation of much of the basics of Lie group theory. Beginning with the second chapter, Gilmore discusses Lie algebras, exponentiation, structure theory, representations, and more, all intertwined with a host of important applications to physics. Chapter 11 presents in great detail the Killing–Cartan classification of simple Lie algebras, both complex and real, with an emphasis on practical algorithms. The next chapter gives a detailed classification of Riemannian symmetric spaces—albeit without defining "Riemannian" or "symmetric space." Then, after a wide-ranging excursion through modern Lie theory and many of its applications to physics, the book returns, in its final chapter, to Lie's original application to differential equations. I find Gilmore's treatment there to be unnecessarily complicated and confusing; for instance, the symmetry-based integration of first-order ordinary differential equations is much easier to do with the integrating factor than with the change-of-coordinates method used by the author. As a result, the concluding chapter fails to do justice to a beautiful and extremely useful aspect of Lie group methods.

In line with his physics roots, Gilmore does not aim for mathematical rigor. If handled adroitly, the nonrigorous approach would be the correct one for the book's intended audience. However, Gilmore's development is not precise enough and can lead to unnecessary pitfalls for the unwary beginner. A particularly egregious example is the lack of attention to connectivity. Often, Gilmore implicitly assumes that Lie groups are connected. For instance, Cartan's theorem that finite products of exponentials cover a group, referred to on page 104, obviously requires the group to be connected, as does the key result, on page 107, that each Lie algebra corresponds to a unique simply connected Lie group. (Note that simply connected does not imply connected.) On the other hand, Gilmore uses the general linear group $GL(n,\mathbb{R})$ and many

other disconnected Lie groups as key examples. As a result, a naive student might falsely conclude that a matrix with negative determinant can be written as a product of real exponentials. Another weakness is that the book often neglects to provide precise definitions of key terms. For instance, the attempted definitions of "semisimple" and "simple" Lie algebras (page 63) are impossibly vague. And nowhere could I find even a hand-waving definition of the fundamental concept of "representation."

Given that much of the book has existed for more than 30 years, I was struck by its rather haphazard organization and lack of careful editing. To give one example, the definition of "group" appears repeated almost word for word on pages 3–4 and on page 24. The index is especially sloppy. For instance, "matrix group" and "Matrix group" appear as separate entries with referrals to different pages. The same holds true for "matrix elements" and "Matrix elements." "Matrix representations" is in the index, but the subcategory "matrix" is not found under the term "representation." With a few exceptions, the bulk of the references, apart from those written by the author, date back to the earlier 1974 book. The bibliography thereby neglects the remarkable wealth of new developments and applications that have emerged during the past 30 years and omits many new texts that could benefit the interested student—particularly one confused by Gilmore's treatment.

Despite my misgivings, I enjoyed much of Gilmore's lively and stimulating exposition. The numerous and varied exercises are a particular strength of the book and lead the motivated reader to explore the diverse connections of Lie groups with a wide range of modern physics. All in all, *Lie Groups, Physics, and Geometry* is a worthy addition to the literature on what Wolfgang Pauli called the *Gruppenpest*, the plague of group theory.

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