Pattern Formation

An Introduction to Methods

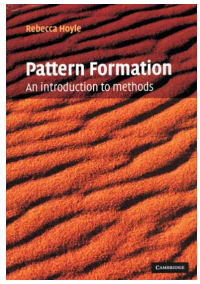
Rebecca Hoyle
Cambridge U. Press, New York, 2006. $85.00 (422 pp.). ISBN 978-0-521-81750-9

Patterns are a feature of everyday life, from coffee stains on a napkin to patterns on the coats of animals to the swirling arms of galaxies. The patterns that form are frequently captivating and are the subject of numerous coffee-table books. My favorite is *The Algorithmic Beauty of Sea Shells* (Springer-Verlag, 1995) by Hans Meinhardt. Over the years, fluid dynamics has motivated much of the basic research on pattern formation, and books are still being published illustrating the variety and beauty of fluid flows. An example of such a book is *A Gallery of Fluid Motion* (Cambridge U. Press, 2003), edited by Mohammad Saminy, Kenneth Breuer, L. Gary Leal, and Paul Steen. Although the above systems are vastly different in both their scales and their physics, they nonetheless share many common features.

In *Pattern Formation: An Introduction to Methods*, Rebecca Hoyle, a senior lecturer in mathematics at the University of Surrey in the UK, focuses on the common aspects of many pattern-forming processes. The book is intended for advanced undergraduate and beginning graduate students, and is written in an informal, even chatty style. The author chooses three physical systems as motivation for her subject: convection in a fluid layer heated from below; patterns formed by gravity–capillary waves on the surface of a liquid in a vertically oscillating container (Faraday waves); and the famous Belousov–Zhabotinsky (BZ) chemical reaction.

The book is intended to describe the mathematical techniques used for analyzing different types of patterns, in both small and spatially extended domains. Thus one will not learn fluid mechanics from the book, nor learn to appreciate the physics of the parametric instability behind the formation of Faraday waves. Neither will one learn the chemistry responsible for the BZ oscillations. These omissions are both a plus since the details of the processes may be quite involved and a minus since one cannot help but feel that one's understanding remains superficial.

The book starts with the arguable definition that patterns form as the result of a spontaneous symmetry-breaking instability, or bifurcation. From this point of view, it is natural to study patterns using techniques that take maximum advantage of the presence of symmetries. These symmetries may be present because of container geometry, as a result of modeling assumptions (researchers often use periodic boundary conditions to discuss large systems) or the mathematical manipulations used to simplify the equations near a bifurcation and put them into normal form. These techniques are the subject of equivariant bifurcation theory. The theory is well summarized in a number of books, the best of which are still the two volumes by Martin Golubitsky, David Schaeffer, and Ian Stewart: *Singularities and Groups in Bifurcation Theory* (Springer-Verlag, 1985 and 1988).



In chapters 2 and 3 of *Pattern Formation*, Hoyle summarizes the essential background from bifurcation theory and group theory. In chapter 4 she turns attention to ordinary differential equations with symmetry, and in chapter 5 she extends this theory to so-called lattice patterns, focusing on spatially periodic patterns in one or two dimensions. The theory of such patterns is much simpler than that of general patterns in the plane, because, as Hoyle makes clear, the symmetry group of a lattice is compact, and only discrete wave vectors are involved in the pattern-formation process. Chapter 6 presents material that is rarely found in texts: superlattice patterns; quasipatterns; hidden symmetries present, for example, in partial differential equations in certain types of domains with Neumann boundary conditions; and pseudoscalar representations of the Euclidean group $E(2)$.

In the second half of the book, chapters 7 through 9 focus on extended domains and use formal asymptotic methods, primarily multiple-scale techniques, to derive envelope equations for the spatial or temporal modulation of patterns on large scales. These equations include the famous Newell-Whitehead-Segel, Ginzburg–Landau, and Kuramoto–Sivashinsky equations. The author then uses these equations to study both the constraints on the wavenumber of stable periodic patterns and the motion of defects in such patterns.

Much of the presentation in the second half treads the path of more advanced texts such as Paul Manneville's *Dissipative Structures and Weak Turbulence* (Academic Press, 1990) or *The Dynamics of Patterns* (World Scientific, 2000) by Mikhail I. Rabinovich, Alexander B. Ezersky, and Patrick D. Weidman—although the emphasis in Hoyle's book is on the use of symmetry principles to derive these equations. Unfortunately, for envelope equations, these techniques are not as compelling because they demand a local description of the system—that is, a description in terms of an envelope function and its derivatives. Although this approach works in many examples, in some cases a formal asymptotic description yields nonlocal or integro-differential envelope equations. In such cases, Hoyle is obliged to make assumptions about the size of certain coefficients, a procedure that calls into question the applicability of a purely local description to real systems.

