Second Sound Propagation

ONE OF THE STRANGEST anomalies thus far exhibited by matter has been the special thermal wave property of liquid helium II, known as *second sound*. Characteristic of no other substance than helium II—that weird form adopted by liquid helium below the λ-temperature of 2.19°K—second sound is essentially an undamped thermal wave propagation. Such thermal waves display all the usual properties of wave phenomena, including resonance and reflection characteristics. This property of heat flow conforming to a wave equation, rather than to the classical diffusive heat flow equation, results in such seemingly paradoxical situations as heat flowing uphill against thermal gradients. Anomalous even in name, second sound never activates microphones and is generated by heat impulses rather than by mechanical impulses. Finally, its behaviour provides perhaps the most effective means for investigating and understanding the true nature of this so-called *quantum liquid*, helium II, and the associated quantum hydrodynamics.

Let us commence our discussion by considering a case of one-dimensional thermal propagation, in which heat pulses are introduced at one end face of a cylindrical enclosure containing helium II. Such heat pulses may be generated electrically and then detected, after a characteristic time delay in transit, by the temperature sensitive opposite end functioning as a bolometer. Using timing techniques analogous to radar, the results can be presented oscillographically as illustrated by the photograph of Fig. 1, where the horizontal time scale provides a direct measure of this delay time in terms of the number of calibrated marker pips (and thus the wave velocity).

The oscillogram of Fig. 1 illustrates rather simply the true wave nature of second sound propagation. That is, following the primary signal representing the directly arriving thermal wave packet, there appears another signal corresponding to roughly three times the initial delay time. This of course represents the heat pulses which have been reflected back from the receiver surface to the transmitter, and return. Fig. 1 thus demonstrates pictorially the reflectivity property for thermal waves in liquid helium II, which of course was inherent in the original thermal standing wave experiments of Peshkov.[1]

Thermodynamics of Pulses

We have introduced the subject of these heat pulses because their study reveals a great deal about the general behavior of second sound. At the risk of boring the reader with a few equations, some of the mathematical relationships for second sound propagation will be formulated on the basis of such pulses. In this manner we obtain the mathematical results without reference to the wave equation, other than to accept the shape-preserving feature of its solutions.

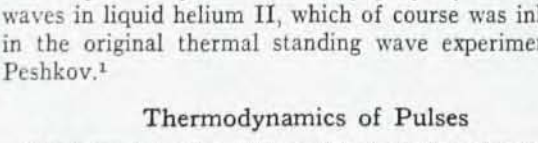

Fig. 1. Oscillogram of Second Sound Pulses. Direct pulse arriving after 9 delay marker intervals is followed by reflected (triple transit) pulse at 27 delay marker intervals.

Let us assume that a sudden one-dimensional heat pulse enters liquid helium II initially at ambient temperature T. If the heat current density $\dot{H}$ is constant during the pulse duration, a resultant square wave region of excess temperature τ will progress through the liquid at a constant wave velocity v_2 characteristic of the temperature T. This pulse is represented by (A) of Fig. 2.

Since the heat delivery $\dot{H}$ (erg $sec^{-1}cm^{-2}$) across any hypothetical normal plane (a-a) must represent the heat transported by the region of excess temperature τ proceeding at velocity v_2 through liquid of density ρ and specific heat capacity C (erg $gm^{-1}deg^{-1}$), we have

$$\dot{H}/\tau = \rho C v_2. \qquad (1)$$

Here we have divided through by τ in order to put the expression in the form of *characteristic thermal admittance*. Relationship (1) provides the basis for treating second sound propagation analogously to electrical or acoustical systems, which can always be done, and expresses the dependence of heat current on temperature variations (from the ambient) rather than on temperature gradient.

"Cold pulses" may also be propagated through liquid helium II. Thus in (B) of Fig. 2 we see such a cold pulse represented by a region within which the temperature is below the ambient T by amount τ. Instead of coinciding with the direction of wave propagation, the heat flows in this case in the reverse direction, toward the source of cold impulses, as indicated. At the pulse front (b) heat current $\dot{H}$ pours from the undisturbed region ahead into the pulse region cooler by amount τ. Since reversibility is inherent to the wave equation [solutions here being of the form $T(x \pm v_2 t)$], this constitutes a reversible heat flow between a source at temperature T and a heat receiver at temperature $T - \tau$. The reverse process occurs at the rear b′ of the pulse where heat actually flows from the cold interior up toward the ambient reservoir temperature following the pulse. Accordingly we may regard the cold pulse (B) as a self-contained thermodynamic unit constituting a reversible *heat engine* at the pulse front coupled to, and thus driving, a reversible *refrigeration* unit at the rear. This coupling is provided by the shape-sustaining property of the thermal pulses.

