Guesstimation

Solving the World's Problems on the Back of a Cocktail Napkin

Lawrence Weinstein and John A. Adam
Princeton U. Press, Princeton, NJ, 2008. $19.95 paper (301 pp.). ISBN 978-0-691-12949-5

Back-of-the-envelope calculations are familiar to most physicists, who typically love to make order-of-magnitude estimates and are well prepared to do so. Physics teachers often encourage their students to develop such skills, which can be powerful aids for building understanding and for guiding research paths. Enrico Fermi was especially well known for making quick numerical estimates using minimal data. Thus questions such as "How many piano tuners are there in Chicago?" are commonly called Fermi questions. Outside the domain of physics, guesstimation skills can enhance understanding and decision making in business and government.

Lawrence Weinstein and John Adam's *Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin* is intended for a general audience. The authors assume that readers do not have the skills of working physicists, and they attempt to provide a general structure for solving estimation problems. Both authors teach at Old Dominion University in Norfolk, Virginia. Weinstein is a professor of physics who writes the column "Fermi Questions" in the *Physics Teacher*, a magazine published by the American Association of Physics Teachers; Adam, a professor of mathematics, is the author of *Mathematics in Nature: Modeling Patterns in the Natural World* (Princeton University Press, 2003).

Given its introductory level, *Guesstimation* has two potential attractions for a physicist: exposure to the clever techniques the authors use to provide order-of-magnitude estimates and a wide selection of fascinating questions. Numerical estimates are quickly obtained for questions that seem at first glance difficult or impossible to answer. Subtle and not-so-subtle humor punctuates the text, and I laughed out loud at times.

An impressive range of topics is covered, including energy and work, energy and environment, Earth and the Moon, the atmosphere, and risk analysis. The first two chapters, on problem solving and dealing with large numbers, can be skipped by scientists; on the other hand, I enjoyed reading them to learn the authors' views on some of the subtleties. Two appendices contain tables showing the lengths, areas, and masses of various small to large objects and the mass densities of materials. A lengthy list of unanswered questions is also available for readers who want to explore further.

The bulk of the book addresses specific questions such as "What is the surface area of a typical human?" The first page of each question contains a thought-provoking drawing by Patty Edwards, an art lecturer at Old Dominion, and gives hints that are printed upside down to deter readers from simply reading them before pondering the problem; a full solution is given on subsequent pages. The format encourages readers to work out each problem before looking at the authors' solution.

The geometric mean is used repeatedly and effectively to estimate numbers based solely on order-of-magnitude bounds. For example, to decide how long a 1.5-V battery will last, one might argue that it is likely to be more than 1 hour but less than 10 hours. The resulting geometric mean is $(1 \times 10)^{1/2} \approx 3$ hours. Some problems are worked in two distinct ways, giving answers of the same order of magnitude; answers to other problems are checked against known data. Such checks build confidence in the techniques used.

Guesstimation is generally well written, with few typos. The main attractions are its potpourri of interesting questions, the way the authors encourage readers to attack them, and the authors' resourcefulness in doing so. A departure from the authors' typically thorough approach is in their comparison of the power consumption of gerbils and humans, in which they miss the importance of the surface-to-volume ratio and the power per surface area. But such occurrences are few.

Although estimation techniques abound on the internet and are broached in many books, few books are devoted to numerical estimations per se. One that is directly physics related is *Back-of-the-Envelope Physics* (Johns Hopkins University Press, 2003) by Clifford Swartz (see the review in PHYSICS TODAY, May 2004, page 58). A more recent one is Douglas Hubbard's *How to Measure Anything: Finding the Value of "Intangibles" in Business* (Wiley, 2007). Although not a physics book, its content complements *Guesstimation*.

I recommend Weinstein and Adam's book to anyone who likes to do back-of-the-envelope calculations that address real-life issues. It can be particularly valuable for physics students, and it provides fun reading and thinking for a wide spectrum of readers.

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Equilibrium Between Phases of Matter

Phenomenology and Thermodynamics

H. A. J. Oonk and M. T. Calvet
Springer, Dordrecht, the Netherlands, 2008. $109.00 (404 pp.). ISBN 978-1-4020-6123-3

Harry A. J. Oonk and M. Teresa Calvet's *Equilibrium Between Phases of Matter: Phenomenology and Thermodynamics* is a welcome addition to the literature. The text will be useful to advanced undergraduates and first-year graduates who have had a solid course in physical chemistry and who want to extend their knowledge of phase diagrams and how those diagrams connect to thermodynamics. It will also be of value to more experienced practitioners who want to broaden their perspective. I have taught and done research on the thermodynamics and statistical mechanics of phase transitions and critical phenomena for more than 40 years; yet in reading through Oonk and Calvet's text, I have learned new things and relearned others that I had once known "in principle" but had not taught over the years.

