

Books

Introduction to the Theory of Games. By J. C. C. McKinsey. 371 pp. McGraw-Hill Book Company, Inc., New York, 1952. \$6.50.

It is trite but true that new fields of applied science induce parallel expansions in mathematics. New developments in operations research and in econometrics are utilizing and stimulating fields of mathematics quite different from the mathematics familiar to the physicist. The theory of games is one such field, closely related to the theory of linear programming and to the more basic theory of convex sets, which is far indeed from the differential and integral equations of modern physics, but which may be of interest to physicists who wish to explore unfamiliar territory.

The subject of "games of chance" provided the initial incentive for the study of the theory of probability by Pascal and Fermat. Here the principle "adversary" to the player was considered to be the chance fluctuations inherent in many games; from a study of the vagaries of thrown dice and shuffled cards the theory was developed to become the basis of actuarial practice and quality control. But in many games chance is not the primary adversary; in chess, for example, the uncertainty as to outcome is produced by the moves of the opponent, which are not randomly directed but which are nevertheless unknown in advance. And so we come to a different mathematical problem, where the outcome of the game is determined by the interrelated decisions of two or more players, each player having to make his decision without knowing all the intentions of his adversaries. Here the analysis of the problem of reaching the "best" or the "safest" decision must be quite unlike that of the theory of random events. Such analysis, only recently developed, is called the theory of games.

The first book on the subject, and the classic in the field, was by Von Neumann and Morgenstern, *The Theory of Games and Economic Behavior*, published in 1944. Almost immediately the application of the theory of two-person games was made to many tactical and strategic problems in warfare and by now this aspect of game theory is a basic technique in military operations research. Development of the theory in the past ten years has been chiefly along the lines of devising algorithms for solving specific problems, a task which is far from completion yet.

The volume under present review is the first general review of the subject since Von Neumann and Morgenstern; it includes most of the more recent developments. Accordingly, it is not surprising that most of the book

is concerned with the two-person game, with its various military applications, and that the discussion of the three-or-more-person game is confined to a review of Von Neumann's earlier work and a discussion of its limitations. There is a full discussion of the general techniques for solving two-person games with a discrete set of possible strategies, together with worked-out examples, and a fairly complete review of the work up to present on the solution of games with a continuous range of strategies.

The book is carefully written and, considering the difficulties of the subject, quite understandable. Cross-references and illustrative examples are many, and the index is adequate. It is recommended for those who wish to delve more deeply into the subject than is possible by reading "popular" accounts of the subject such as McDonald's *Strategy in Poker, Business and War* or the soon forthcoming Williams', *The Complete Strategist*.

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Statistical Theory with Engineering Applications. By A. Hald. 783 pp. John Wiley & Sons, Inc., New York, 1952. \$9.00.

One of the Wiley publications in statistics, this book maintains the high standard set by its predecessors in the series. It should prove a boon to all who seek knowledge of modern statistical techniques. Using only standard differential and integral calculus, the author, a professor of statistics at the University of Copenhagen, discusses the elements of the theory of probability and then gives a thorough exposition of the statistical theory developed during the past fifty years which has proved to have practical value.

The exposition is clear and concrete. It is based on the excellent premise that in teaching, theory and practice must go hand in hand and that the full significance of a theorem first dawns upon a student when he sees a practical application of it. Because of this, the book is full of examples, and in fact the author is quite willing to use ten pages discussing a simple example which will serve to introduce a general method or theorem. The examples are mostly from engineering applications. This is a refreshing change from the usual "plots" and "yield" data of the agricultural experiments so often quoted in statistical books.

Some of the topics treated in the book are the following: the fundamental concepts of probability and their applications to the binomial, multinomial, and hypergeometric distributions; the graphical and tabular representation of observations; definitions and properties of empirical and theoretical distributions; skew distributions such as the logarithmic normal; the fundamental limit theorems of probability such as the Central Limit Theorem and the Law of Large Numbers; the distribution of the mean, the variance and chi-square; statistical quality control; the analysis of variance and the design of experiments; linear regression; the Poisson distribution; and sequential analysis. Of