Such a system of heat flow into and out of a colder region is consistent with the second law of thermodynamics provided an appropriate amount of mechanical energy appears, and is in turn consumed, as the pulse passes. This requires the entirely new concept of a mechanical energy content in a region (to first order) of zero net mass flow and zero pressure fluctuations! Such a radical form of energy cannot be rationalized in

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n LIQUID HELIUM II

By John R. Pellam

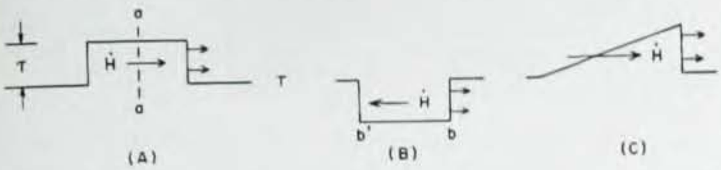

Fig. 2. Thermal Pulses. Temperature distributions (one-dimensional) in various thermal pulses progressing through liquid helium II; in each case shown here, the pulse is moving to the right. (A) Square-wave heat pulse, (B) square-wave cold pulse, and (C) saw-tooth heat pulse, showing uphill heat flow.

classical materials and, as we shall see, requires a new concept of liquid structure for visualization. We will not attempt for the moment to visualize the exact form taken by such energy, deferring this to the discussion of the two-fluid model. However we can at this point deduce from thermodynamic considerations the amount of this energy, and provide direct experimental verification of its existence.

Referring to (B) in Fig. 2, the fraction of heat flow $\dot{H}$ converted to such mechanical energy at the pulse front (b) must equal the usual ratio of temperature difference to absolute temperature. This rate of conversion constitutes a complex packet of mechanical energy flow γ (erg sec^{-1}cm^{-2}) transported with the pulse

$$\gamma/\dot{H}=\tau/T; \quad \text{or} \quad \gamma=\tau\dot{H}/T \tag{2}$$

(even though for cold pulses heat flows counter to the pulse propagation, this mechanical energy progresses with the pulse). Similarly at the rear of the pulse the reverse process occurs, corresponding to a refrigerator returning its working substance to the higher temperature T. In complete analogy with other wave propagations, the quantity γ representing energy flow is essentially the Poynting vector for heat waves! On this basis it is not difficult to see for example that within a saw-toothed shaped heat pulse, such as (C) in Fig. 2, the uphill heat flow is not a violation of the second law of thermodynamics, but rather a result of it. (Direct application of the second law to second sound waves was first made by Gogate and Pathak.[2])

The parallelisms to other types of wave propagation are manifest in many ways. For example, one of the thermodynamic requirements of such thermal wave propagation is that the peculiar mechanical energy of such a travelling second sound wave be divided equally between a kinetic energy form (depending on $\dot{H}^2$) and a potential energy form (depending on τ^2); this is in complete analogy with ordinary mechanical wave propagation. Such equidivision of thermal wave energy may be deduced easily by a modification of Rayleigh's early method for the equivalent acoustical case. That is, one examines the juxtaposition of identical square-wave heat pulses approaching from opposite directions and reconciles the quadratic dependence of γ on $\dot{H}$ [from (1) and (2)] with the conservation of mechanical energy.

If we divide energy flow γ of expression (2) by wave velocity v_2, we obtain simply the mechanical energy density. Then considering the equidivision of this energy between the two forms (i.e. one-half kinetic) we have for the kinetic energy density KE

$$KE=\tfrac{1}{2}(\dot{H})^2/(\rho c v_2{}^2 T) \tag{3}$$

(in units of erg cm^{-3}). Although this result is thermodynamically independent of any fluid model, we shall see shortly that the two-fluid model provides an excellent basis for visualizing both the kinetic and potential energy forms existing within second sound waves.

