Books

Introduction to the Theory of Games. By J. C. C. McKinsey. 371 pp. McGraw-Hill Book Company, Inc., New York, 1952. $6.50.

It is trite but true that new fields of applied science induce parallel expansions in mathematics. New developments in operations research and in econometrics are utilizing and stimulating fields of mathematics quite different from the mathematics familiar to the physicist. The theory of games is one such field, closely related to the theory of linear programming and to the more basic theory of convex sets, which is far indeed from the differential and integral equations of modern physics, but which may be of interest to physicists who wish to explore unfamiliar territory.

The subject of "games of chance" provided the initial incentive for the study of the theory of probability by Pascal and Fermat. Here the principle "adversary" to the player was considered to be the chance fluctuations inherent in many games; from a study of the vagaries of thrown dice and shuffled cards the theory was developed to become the basis of actuarial practice and quality control. But in many games chance is not the primary adversary; in chess, for example, the uncertainty as to outcome is produced by the moves of the opponent, which are not randomly directed but which are nevertheless unknown in advance. And so we come to a different mathematical problem, where the outcome of the game is determined by the interrelated decisions of two or more players, each player having to make his decision without knowing all the intentions of his adversaries. Here the analysis of the problem of reaching the "best" or the "safest" decision must be quite unlike that of the theory of random events. Such analysis, only recently developed, is called the theory of games.

The first book on the subject, and the classic in the field, was by Von Neumann and Morgenstern, *The Theory of Games and Economic Behavior*, published in 1944. Almost immediately the application of the theory of two-person games was made to many tactical and strategic problems in warfare and by now this aspect of game theory is a basic technique in military operations research. Development of the theory in the past ten years has been chiefly along the lines of devising algorisms for solving specific problems, a task which is far from completion yet.

The volume under present review is the first general review of the subject since Von Neumann and Morgenstern; it includes most of the more recent developments. Accordingly, it is not surprising that most of the book is concerned with the two-person game, with its various military applications, and that the discussion of the three-or-more-person game is confined to a review of Von Neumann's earlier work and a discussion of its limitations. There is a full discussion of the general techniques for solving two-person games with a discrete set of possible strategies, together with worked-out examples, and a fairly complete review of the work up to present on the solution of games with a continuous range of strategies.

The book is carefully written and, considering the difficulties of the subject, quite understandable. Cross-references and illustrative examples are many, and the index is adequate. It is recommended for those who wish to delve more deeply into the subject than is possible by reading "popular" accounts of the subject such as McDonald's *Strategy in Poker, Business and War* or the soon forthcoming Williams', *The Compleate Strategyste.*

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Statistical Theory with Engineering Applications. By A. Hald. 783 pp. John Wiley & Sons, Inc., New York, 1952. $9.00.

One of the Wiley publications in statistics, this book maintains the high standard set by its predecessors in the series. It should prove a boon to all who seek knowledge of modern statistical techniques. Using only standard differential and integral calculus, the author, a professor of statistics at the University of Copenhagen, discusses the elements of the theory of probability and then gives a thorough exposition of the statistical theory developed during the past fifty years which has proved to have practical value.

The exposition is clear and concrete. It is based on the excellent premise that in teaching, theory and practice must go hand in hand and that the full significance of a theorem first dawns upon a student when he sees a practical application of it. Because of this, the book is full of examples, and in fact the author is quite willing to use ten pages discussing a simple example which will serve to introduce a general method or theorem. The examples are mostly from engineering applications. This is a refreshing change from the usual "plots" and "yield" data of the agricultural experiments so often quoted in statistical books.

Some of the topics treated in the book are the following: the fundamental concepts of probability and their applications to the binomial, multinomial, and hypergeometric distributions; the graphical and tabular representation of observations; definitions and properties of empirical and theoretical distributions; skew distributions such as the logarithmic normal; the fundamental limit theorems of probability such as the Central Limit Theorem and the Law of Large Numbers; the distribution of the mean, the variance and chi-square; statistical quality control; the analysis of variance and the design of experiments; linear regression; the Poisson distribution; and sequential analysis. Of

course, not all topics are treated with the same amount of detail but enough is given to introduce the subjects and to enable the reader to use it in simple applications.