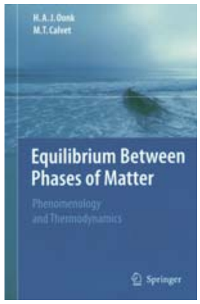
The authors divide the book into three parts. The first presents a phenomenological overview of phase equilibria with a minimum of thermodynamic analysis. Results are introduced simply as facts. The second deals with formal thermodynamics and its application to phase diagrams in which the properties of solutions, particularly non-ideal solutions, do not play a significant role. The third part covers the thermodynamics of solutions and is developed in some detail. The authors consider partial molar quantities and composition dependence of chemical potentials, and they explore the consequences of those concepts for phase diagrams. Extensive problems and their detailed solutions are a solid asset. I worked through several and found them well-posed and instructive.

The authors' use of simple model free energies to explicate a variety of phase behavior is a strong point of the text. For example, they employ linear free energies to illustrate the distinction between monotrophism and enantiomorphism (pages 156 and 157). But I think that more care is needed to qualify the conclusions: If both the $\alpha \rightarrow \gamma$ and $\alpha \rightarrow \beta$ transitions were sufficiently slow, it would seem that the transition $\alpha \rightarrow \delta$ shown in figure 110.8b could sensibly take place reversibly. Another strength of the text is that the authors approach the problem of understanding phase equilibrium from several different directions. In addition to employing simple model equations of state, they use geometric reasoning about the general shape of free energies as functions of mole fraction, temperature, and pressure as well as the numerical solution of nonlinear equilibrium equations. As a result, readers learn to think about a problem in phase equilibrium from various perspectives and determine which approach works best.

I have, however, serious concerns about the notation in the second part of the text. The authors use ΔQ and ΔW to represent the integral heat absorbed by and the work done on a system in a finite process. Their choice is unfortunate from a pedagogical viewpoint because the notation strongly suggests that those are changes in something—namely a state function—rather than simply the amounts of heat and work absorbed along a path. I can see no advantage to their notation over the simpler choice of Q and W.

Another problem for students—for most US students at least—is the use of q and w to denote differential heat and work absorbed in an infinitesimal process. That choice leads to such equations as $\int q + \int w = \Delta U$ and $q + w = dU$. In their differential notation, though, the authors are on more solid, theoretical ground; they have at their disposal both the impressive precedent of Edward Guggenheim's classic *Thermodynamics: An Advanced Treatment for Chemists and Physicists* (North-Holland, 1949) and the mathematicians' and mathematical physicists' notation denoting general differential forms, of which differential heat and work are specific "1-form" examples.

They also have the compelling argument that dq and dw—commonly used in the US if not elsewhere—are easily mistaken for exact rather than inexact differentials. Other authors have used δq or $đq$ to emphasize that the differential quantity in question is an inexact differential, and those notations seem to me preferable as representations of an infinitesimal quantity. Nevertheless, the authors might reply in their defense, to paraphrase Humpty Dumpty, that when they use a symbol, it means just what they choose it to mean—neither more nor less. Oonk and Calvet define their symbols and clearly know how to use them correctly, but I suspect that the mismatch between their notation and that commonly used in physical chemistry and thermodynamics textbooks in the US will limit their book's adoption for classroom instruction.



In section 2 of the first part of the text, the authors use a simple model free energy to extract the dilute-solution laws. Although that model is an elegant and efficient way to extract the properties, I was disappointed that they did not emphasize more strongly that those are general laws of nature for nondissociating and nonassociating solutes that follow from the molecularity of the solute in dilute solution. Their powerful thermodynamic analysis of phase diagrams near the pure solvent depends on the generality of those laws.

I was pleased to see the authors' treatment of the 180° rule and other rules concerning the placement of phase boundaries in phase diagrams—both in the early phenomenological section where the rules were simply proclaimed and in later sections where they were argued for in terms of the behavior of free energies. However, I was disappointed that they did not show the reader that those are rigorous theorems of thermodynamics under mild assumptions about the nature of the triple point and that the assumptions are necessary for the theorems to hold.