The Two-Fluid Hypothesis

Thus far we have deliberately avoided the two-fluid hypothesis in our discussion of second sound waves in liquid helium II. This was for the purpose of presenting the concept of second sound propagation on as purely a thermodynamic basis as possible. In this way we have been able to recognize some of its properties as quite general, and not dependent on any particular fluid model.

We must now introduce the two-fluid concept, not only as a means of visualizing such quantities as the kinetic and potential energy densities, but also for deriving an expression for the wave velocity of second sound. The two-fluid hypothesis was originally proposed by F. London [3] as a Bose-Einstein condensation phenomenon. This model enabled Tisza [4] to predict the existence of second sound and to foretell correctly some of its properties. Some time later Landau,[5] employing a somewhat different two-fluid hypothesis, independently predicted second sound, and deduced in fact the correct velocity behavior for temperatures all the way down to a few tenths of a degree above absolute zero.

The two-fluid hypotheses presuppose liquid helium II to be made up of two component liquids occupying the same space at the same time. One of these, the so-called *normal fluid* component, is responsible for all of the entropy of liquid helium II and also its viscosity; the *superfluid*, on the other hand, is considered totally devoid of both entropy and viscosity. The absence of viscosity in superfluid is so complete, in fact, that this component can actually flow through the normal fluid component without friction or interference! This situation may be handled most conveniently by ascribing separate flow fields to the two component fluids, so that momentum $\rho_n v_n$ is associated with normal fluid flow and $\rho_s v_s$ with superfluid flow; where ρ_n and v_n refer to density and particle velocity respectively for normal fluid, and ρ_s and v_s refer to superfluid.

We can thus write, in terms of ρ and v for the liquid as a whole,

$$\rho=\rho_n+\rho_s,$$

and, for second sound waves,

$$\rho v = \rho_n v_n + \rho_s v_s = 0$$
$$v_s = -\left(\frac{\rho_n}{\rho_s}\right) v_n. \quad (4)$$

The density equation simply expresses the composite density ρ as the sum of the component densities. Equation (4) states the condition of zero net momentum associated with second sound waves (recalling that microphones are not affected), and specifies the remarkable condition that within these thermal waves the two fluid components are actually flowing directly through each other in opposite directions! This "internal counterflow" occurs along the direction of wave propagation (as does also the heat flow) so that in this sense second sound may be considered "longitudinal".

We may now specify the quantities discussed earlier more definitely in terms of this two-fluid system. For example, the kinetic energy density KE may be written directly in terms of the component particle velocities and densities

$$KE = \tfrac{1}{2}\rho_n v_n^2 + \tfrac{1}{2}\rho_s v_s^2. \quad (5)$$

Furthermore, since the entropy of liquid helium II resides entirely within the normal fluid component, v_n may be related directly to heat current density $\dot{H}$. Referring once again to (A) of Fig. 2, the entropy flow $\dot{H}/T$ supported within the pulse by this normal fluid component is given by the London expression

$$\dot{H}/T = \rho S v_n, \quad (6)$$

where S is the entropy (in erg gm^{-1}deg^{-1}) of liquid helium II. For this we have visualized the flow of heat as a mass transport process associated with the motion of normal fluid component, but completely unaffected by the counterflowing and "thermally empty" superfluid.

Finally we can write for the "mechanical" expression for kinetic energy density KE

$$KE = \frac{1}{2}\frac{\rho\rho_n}{\rho - \rho_n}(\dot{H}/\rho S T)^2. \quad (7)$$

As we shall see, this relationship can be equated to our earlier "thermodynamic" expression (3) for this same quantity, to give an expression for second sound velocity. Before going on to this, however, we shall consider an experiment bearing on the above subject.

We have already noted that the existence of this kinetic energy density within such thermal waves can be verified by a direct mechanical test. Thus far we have treated second sound propagation as a purely thermal phenomenon, both in excitation and detection. We have in fact emphasized that ordinary acoustical devices, such as vibrating sources and microphones, are ineffective for dealing with these waves. Nonetheless, as we have also seen in the foregoing, a mechanical energy content is fundamental to the existence of such waves, and can be observed mechanically by appropriate methods.

