letters

Boundary layers, Prandtl's and others

As John D. Anderson Jr pointed out in his excellent article (PHYSICS TODAY, December 2005, page 42), Ludwig Prandtl was a leader in developing the concept of two-dimensional boundary layers, so important in aerodynamics and related fluid problems in which the flow does not change direction with distance from the boundary. However, the equally important and slightly earlier work of Vagn W. Ekman[1] deserves equal exposure and recognition. Ekman was the first to develop the concept of 3D boundary layers, those in which rotation (or curved flow) in a viscous fluid causes a boundary layer with a well-defined depth. The depth of such boundary layers is quite generally $(\nu/\Omega)^{1/2}$, the Ekman depth, where ν is the kinematic viscosity (or an appropriate eddy viscosity) and Ω is some appropriate rotation rate.

Ekman was inspired by reports that icebergs in the Northern Hemisphere generally drifted at an angle to the right of the wind direction, and he sought an explanation in the effect of Earth's rotation. Prior to Ekman's discovery, oceanographers had often assumed that the wind acting on the ocean would produce a surface current in the wind direction; if the wind persisted long enough, they thought, it would lead to a linear decrease of the current from the top of the ocean to the bottom. Ekman discovered that because of the Coriolis force the effect of wind stress would be limited to a boundary layer around 5 to 50 meters deep, depending on wind speed and turbulence. Moreover, for a steady wind stress the net transport in the boundary layer would be exactly 90 degrees to the wind stress and independent of the amount of turbulence, to the right in the Northern Hemisphere and to the left in the Southern.

For the ideal condition of constant viscosity, Ekman found an exact analytical solution for his spiral boundary-layer flow. He also found a second spiral solution for the case in which the mixing coefficient is proportional to the square of the rate of gliding. Curiously, that second solution has a defined finite depth below which the wind has no direct effect. Ekman also conducted laboratory experiments with wind over a rotating tank of water; they clearly showed the predicted effect of the Coriolis force. In an ordinary laboratory case, a laminar Ekman boundary layer is about 1 mm deep. Ekman also recognized that similar turbulent boundary layers occur in the atmosphere and at the bottom of the ocean due to flow over rigid boundaries. Theodore von Karman and U. T. Boedewadt[2] also found analytical solutions to 3D boundary layers: von Karman to the flow due to a rotating disk in a stationary fluid and Boedewadt to the boundary layer beneath a vortex in solid rotation over a stationary boundary. Both of these spiral boundary layers are rather like the Ekman spiral and sometimes are loosely referred to as Ekman layers.

The stability and transition to turbulence in 3D boundary layers is today perhaps of more general application (airfoils, curved pipes, rotating machinery, curved rivers, and so on) than Prandtl's 2D boundary layer. In the geophysical sciences, the wind-driven Ekman transport in the surface Ekman layer is fundamental to all theories of ocean circulation, and in the atmosphere the Ekman spiral and transport toward low pressure are fundamental to theories of hurricanes and all atmospheric vortices.

In past years when I discussed my studies of the Ekman boundary layer with friends in the physics community, the response frequently was "oh yes, the Einstein teacup effect," as though Einstein was the first in this area of study. But when one reviews Einstein's paper[3] it is clear that the notion of a thin boundary layer was absent from his work. So perhaps this note will correct some misimpressions.

References

1. V. W. Ekman, *Archive Math. Astron. Phys.* **2**, 11 (1905).
2. T. von Karman, *Z. Angew. Math. Mech.* **1**, 233 (1921); U. T. Boedewadt, *Z. Angew. Math. Mech.* **20**, 241 (1940).
3. A. Einstein, *Ideas and Opinions*, Crown Publishers, New York (1954), p. 250.

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The concept of the boundary layer is illustrated on a spectacular scale in the circulation of the oceans. While Ludwig Prandtl's boundary layers are regions of slow flow (relative to the boundary), the peculiarities of dynamics on a rotating sphere allow for a viscous boundary layer consisting of an intense, relatively narrow jet at the western edge of the ocean.[1] This is the explanation for major currents such as the Gulf Stream in the Atlantic Ocean and the Kuroshio in the Pacific. Each current is about 100 km wide and 1000 km long, and transports more than 30 million tons of water per second along the coast at speeds about 100 times greater than the average speed outside the jet. Such jets are important ocean features that affect Earth's climate.

Reference

1. See, for example, J. Pedlosky, *Ocean Circulation Theory*, Springer, New York (1996).

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John Anderson's article on Ludwig Prandtl's boundary layer is both interesting and informative. More recently, beginning in 1961, the boundary layer concept has been applied to flow about a type of surface called "continuous."[1] A characteristic of flows over continuous surfaces is that, for any given period, any two solid surface elements exhibit different drag-time histories, as contrasted with finite-surface flows, in which all surface elements exhibit equal drag-time histories. As a result the formation and termination of the boundary layer are not identified with any part of the surface, but are determined by the system's boundaries.