One of the outstanding attractions of the book is its emphasis on practical applications. This is shown by the author's devoting an entire chapter to the distribution of the range, a topic which lacks mathematical elegance but which is useful in practice. Throughout the book, it is emphasized that the standard statistical techniques are derived for the normal distribution only but that in many cases skew distributions are required. Consequently, the author is careful to indicate what techniques can be used for skew distributions and more important to show how a skew distribution can be transformed to the normal by suitable transformations.

There are some defects. In a few cases the author's notation is not standard and this may prove a source of confusion to the student. For example, the author uses $U \dot{+} V$ for the logical sum of the classes and $U + V$ for the sum with the common part omitted. The standard notation for the first concept is $U + V$ and it would have been just as easy for the author to use the dot to indicate the second concept. Again, in the analysis of variance, the author uses u and v^2 instead of the well-known z and F. A more serious criticism is that, because of the arrangement of topics, some concepts are introduced before they are explained. For example, on page 36 an example involving tolerance limits is discussed while the concept is not formally introduced until page 311. Also, the "Studentized" range is mentioned before Student's distribution is treated. However, these examples of foreknowledge are few. On the whole, the reviewer feels that the book can be highly recommended.

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Physics and Medicine of the Upper Atmosphere. A Study of the Aeropause. Edited by Clayton S. White and Otis O. Benson, Jr. 611 pp. The University of New Mexico Press, Albuquerque, New Mexico, 1952. $10.00.

The fact that able scientists, including a Nobel laureate, would devote an appreciable amount of time to the problems of space travel and to the physics and medicine of the upper atmosphere is a striking indication of the rate at which the field of aviation and aviation medicine is progressing. This volume is the outgrowth of a symposium on the physics and medicine of the upper atmosphere held somewhat over a year ago.

While only a small amount of new factual data have been obtained, the bulk of the material in this volume can be regarded as authoritative. In any consideration of travel in the upper atmosphere, theory and conjecture hold primary sway. Nonetheless, as our knowledge of the ionosphere is widened by the use of rockets and guided missiles, a certain amount of reality can be interspersed. Problems are manifold, and how living organisms, including man, will behave under the conditions of pressure, temperature and variations of gravitational field that will occur is still in large part to be determined.

The medical department of the U. S. Air Force is to be congratulated in taking an active part in these extraordinary pioneering studies which may lead eventually to readjustment of man's relation to the universe.

This volume represents the thinking of adventurous-minded pioneers who have at the same time the moderating effect of their scientific knowledge.

Inevitably in a symposium a certain amount of duplication of material occurs and the various chapters in this volume are no exception. However, this is not as serious a handicap as it would be, were the concepts under consideration not so novel. While the volume unavoidably has a certain flair of Jules Verne and the comic strip, nonetheless the documentation is sound and the speculation restrained. Few interested in problems of flight and in the wider-flung frontiers of human knowledge can afford to neglect it.

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Electrodynamics. Lectures on Theoretical Physics. Volume III. By Arnold Sommerfeld. 371 pp. Academic Press Inc., New York, 1952. $6.80.

"I can best give you an idea of the sweeping changes in viewpoint brought about by the theory of Faraday and Maxwell by telling you of the time I spent as a student . . ."—so begins Part I (54 pp.) of the four parts that make up the text. After a brief historical review and discussion of basic concepts, Maxwell's equations in integral form are presented as the axiomatic foundations of electromagnetics. Auxiliary concepts and postulates are then introduced and their principal consequences deduced. Part I ends with a discussion of units and the selection of the rationalized MKSQ system for the major part of the text.

In Part II (156 pp.), the fundamental equations are used to deduce the basic phenomena of the static, stationary, quasi-stationary, and rapidly varying fields. The section on statics treats charging and induction problems, the sphere in a uniform field, capicitors, the field of the permanent bar magnet, and the field within an ellipsoid of revolution; here there are departures from the phenomenological viewpoint to discuss polarization and magnetization on the molecular level. Next, stationary currents and their magnetic fields, mutual and self induction, and the magnetic double layer are discussed; the fields of a straight wire and coil are treated in detail. This leads into a brief discussion of quasi-stationary currents: the free and forced vibrations of a circuit, the Wheatstone bridge, coupled circuits, and the telegraph equation. Part II ends with the treatment of rapidly varying fields, the retarded potentials, the Hertzian dipole, and cylindrically symmetric fields. Detailed applications to the field of a circularly cylindrical wire and coil, to waves on wires, to wave guides, and to the Lecher two-wire line are given.

Part III (67 pp.) deals with special relativity and the