

Pieces of tumbled string form topologically simple and complex knots

One day two years ago, Douglas Smith walked into his biophysics lab at the University of California, San Diego, and found two of his undergrads discussing knot theory. They'd downloaded the lecture notes of a math course and were weighing whether to take it. Not wanting his students to spend less time in his lab, Smith, half in jest, challenged them: "If you're going to study knot theory and you're going to work in this lab, there'd better be knot experiment too!"

But what is a knot experiment? At first, Smith and Dorian Raymer, the undergrad who took up the challenge, considered studying the knots found in loops of viral DNA. But working with the tiny polymers is difficult and expensive. Instead, they opted for something far simpler and cheaper: tumbling a piece of string in a box.

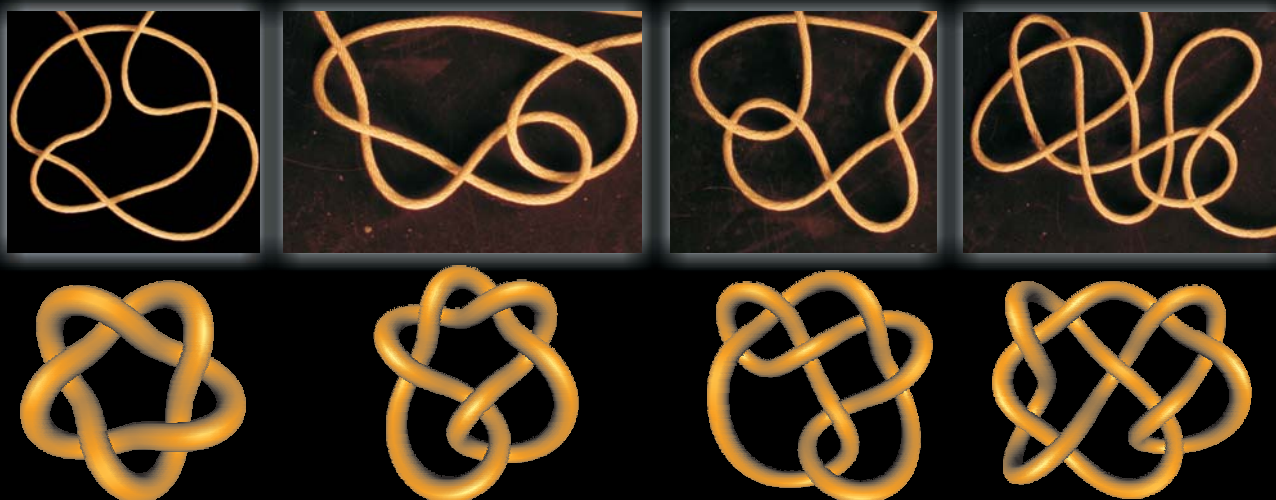
The experiment addressed two questions. What determines knotting probability? What kinds of knot form? The answers Raymer and Smith obtained were a mix of common-sense expectations and subtle surprises.¹

Raymer and Smith's box measured 30 cm on a side. Their

plex calculations to yield the knot's topological fingerprint—the Jones polynomial. And using the Jones polynomial, he could look up the knot type in an online database.

The top row of the figure shows four knots that formed in Raymer and Smith's box. Below each knot is an image of its topological abstraction. Raymer and Smith found that the probability P of forming any type of knot depended on string length L . If the string was too short, an end spent too little time around the rest of the string to knot. That threshold was 46 cm. As L increased, P rose rapidly until L reached about 1.5 m. At that length, P appeared to saturate at about 0.5. Slowing the tumbling rate from the usual 1 Hz to 0.3 Hz, while preserving the number of rotations, didn't affect P , whereas tripling the tumbling rate to 3 Hz reduced P ; centrifugal force pressed the string against the walls of the box.

Those not unexpected findings are consistent with previous experiments on agitated chains.² In those experiments, only the simplest knot appeared, the granny or trefoil, which, in its topological abstraction, has three crossings. To their surprise,



string, bought from a hardware store, was 3.2 mm thick and about as stiff as a strand of half-cooked spaghetti. Each run began with dropping a length of string into the box, closing the lid, then spinning the box about a horizontal axis to tumble the string and give it the chance to tangle with itself. Raymer ran the experiment thousands of times to gather statistics over a range of conditions.

Topologically speaking, an open-ended string is just like a straight, unknotted line, no matter how tangled the string appears. After tumbling a piece of string, Raymer would therefore attach its ends together to make a knot—or not: Knots formed more or less half the time.

Raymer's knot-theory studies paid off when it came to characterizing the knots. Just as a coffee mug is topologically equivalent to a torus, quite different-looking knots can share the same topology. To determine a knot's topological type, Raymer first laid each tangled string on a flat, black surface, smoothed the loops and crossings, and took a digital photo. Then, using a computer program he'd developed, he traced the knot's image with the cursor, classifying each crossing in turn: under or over, left or right. His program carried out com-

plex calculations to yield the knot's topological fingerprint—the Jones polynomial. And using the Jones polynomial, he could look up the knot type in an online database.

Knots nucleate at one end of a string. Noticing that their tumbling strings tended to coil like a garden hose, Raymer and Smith modeled knot formation as a random braiding process. When an end finds itself next to a coiled segment, it has an equal chance of moving under or over the segment and an equal chance moving up or down to the next segment. Longer strings, having more coils, will knot more readily than shorter strings.

The simple model could account for the observed knotting behavior, including the dependence of P on L . It could also explain which knots Raymer and Smith saw. In 1923, topologist James Waddell Alexander proved that braiding yields every type of knot.

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References

1. D. M. Raymer, D. E. Smith, *Proc. Natl. Acad. Sci. USA* (in press).
2. E. Ben-Naim et al., *Phys. Rev. Lett.* **86**, 1414 (2001); A. Belmonte et al., *Phys. Rev. Lett.* **87**, 114301 (2001).