Direct observation of this kinetic energy is made possible by a device invented by Lord Rayleigh for measuring acoustic energy. It is noteworthy that this Rayleigh disk, developed in the century before microphones, is capable of detecting the internal counterflow of quantum hydrodynamics to which modern microphones are deaf. Rayleigh [6] suspended a small disk within the sound field of a resonant acoustic cavity and oriented at an angle $\pi/4$ to the axis of wave propagation. For conditions of resonance this sensitive disk was deflected slightly by a torque tending to swing it cross-wise to the direction of wave-motion. König showed by integrating the Bernoulli pressure over the surface of such a disk of radius a that this torque was given by $(4a^3/3)\rho v^2$, in terms of undisturbed fluid velocity v.

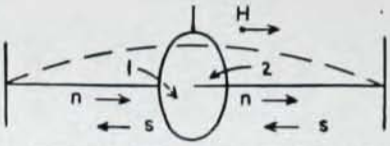

Fig. 3a. Thermal Rayleigh Disk Experiment. Thin disk suspended by sensitive fiber within resonant second sound field swings slightly crosswise to propagation axis from equilibrium 45° orientation. Heat-flow distribution $\dot{H}$ indicated by dotted line; internal counterflow of normal fluid component (n) and superfluid (s) around disk develops torque-producing "internal stress" in liquid.

For the purpose of illustrating this mechanism we can refer to Fig. 3a. Although this diagram represents the present application to second sound, it enables us to visualize the process for Rayleigh's classical application also. Thus during the portion of the cycle during which the particle flow is from left to right, a stagnation point is formed where particles encounter the disk on the far left side (1). Similarly a stagnation point occurs on the near right side (2). At the same time, however, unimpeded streamline flow takes place tangentially past the corresponding opposite sides of the disk. The resultant Bernoulli pressure difference across the thickness of the disk provides a torque as shown. It is easily seen that an identical pressure distribution is set up during the opposite half of the cycle. The quadratic nature of the König expression requires this condition so that the disk acts as an acoustic detector.

The special application [7] to thermal waves in liquid helium II stems from this quadratic dependence on particle flow. Thus we can see from Fig. 3a that each of the two counterflowing fluid components should exert its contribution to torque independently of the other. That is, both terms of the kinetic energy expression (5) exert their separate influences on the disk, giving

$$\text{Torque} = \frac{4}{3}a^3\rho_n v_n^2 + \frac{4}{3}a^3\rho_s v_s^2$$
$$= \frac{4}{3}a^3\frac{\rho\rho_n}{\rho - \rho_n}(\dot{H}/\rho S T)^2. \quad (8)$$

Thus in terms of the two-fluid concept the resultant torque produced by second sound on the Rayleigh disk is visualized as the sum of the torques exerted by normal fluid and superfluid separately as they stream past the disk in opposite directions! And this occurs in the complete absence of any detectable acoustic-type pressure fluctuations or momenta, in an otherwise perfectly quiescent medium.

Experimental confirmation of the König formulation extended to thermal waves is shown in Fig. 3b, where

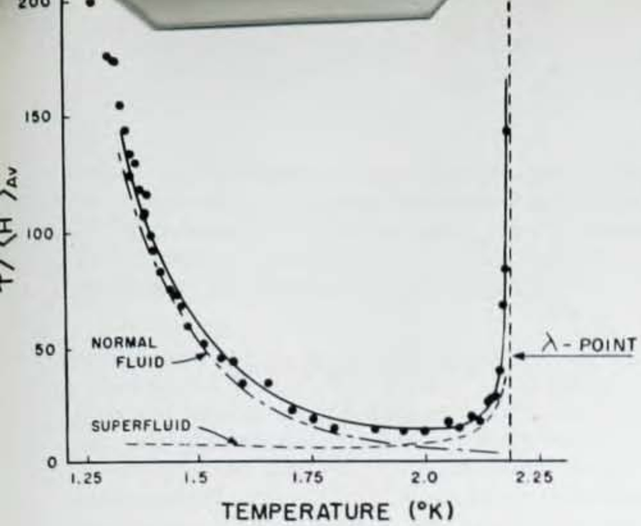

Fig. 3b. Torque on Disk versus Temperature. Torque ratio ($\tau/<H^2>_{AV}$) versus temperature T(°K) for thermal Rayleigh disk in liquid helium II. Circles represent experimental values, and solid line gives the theoretical value, equation (8). Dotted lines represent separate contributions of the component fluids as indicated.

