Approaches for improving students' understanding of quantum mechanics

In the first sentence of their article "Improving Students' Understanding of Quantum Mechanics" (PHYSICS TODAY, August 2006, page 43), Chandralekha Singh, Mario Belloni, and Wolfgang Christian refer to Richard Feynman's well-known assertion that nobody understands quantum mechanics. But thereafter they ignore it, and apparently assume that student misconceptions when learning quantum mechanics are not connected with foundational issues. I argue the contrary, that Feynman's statement should be a central concern in all efforts to improve quantum pedagogy. If we teachers do not understand a topic, we pass our own misconceptions on to our students and make the subject much more difficult for them.

Many conceptual difficulties, including those Feynman was referring to, arise from the problem of introducing probabilities into quantum theory in a useful and consistent way. Textbooks avoid the problem by assigning probabilities to macroscopic measurement outcomes rather than to microscopic quantum systems. Although that approach avoids inconsistencies, it gives rise to some serious misconceptions: Measurements are somehow special and unrelated to other quantum phenomena; they require a "classical" apparatus that functions outside the scope of quantum mechanics (where is one to find such a thing nowadays?); they produce physical effects at long distances; one can say nothing sensible about what a quantum system is doing in the absence of measurements; measurements can be used to predict the future of the measured system but tell us nothing about its past; and so forth. These misconceptions are not unrelated to those that Singh and coauthors have reported.

Letters and opinions are encouraged and should be sent to Letters, PHYSICS TODAY, American Center for Physics, One Physics Ellipse, College Park, MD 20740-3842 or by e-mail to ptletter@aip.org (using your surname as "Subject"). Please include your affiliation, mailing address, and daytime phone number. We reserve the right to edit submissions.

Students learning quantum mechanics could benefit greatly if their instructors used advances in our understanding that have occurred during the 40 years since Feynman unashamedly confessed his perplexity. He seems to have reacted favorably to a preliminary version of the new ideas (see the letter from Murray Gell-Mann and James Hartle in PHYSICS TODAY, February 1999, page 11), so he might have appreciated the more mature form now available. In brief, we now know how to consistently assign probabilities directly to microscopic systems without referring to measurements, and we can show that under appropriate conditions a properly constructed measurement apparatus, described in fully quantum terms, will reveal properties the measured system possessed *before* the measurement took place. In such circumstances the probabilities of measurement outcomes are the same as those of the measured properties, and measurements are no longer an essential conceptual tool: One can think directly in physical terms about what the quantum system is doing at different times. This gets rid of a major source of student difficulties and misconceptions.

Consider the example reported by Singh and coauthors in which students used a calculation employing $\langle A \rangle = \langle \psi | A | \psi \rangle$ to find the expectation of an observable A, rather than simply using a probability distribution they had worked out previously. (In that article, A was the energy, but the same principle applies to any observable.) This failure is not surprising given that textbooks lack a good discussion of how to assign probability distributions to observables. So the student memorizes an independent formula $\langle A \rangle = \langle \psi | A | \psi \rangle$, which is a good way of calculating something that comes up in homework and exams, but whose physical significance is not particularly clear (to student or instructor). What the student should be taught is that A is the quantum counterpart of a random variable in ordinary probability theory, and its average can be obtained from its probability distribution in exactly the same way. Defining $\langle A \rangle$ in this manner before introducing $\langle \psi | A | \psi \rangle$ as a convenient formula for calculating it would make things clearer. But quantum textbooks do not contain the necessary tools, and for good reason. With two noncommuting observables A and B, it is easy to poke either of them into the $\langle \psi | A | \psi \rangle$ formula, whereas assigning probabilities leads into a vast swamp, which work on quantum foundations has shown to be filled with nasty paradoxes ready to bite the unwary. Retreating to macroscopic measurements allows textbook writers to avoid the swamp, but with a serious loss in clarity of thought and physical intuition. It is better to drain the swamp of its root cause: a failed attempt to meld classical and quantum modes of reasoning, instead of consistently applying quantum concepts at all levels, microscopic and macroscopic, which is something we now know how to do.

Another misconception reported in the PHYSICS TODAY article, that measurement of a physical observable causes the system to be stuck forever in the measured eigenstate, is hardly surprising when students are taught that measurements and wavefunction collapse are part of the axiomatic, and thus unanalyzable, structure of quantum theory. Instead, they need to think about measurements as quantum physical processes, governed by the same laws as the rest of the quantum world, and learn how to use conditional probabilities to relate measurement outcomes to the past as well as the future behavior of a measured system. Once again, outdated ideas make the subject harder to learn.

For 10 years I have been teaching advanced undergraduate and beginning graduate quantum mechanics courses and courses in quantum information, using the new perspective in which quantum mechanics is based on probabilistic laws of universal validity, with measurements being only one application. Reactions have generally been positive, though the students show signs of shock when I tell them that by the end of the course, provided they do their homework, they will understand some aspects of quantum mechanics better than Feynman did. Presenting the new ideas takes somewhat longer than the material they replace, but not enormously so. Some time will be regained in courses that include an introduction to quantum entanglement and Einstein-Podolsky-Rosen, since circuitous arguments invoking Bell's inequality and the like, which can leave students quite confused, are replaced by a short, clear treatment of the essentials.