The text does have the kinds of typos and errors to be expected in any first edition. I was impressed, however, at how few I found. In problem 202.6 on page 189, the last datum for ρ is almost surely in error. The value 0.1972 agrees much better both with other data in the text and with literature values than the 1.772 that the authors give. An amusing faux pas occurs on pages 54–55. On page 55, the authors scrutinize the quality of phase diagrams in other textbooks, noting that some "drawn in a freehand manner" can lead to error; but on page 54 they provide—inadvertently, I assume—an example. The tie lines in their three-component phase diagram are essentially parallel, which leads to a contradiction at one side of the diagram. The authors, who are not native English speakers, request that the reader "enjoy the science and wink at the linguistic shortcomings." Those shortcomings are numerous, and occasionally distracting, but nowhere did I find them a serious impediment to understanding the text.

Despite such quibbles, I heartily recommend *Equilibrium Between Phases of Matter* if you want to enrich your knowledge of phase equilibrium and phase diagrams and their relation to thermodynamics. You are remarkably knowledgeable about the subject if you do not learn something from this book.

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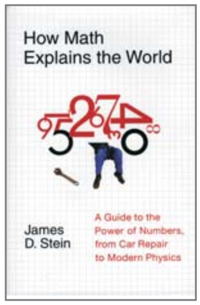
How Math Explains the World

A Guide to the Power of Numbers, from Car Repair to Modern Physics

James D. Stein
Smithsonian/Collins, New York, 2008. $24.95 paper (264 pp.). ISBN 978-0-06-124176-5

I wanted to review James Stein's *How Math Explains the World: A Guide to the Power of Numbers, from Car Repair to Modern Physics* because I expected it to be a modern source of information about Fermi problems, which I use in my classes. I was looking for something like John Allen Paulos's now rather dated *Innumeracy: Mathematical Illiteracy and Its Consequences* (Vintage, 1990). I was wrong.

My undergraduate students, primarily from liberal-arts disciplines, use the phrase "do the math" to mean "put in the numbers"; they reserve the longer phrase "do the mathematics" for formal reasoning and actually doing proofs. So from that viewpoint, Stein's book is really not about math and the power of numbers to explain the world; instead, it is about mathematics on the one hand and explaining the physical world on the other. The problem with the book is that those two topics are not synthesized; they are interlaced and appear in separate segments that do not connect the mathematics and the physics. The author also spends significant time and effort relating the great mathematicians and some famous physicists to the fruits of their labor. He often tells well-known stories but occasionally adds some interesting new elements that give insight into a source of the accomplishments or an indication of the eccentricities of those people he mentions.



I can't easily guess the purpose of Stein's book, and I would not recommend it as the kind of class supplement I was looking for. It certainly does not use the usual Fermi-problem inference and estimation techniques to clarify everyday situations. The book also cannot serve as a text on modern physics for poets. Stein is a mathematician, and his understanding of physics clearly comes from second- and third-hand descriptions for laymen. The result is like the telephone game, in which a sentence is passed from player to player and modified at each transfer. Therefore, although the author uses many analogies to attempt to communicate the implications of modern physics, those descriptions are sometimes incorrect and only burden the reader who wants to understand microscopic phenomena.

The book could serve as reading material for a mathematics course, but that course would have to deal with modern fundamental issues of mathematics—not the usual fare for students who won't be majoring in physics or mathematics. Much valuable space is spent on biography. The text could be a supplement to a mathematics-for-poets course; it is an easy read, except that some of the concepts and jargon are introduced without reference, and the index is far from complete. Some good editing would have helped. I was also offended by the author's bait and switch. True, the book opens with a discussion on scheduling for a car repair and closes with modern physics and preference voting. The bulk of the text, however, describes important developments of mathematics and the lives of mathematicians but does not indicate the relevance of mathematics in explaining the world. By the way, I am always bothered when the word "explain" is applied to physics. The way I look at it, physicists do a lot of experiments, and some people thoughtfully try to find a concise, consistent description of all those results—a mathematical description. But if what we do is called an "explanation," then I respectfully disagree.

So, to whom would I recommend *How Math Explains the World*? Someone who is a nonexpert traveling on a cross-continental flight could finish and enjoy it. I regularly teach a course in modern physics to very bright, mostly liberal-arts honors students, and I require that they read a book with some relationship to modern physics and prepare a response for a grade. They can select from a large list of books, and I will add Stein's book to that list. I hope those students who choose it catch some of the physics problems that bothered me.

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