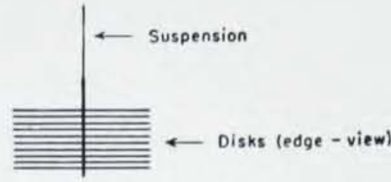

Fig. 4. Andronikashvilli Experiment. Parallel disks suspended for rotational oscillation in liquid helium II.

Torque/$(\dot{H})^2_{AV}$ is plotted versus temperature T. It may be observed that the torque exerted by superfluid is greatest near the λ-point, where superfluid is scarce; and similarly, the effect of normal fluid is greatest at the lowest temperatures where its concentration is low. These effects are the direct result of the zero momentum condition (4), requiring the minority component to travel faster, plus the quadratic dependence of torque on particle velocity (8) more than off-setting the decreased density.

Wave Velocity of Second Sound

As in other forms of wave propagation, the velocity of second sound is the most readily measurable quantity associated with the phenomenon. It is also perhaps the most physically significant. We already have the expressions from which this wave velocity v_2 may be written; combining equations (3) and (7) for the thermodynamic and mechanical expressions for mechanical energy *KE*, we have the Tisza-Landau equation

$$v_2^2=\left(\frac{\rho-\rho_n}{\rho_n}\right)\frac{S^2T}{C}. \qquad (9)$$

That is, second sound velocity v_2 is the quantity relating these two energy expressions.

We note that equation (9) is essentially a thermodynamic expression, giving us a great deal of insight to the behavior of second sound. For example, near the λ-point where superfluid disappears ($\rho-\rho_n \rightarrow 0$) the wave velocity drops to zero. At temperatures in the 1°K–2°K range, the value of $(\rho-\rho_n)/\rho_n$ may be determined by an independent mechanical measurement, the Andronikashvilli experiment, thus affording a check of (9). Finally at temperatures below 1°K, where v_2 can still be measured directly but ρ_n cannot, expression (9) provides an indirect evaluation of ρ_n. We next consider Andronikashvilli's direct measurement of ρ_n.

Andronikashvilli [8] suspended a set of closely-spaced disks in liquid helium II on a torsion fiber, as shown in Fig. 4, and observed the dependence upon temperature of the angular rotation period of the system. Now it is well known by experiment that the exceptional heat flow properties of liquid helium II are suppressed in narrow channels (presumably a close correlation between entropy and viscosity). Andronikashvilli spaced these plates so close together that the heat content, and thus the normal fluid, would necessarily be carried with the disks during their angular oscillations. At the same time, the completely non-viscous superfluid component would ignore the motion of the disks and remain stationary. Accordingly, by observing the period of this torsion pendulum he was able to measure the effective mass ρ_n associated with the normal fluid component of the helium (subtracting of course the background moment of the torsion pendulum itself).

Actually what is really involved in applying these results for ρ_n to equation (9) is to express second sound velocity in terms of Andronikashvilli's observed entropy moment, *viz.*

$$v_2^2=\left(\frac{I_0-I}{I}\right)\left(\frac{S^2T}{C}\right). \qquad (10)$$

Here I represents the effective moment of inertia of the system attributable to normal fluid density, and I_0 the moment at the λ-point (i.e. where the liquid is entirely normal fluid). The known correctness of expression (10) in the 1°K–2°K range thus merely expresses a consistent relationship between two different types of thermo-mechanical experiments. The role of the two-fluid concept here has really been to provide a vehicle for relating such experiments, and formulation (10) is the truly basic one.

The over-all second sound behavior is illustrated in Fig. 5 where wave velocity (m/s) is plotted *vs* temperature (°K) from the λ-point down to a few hundredths of a degree Kelvin above absolute zero. The solid curve (Peshkov-Pellam-Herlin) in the region above 1°K shows the velocity behavior in the upper temperature range where Tisza's and Landau's results agree [given by (9) and/or (10)], and illustrates the rapid decrease to zero near the λ-point as the liquid becomes all normal fluid.