Chapter 10 contains a limited discussion of spiral waves, and chapter 11 provides a welcome summary of recent developments in the phase description of large-amplitude, slowly varying patterns based on the Cross–Newell equation and its adumbrations. The approach has so far been missing from textbooks.

Hoyle's text is the first on the subject that includes both group-theoretic and the more formal multiple-scale techniques. It is regrettable, however, that she has not used this opportunity to show how well these techniques can do, quantitatively, to describe real-world patterns. Yuanming Liu and Robert E. Ecke's 1999 *Physical Review E* paper on the Eckhaus instability of wall-confined traveling-wave convection in a rotating cylinder is one good example that could have been used. The beautiful computations of the von Kármán flow (the "French washing machine") by Caroline Nore, Laurette S. Tuckerman, Olivier Daube, and Shihe Xin in the 2003 issue of *Journal of Fluid Mechanics* could have been mentioned in connection with the discussion of structurally stable heteroclinic cycles in chapter 6.

The other concern I have is that students will not learn from this book how to compute the coefficients present in the equations studied in terms of physical parameters, and it is precisely such calculations that must be done to relate the otherwise abstract theory to real-world experiment. For example, Hoyle makes no mention of how to determine, in a given system, whether a Hopf bifurcation is sub- or supercritical. In addition, because of its chatty style the book contains statements that are, strictly speaking, incorrect: "[T]he point $\mathbf{x}_0$ is called a *sink* if all the eigenvalues have strictly negative real part, a *source* if all the eigenvalues have strictly positive real part, and a *saddle* otherwise" (page 29). Likewise, the notion of a normal form (pages 47 and 140) is discussed incorrectly, and as a result the statements about the origin of time-translation invariance in envelope equations (pages 233 and 236) are misleading. It is also regrettable that key results such as the Fredholm alternative theorem (page 214) are not stated precisely. Even figure 4.10, representing standard results for the steady-state bifurcation with D_4 symmetry, is misleading. The names of Yoshizawa (page 16), Bolton (page 285), and Sivashinsky (pages 289 and 420) are all misspelled. Other misprints could have been caught by any alert technical editor, who surely should know the difference between the Greek letter ν and italicized v.

However, despite the reservations just expressed, I have recommended this book to all my graduate students. It provides a useful starting point for those of us interested in the theory of pattern formation and is well suited as a text for a first course on pattern formation.

Edgar Knobloch
University of California, Berkeley

Exploring the Quantum
Atoms, Cavities, and Photons

Serge Haroche and Jean-Michel Raimond
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In 1952 Erwin Schrödinger wrote in the *British Journal of the Philosophy of Science*, "We never experiment with just one electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences." One cannot help but wonder how Schrödinger would react to the fact that single atoms, ions, and photons can now be isolated, manipulated, and detected almost routinely, and with exquisite control.

Those developments are quite astounding when we really think about them. After all, it was not until the beginning of the 20th century that the existence of the atom was definitely established. Until the 1970s researchers could only handle atoms in vast quantities. The origins of single-particle trapping and control are found in the pioneering experiments of Hans Dehmelt and coworkers, who succeeded in 1973 in trapping a single electron, eventually for several months at a time, an achievement that led to a revolution in precision measurements. The story has it that Dehmelt became tired of his teachers and colleagues telling him to "consider a single electron" and then chalking a dot on the blackboard; he decided that he wanted to really see one. I also recall the astonishment of the atomic-physics and quantum-optics community at the first images of fluorescence from a lone barium ion trapped by Peter Toschek and his colleagues. Suddenly, it became clear that the thought experiments dreamt up by Schrödinger, Albert Einstein, and the other fathers of quantum mechanics would now become possible: We would soon see whether those experiments did entail ridiculous consequences.

Exploring the Quantum: Atoms, Cavities, and Photons by Serge Haroche and Jean-Michel Raimond describes much of the remarkable progress that has occurred in the trapping and controlling of single atoms, ions, and photons in the past 30 years. The book covers the new and sometimes surprising directions in which these developments are taking us, particularly in the emerging field of quantum information science. The authors, grand masters at that game, are eminently qualified to write such a