Although I can see the value of computer simulations of Schrödinger's equation, I think it is more effective to first introduce students to basic quantum dynamics, both unitary and stochastic, through the use of "toy models." I included various examples in *Consistent Quantum Theory*.[1] The properties of such models are easily worked out with a pencil on a small sheet of paper, like the back of an envelope. Working through them helps students master new concepts and get rid of certain misconceptions about quantum measurements.

The fact that students in my courses have been able to learn how to apply probabilities consistently to microscopic systems, in a way that disposes of numerous difficulties and conceptual paradoxes, suggests it might be worthwhile for other teachers to invest some time in learning post-Feynman ideas. The main difficulty is the absence of a textbook. I have used reference 1 as a supplement, though it is not ideal. It has no exercises, although a few are available on the corresponding website. I would be happy to hear from anyone skilled in textbook writing who wants to revise an older one or start something new.

In conclusion, I strongly favor every effort to improve students' understanding of quantum mechanics, and I consider the research reported by Singh and coauthors a valuable contribution to that end. However, if we want our students to genuinely understand quantum mechanics and not simply calculate things, I believe a much bigger step forward is possible by combining the efforts reported in the article with advances in quantum foundations.

Reference

1. R. B. Griffiths, *Consistent Quantum Theory*, Cambridge U. Press, New York (2002). Some chapters and a few exercises are available at http://quantum.phys.cmu.edu.

Robert B. Griffiths
(rgrif@cmu.edu)
Carnegie Mellon University
Pittsburgh, Pennsylvania

In their article, Chandralekha Singh, Mario Belloni, and Wolfgang Christian focus exclusively on "functional understanding of quantum mechanics," which they claim "is quite distinct from the foundational issues alluded to by Feynman."

But are the foundational and the functional really so distinct? The work of other physics education researchers suggests not. For example, in a classic article, Alan Van Heuvelen discusses students' prevalent and frustrating use of "primitive formula-centered problem-solving strategies"[1] and suggests that physical, intuitive understanding developed through qualitative diagrams and models "must come before students start using math in problem solving. The equations become crutches that short-circuit attempts at understanding." Van Heuvelen also urges that "instead of thinking of [problems] as an effort to determine some unknown quantity, [teachers] might . . . encourage students to think of the problem statement as describing a physical process—a movie of a region of space during a short time interval or of an event at one instant of time." I suspect Singh, Belloni, and Christian would agree with this advice. They comment that such "qualitative understanding of quantum mechanics is much more challenging than facility with the technical aspects."

But isn't the main barrier to such intuitive, qualitative understanding the nature of quantum mechanics itself—at least, the version of the theory advocated by Niels Bohr, Werner Heisenberg, and virtually every textbook writer since? Why should we expect students to invest the time and energy necessary to, say, visualize the time-dependence of $|\psi|^2$ when we also preach the ambiguous and contradictory Copenhagen dogma that ψ does not represent anything physically real, yet still provides a complete description of physical reality? Why are we surprised that students are confused about, and don't take seriously, something that we assure them is, at best, some kind of algorithmic fantasy? Is there really any difference between "shut up and calculate" and "plug and chug"?

Why not teach them Bohmian mechanics—an alternative (deterministic) version of quantum theory in which particles are particles (and really exist, all the time) and the same dynamical laws apply whether anyone is looking or not?[2] About this alternative theory John S. Bell asked, "Why is [it] ignored in text books? Should it not be taught . . . as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism are not forced upon us by experimental facts, but by deliberate theoretical choice?"[3]

If we really want to help students understand quantum mechanics, the first step is to reject the confusion-spawning foundational vagueness, ambiguity, and philosophical absurdity of Copenhagen quantum theory, and adopt a clearer, more scientific, less fuzzy version. (See Sheldon Goldstein's two-part article "Quantum Theory Without Observers," PHYSICS TODAY, March 1998, page 42, and April 1998, page 38.) The first step, in short, is to present them with a theory that *can* be understood.

References

1. A. Van Heuvelen, *Am. J. Phys.* **59**, 891 (1991).
2. S. Goldstein, in *Stanford Encyclopedia of Philosophy,* http://plato.stanford.edu/entries/qm-bohm.
3. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics: Collected Papers on Quantum Philosophy*, Cambridge U. Press, New York (2004), p. 173.

Travis Norsen
(norsen@marlboro.edu)
Marlboro College
Marlboro, Vermont

A quite different approach from the one presented by the authors of "Improving Students' Understanding of Quantum Mechanics" may be appropriate at least for some classes of students. It might be called the pragmatic approach, teaching students to deal with a wide variety of problems while minimizing philosophical discussion. I took this approach for several years while teaching a course for graduate engineers at Stanford.[1] The resulting course was surprisingly orthogonal to the traditional quantum course. Solving the Schrödinger equation became a minimal part of the subject; rather, tight-binding expansions allowed the student to use simple algebra to obtain a meaningful understanding of atoms, molecules, and solids. Transition rates and shake-off excitations provided understanding of a wide variety of phenomena.

I took the defensible stance that all of quantum mechanics is the direct consequence of a single assertion, wave–particle duality. The uncertainty principle and the Pauli principle are consequences, not independent conjectures. Quantum theory does not tell us that there will be a particle of spin ½ with the mass and charge of an electron, but it indicates how such a particle will behave if there is one. When the consequences seem puzzling, it is fair to say