The results in the lower half of the temperature range confirm the qualitative correctness of Landau's early prediction [5] that second sound velocity would *in-*

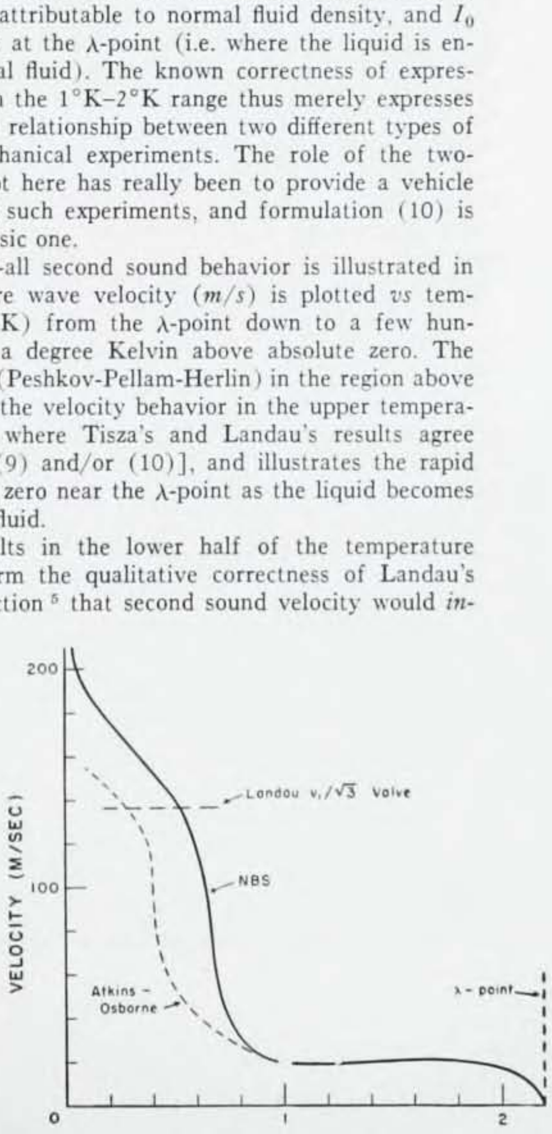

Fig. 5. Second Sound Wave Velocity. Second sound velocity (m/s) versus temperature T from the λ-point (2.19°K) down to a few hundredths of a degree above absolute zero. In the upper temperature range (1°K–2°K roughly) the curve (Peshkov-Pellam-Herlin) agrees favorably with both Landau's and Tisza's predictions. In the range below 1°K the velocity rises as predicted by Landau. The dotted curve represents Atkins's and Osborne's data; the solid curve below 1°K represents more

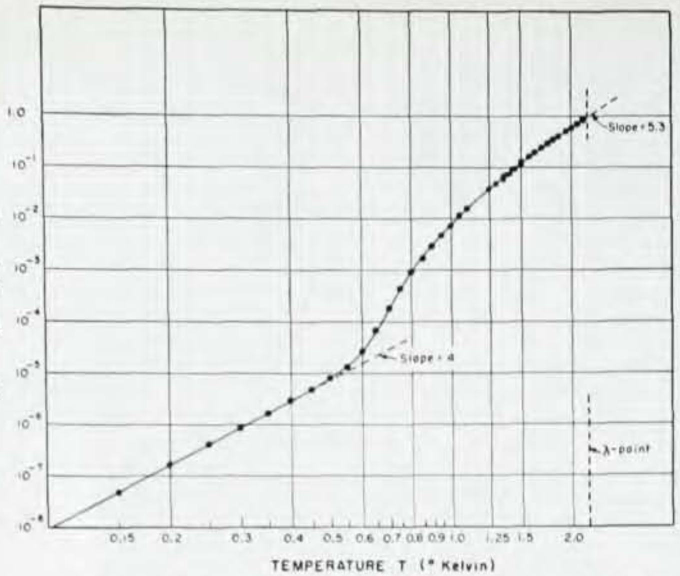

Fig. 6. Normal Fluid Concentration. Log-log plot of $\rho n/\rho$ for liquid helium II vs temperature T. Above 1.2°K data are Andronikashvilli's direct measurements; below 1.2°K results are deduced indirectly from velocity measurements, using Eq. (6). Below about one-half degree Kelvin, $\rho n/\rho$ obeys the T^4 behavior predicted by Landau.

crease drastically at temperatures just below 1°K. Measurements at these extreme low temperatures had to await the application to the problem of cooling by adiabatic demagnetization. First steps in this direction were taken at the National Bureau of Standards in 1949, when a sample of liquid helium II was cooled sufficiently to observe a doubling in wave velocity. The pulse technique was used and a velocity increase from 18.4 *m/s* to 34 *m/s* measured at temperatures well below 1°K, thus strongly favoring Landau's treatment. Some time later Atkins and Osborne [9] extended such investigations down to much lower temperatures, observing a gross velocity increase to values apparently tapering off at about 150 *m/s*, leaving little doubt about the over-all correctness of Landau's predictions. The dotted line of Fig. 5 gives these results and provides the general shape of the velocity curve.

