

Wrinkles provide the means to measure the elasticity of thin floppy films

Bending a pencil eraser stretches the rubber on the outside of the bend and squashes the rubber on the inside. Both distortions cost energy. But the thinner the rubber, the less energy you need to stretch and squash. Bending becomes easier.

That thin elastic sheets bend so readily endows them with a way of accommodating stress that thick slabs lack. If you place two fingers on your arm and move them apart, your skin will respond like any other piece of elastic material, whether sheet or slab: It stretches. But if you move your fingers together, your skin, rather than compressing, will buckle out of plane: It wrinkles.

Despite wrinkling's ubiquity and seeming simplicity, the underlying mechanism is physically subtle and mathematically formidable (see the article by Michael Marder, Robert Deegan, and Eran Sharon, *PHYSICS TODAY*, February 2007, page 33). The governing equations, derived in 1907 by August Föppl and in 1910 by Theodore von Kármán, are highly nonlinear and feature derivatives up to fourth-order.

Obtaining a general solution is all but impossible. Nevertheless, in 2002 Enrique Cerda, Krishnaswamy Ravi-Chandar, and L. Mahadevan derived scaling laws for the number and wavelength of wrinkles induced when a thin rectangular sheet is clamped at its ends and stretched.¹ According to their analysis, the stress accommodated by the wrinkles depends on the fourth power of the wavelength and on the square of the sheet's thickness. That sensitive dependence could be exploited, the three theorists pointed out, to measure the sheet's elastic properties.

Now, a team of experimenters from the University of Massachusetts Amherst has demonstrated that wrinkling does indeed provide an accurate and reliable way to measure the Young's modulus and the thickness of thin elastic films.² What's more, the method provides a simple, controllable way to study the wrinkling phenomenon itself.

The UMass project began when Narayanan Menon sought a new way to study wrinkling, crumpling, and other manifestations of two-dimensional elasticity. Ideally, the sheets should be ultrathin, easy to handle and observe, and have adjustable properties. His UMass colleague Tom Russell suggested polymer films.

Making the films is straightforward. The polymer, dissolved in a volatile solvent, is dripped onto a glass slide fixed to a turntable. The faster the turntable spins, the farther the solution spreads and thins before the solvent evaporates. When a film has set, the slide is dipped into water, which lifts the film from the substrate. Surface tension ensures the film lies flat on the water. In their experiments, the UMass researchers worked with polystyrene films of nine different thicknesses from 31 nm to 233 nm cut to a diameter of 22.8 mm. Elasticity was adjusted by adding varying amounts of a plasticizer.

Relating observations to theory requires measuring the films' thickness, Young's modulus, and other elastic properties.

X-ray reflectivity, a standard thin-film technique, can determine thickness. To determine elastic properties, Menon first tried laying the film over a solid substrate that had a hole in it. Blowing air onto the film forced the film through the hole, visibly straining it. But the substrate also stressed the film by a hard-to-measure amount.

A better way originated in an observation that Menon and undergraduate Megan Juskiewicz made when they first started working with the films. If a drop of water happened to lie on top of a film, it caused radiating wrinkles like the ones that surround your finger when you poke a pillow. And wrinkling, as predicted by Cerda, Mahadevan, and Ravi-Chandar, could provide a way of determining elastic properties. Menon and Russell assigned the wrinkle project to their graduate student Jiangshui Huang.

The drops induce wrinkling not through their weight but through surface tension. At 88°, the contact angle between water and polystyrene is nearly vertical. The sides of the drop pull the film vertically, while water underneath resists, compressing the film horizontally and inducing wrinkles.

Huang observed the wrinkles through an optical microscope and imaged them with a digital camera. The accompanying figure shows two examples of different drop sizes and film thicknesses. The figure also shows that the wrinkles are distinct and of well-defined length. By adding water through a pipette in increments of 0.2 mg, Huang could reproducibly determine the equilibrium number of wrinkles for a given drop radius a and film thickness h . The number of wrinkles N varied as $a^{1/2}h^{-3/4}$. The thicker the film or the smaller

the load, the bigger the role played by compression in relieving the stress and the fewer wrinkles.

Cerda, who's at the University of Santiago in Chile, adapted his theory to the circular geometry and fluid substrate of the UMass experiment. His expression for N yielded values of the thickness that matched those determined by x-ray reflectivity and values of Young's modulus determined by indentation and buckling. He can also predict the length of the wrinkles.

The UMass team also observed hysteresis. In equilibrium, a sample polystyrene film 51 nm thick develops 84 wrinkles to support a drop of radius 0.41 mm; it develops 85 wrinkles to support a drop of radius 0.42 mm. Adding that extra wrinkle entails rearranging the entire pattern and constitutes an energy barrier. By adding water gradually, Huang could hit each successive equilibrium state. But as a drop evaporates, N doesn't fall smoothly in one-wrinkle steps. Rather, N lingers and then jumps down several steps at once. The dynamics of that process, which Menon plans to study, are nonlinear, fourth-order, and nonequilibrium.

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References

1. E. Cerda, K. Ravi-Chandar, L. Mahadevan, *Nature* **419**, 579 (2002).
2. J. Huang et al., *Science* **317**, 650 (2007).