The solid line below 1°K in Fig. 5 represents second sound velocity measured relatively recently [10] at NBS (de Klerk, Hudson, and Pellam) under conditions more closely approaching temperature equilibrium (i.e. warm-up times of one-half hour). At roughly one-half degree Kelvin there is a partial levelling-off in the neighborhood of Landau's predicted upper limit $v_1/\sqrt{3}$, where v_1 is the first (ordinary) sound velocity. However, it is also evident that at still lower temperatures the velocity continues to rise, apparently to an eventual value nearer to v_1 than the "Landau velocity" $v_1/\sqrt{3}$. Thus although Landau was essentially correct in his prediction of sharply increased velocity below 1°K, his treatment clearly requires some refinements to account for the continued velocity increase.

As mentioned earlier, at temperatures below 1°K where an Andronikashvilli measurement would be futile (ρ_n/ρ is less than 1 percent below 1°K), the second sound velocity determinations lead to an indirect evaluation of normal fluid concentration. Using these velocity results in conjunction with equation (9), ρ_n/ρ is given in Fig. 6 down to about 0.1°K. These determinations are plotted versus temperature on a log-log basis to illustrate the approach below about one-half degree Kelvin to the T^4 behavior predicted by Landau (to be discussed shortly). Note the extremely low normal fluid concentrations which can be determined by these second sound measurements—ρ_n/ρ is but one part in 100 million at 0.1°K!

The Phonon Gas Concept

Landau's correct anticipation of the increase in second sound velocity below 1°K was based primarily upon his special consideration of the phonon gas behavior at temperatures approaching absolute zero. Phonons may be regarded as quantized sound excitations, conforming in many ways to the behavior of photons. Thus an assemblage of phonons may be pictured as a (sound) radiation gas and as such displays the same T^4 total heat content of black body radiation, with the attendant Debye T^3 behavior of entropy and specific heat. There is an important difference, however, in regard to interactions between phonons. Whereas photons do not interact directly, maintaining equilibrium distribution rather through interaction with the container wall, phonons suffer direct collisions with each other. Thus phonons may be regarded on a "particle" basis, and the influence of effects attributable to "mean-free-path lengths" between collisions may become important.

Perhaps the basic viewpoint most contributing to Landau's success in predicting second sound behavior near absolute zero was his recognition of the "radiation mass" of these phonon excitations as normal fluid density. This was consistent with Landau's insistence on treating the partition of the liquid in terms of thermal energy, associating all thermal excitations of any kind with a normal fluid, rather than to regard specific groups of atoms as superfluid and other specific groups as normal fluid, as did Tisza. Landau essentially deduced the moment of inertia associated with a hypothetical Andronikashvilli experiment near absolute zero by computing the net momentum associated with the phonon gas carried between hypothetical disks. This involved an integration over the classical sound momentum $p = \epsilon/v_1$ for phonons of energy ϵ (and first sound velocity v_1) obeying a Bose-Einstein energy distribution. We shall not go into the details of this rather involved integration here, but the resulting effective density ratio due to the phonon gas becomes

$$\frac{\rho_n}{\rho} \sim \frac{4}{3}\frac{E_{ph}}{v_1^2}, \tag{11}$$

where E_{ph} is the energy (per gram). Although we refer to this quantity as the ratio of normal fluid density to liquid helium density, we tacitly understand that it really represents the ratio I/I_0 of an imaginary Andronikashvilli experiment.

Relationship (11) for ρ_n/ρ (or I/I_0), plus the dependence of E_{ph} on the fourth power of temperature, are sufficient for evaluating equation (9)—or (10)—for second sound velocity. Thus $C_{ph} = 4E_{ph}/T$ and $S_{ph} = 4E_{ph}/3T$ (where the symbols C_{ph} and S_{ph} represent phonon specific heat and entropy, respectively), and substitution into (9) leads directly to the well-known Landau velocity.

$$v_2 = v_1/\sqrt{3} \quad (12)$$

While appearing somewhat remarkable at first encounter, this Landau velocity is actually almost a fundamental requirement of thermal propagation in a phonon gas. For with constant phonon velocity v_1 (phonons all must travel at the velocity of sound!) the average individual messenger velocity along any particular axis becomes $v_1/\sqrt{3}$, and it is indeed difficult to see how a signal could be transmitted at any other velocity! The situation is easier to visualize than the propagation of ordinary sound in air, where the Newton velocity related to mean particle speed requires the Lagrange correction for variation in molecular velocities. (A simplified derivation of the Landau velocity based on the phonon gas picture has been given by Ward and Wilks,[11] following a treatment by de Hoffmann and Teller[12] for sound propagation in a relativistic gas.)

Role of Particle Statistics

Thus far we have discussed the entire subject of liquid helium II without direct reference to the role played by the fundamental particle statistics. Although we have relied upon the two-fluid model often during the foregoing, we have not considered the question of *why* there should be two fluid components in liquid helium II. In fact, even in discussing Landau's essentially correct computations of second sound velocity it did not appear necessary to introduce the subject of particle statistics. This seems somewhat surprising, particularly in view of the marked differences known to exist between the liquid properties of helium 4 and the rarer isotope helium 3.

In London's original proposal of the two-fluid model of liquid helium II as an example of a Bose-Einstein condensation, he intimately related the properties to the even-particle nature of helium 4. And by the same token, the possibility of direct experimental verification of this straight-forward hypothesis was provided in terms of the properties of liquid helium 3; for the odd-particle helium 3 should display Fermi-Dirac behavior, and thus no λ-point nor superfluidity properties. The subsequent liquefaction of helium 3 and verification[13] that no transition of the helium I–helium II type occurred, at least down to a few tenths degree Kelvin, substantially supported the London hypothesis.

Accordingly we may justifiably ask at this point how Landau made his correct predictions regarding second sound behavior on the basis of a theory apparently independent of the Bose-Einstein condensation hypothesis. The answer is probably that, by properly associating normal fluid density with *all* thermal excitation in liquid helium II, the generally correct behavior can be associated with *any* two-fluid model, given sufficient flexibility in arbitrary parameters. In Landau's case, he appears arbitrarily to have chosen a two-fluid model in which phonons contribute to the excited state at the lowest temperatures, and in which he ascribed the rapidly augmented energy content of the excited state above one-half degree Kelvin to *rotons*. By assigning a convenient energy gap Δ to these *rotons*, and introducing other arbitrary parameters, he was able to fit the corresponding entropy expressions to known values, and thus produce a mathematically valid result.

The success of the Landau treatment need not detract at all from the validity of London's condensation hypothesis. Tisza's calculation notwithstanding, there is every reason to believe that the entire problem can be treated on the basis of the Bose-Einstein hypothesis. Actually in his original derivation London mentioned the existence of Debye waves in the Bose-Einstein liquid, and the picture presented earlier of thermal signals transmitted by phonon collisions should apply with the same resultant velocity $v_1/\sqrt{3}$ near absolute zero; the only requirement is the truism that phonons travel with the velocity of sound!

The London hypothesis has the special advantage of not requiring any new concepts, such as rotons, or other devices to explain the drastic increase of specific heat with temperature between one-half degree Kelvin and the λ-point, because this behavior is inherent in a condensation model. Perhaps the weakest feature of the condensation model to date has been the arbitrary application of an essentially gas model to a liquid state. While the basic condensation property of an even-particle substance should persist as well for the liquid state, other considerations must be introduced to make the situation more physically realistic.

Of these probably the most important concerns the zero-point energy of liquid helium which is credited with preserving the liquid state down to absolute zero for either helium 3 or helium 4. A more detailed quantum-mechanical treatment of liquid helium II should take into account not only the effects of the particle statistics but also such background effects associated with the liquid state itself as this zero-point energy. It could reasonably be expected that an analysis of this nature, with the corresponding modifications in distribution function, ought to result in the same numerical results as the Landau treatment but with determined, rather than adjusted, constants.

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