Flows over continuous surfaces constitute a new class of boundary-layer problem. Although the differential equations governing flow around the

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continuous and finite surfaces are the same, the boundary conditions are different, which results in substantially different solutions for the two types of surfaces. Continuous surfaces are primarily encountered in industrial processes—for example, in fiber spinning,[2] sheet casting,[3] and film coating[4]—where the production of such surfaces is technically feasible and economically desirable.

References

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3. P. J. Abraham, E. M. Sparrow, *Int. J. Heat Fluid Flow* **26**, 289 (2005).
4. S. J. Weinstein, K. J. Ruschak, *Annu. Rev. Fluid Mech.* **36**, 29 (2004).

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Anderson replies: The letters from Alan Faller, Barry Klinger, and Byron Sakiadis are a welcome addition to my article on the boundary-layer concept. Faller correctly points out the work of Vagn Ekman on three-dimensional boundary layers in rotating fluids. Ekman's discovery, and his subsequent experimental and analytical work, was contemporary with Ludwig Prandtl's and is an interesting example of an idea or concept whose time had come.

The history of science and technology is replete with such examples. The invention of the first successful heavier-than-air, pilot-controlled flying machine was an idea whose time had come at the beginning of the 20th century. The Wright brothers were simply the first to make it happen.

Both Faller and Klinger point out the role of the boundary-layer concept on the study of the motion of oceanic flows and circulation, a dimension of the concept not addressed in my article, written by an aerodynamicist from an aerodynamicist's point of view. Sakiadis discusses an even more general application to flows over continuous surfaces.

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Saving our view of the night skies

Of the many forms of pollution caused by humans, none is more obvious than the misuse of lighting. Unwanted and unwelcome light trespass began with the invention of the electric light and has expanded in proportion to the electrification of the planet (see PHYSICS TODAY, June 2005, page 24). The major sources of light pollution include street lighting, outdoor sports arenas, the promotional lighting of buildings and monuments, and car sales lots.

Simply put, light pollution is blinding ground-based astronomy. Extensive light pollution is gradually and inexorably reducing the quality and utility of astronomical observations made from major observatories all over the planet. To add insult to injury, more recently a number of medical researchers have discovered a correlation between extensive light pollution and an emerging number of serious human disorders.[1]

The International Dark-Sky Association (IDA) was founded by Kitt Peak astronomer David Crawford more than 18 years ago, and is made up of concerned astronomers and others who are accepting the challenge to slow down the continually advancing march toward brighter and brighter night skies. The IDA has taken the unusual stance of enlisting the major utilities, lighting manufacturers' associations, professional lighting engineering societies, and municipalities in finding better ways to achieve the objectives of lighting for safety and commerce without denigrating the natural dark sky environment. Organizations that have joined in supporting the IDA include the Illuminating Engineering Society of North America, the International Association of Lighting Designers, the Illuminating Engineering Society of Australia and New Zealand, and the Vienna-based International Commission on Illumination.

Although adopting coherent and effective lighting ordinances can provide immediate gains—including less energy consumption, improved security, and reductions in glare and light trespass—the improvement in preserving our dark skies for astronomical research is insufficient. The institution that has the responsibility to advance astronomical sciences in the US is the venerable American Astronomical Society. However, that organization is failing to address the problem. Although AAS has always agreed with the goals of the IDA and has written numerous papers and articles in support of the IDA's efforts to reduce light pollution, it has failed to aggressively encourage its members to join and support the IDA. Of all the members of the physics community, AAS has the most to lose by not directly supporting the IDA.

I call on the US physics community and all readers of PHYSICS TODAY to support the IDA, with your time and treasury, and to actively engage in any of the IDA's many sections throughout the world. Failure to act today will doom future generations from ever experiencing the wonder of seeing the stars from a dark and natural site and will ensure that the dark skies can only be experienced through planetariums and other artificial media. Astronomers of the future will only speculate on why this generation did not take the lead in protecting our dark skies when the opportunity was so clearly and readily available.

Reference

1. See, for example, B. Harder, *Science News* **169**, 8 (2006).

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[Editor's note: We invited a response from Craig Wheeler, president of the American Astronomical Association, and Kevin Marvel, AAS's executive officer.]

Wheeler and Marvel reply: The International Dark-Sky Association does valuable work to preserve precious dark skies. Through education, outreach, and consultation with lighting designers and policymakers, the association promotes more efficient use of nighttime lighting to reduce light pollution. We endorse the association's activities. The American Astronomical Society has been an affiliated organization of the IDA for years, one of us is a lifetime member, and many AAS members and astronomy institutions are active in the association as well. However, we believe that the IDA will draw members principally through its accomplishments and the value they provide.

The membership of the IDA is growing rapidly. The association currently has nearly twice as many members as the AAS. Even if all US AAS members joined, IDA membership would only increase by about 20%. Roberts correctly states that the IDA is composed of "astronomers and others." What he does not make clear is that the others far outnumber the astronomers who are members, and rightly so. Light pollution predominantly affects the general public. Poor nighttime lights threaten our health and safety, put migrating wildlife at risk, waste energy and money, and deprive countless millions of the beauty of the nighttime sky.

Light pollution, radio-frequency interference, and space debris are all issues of concern to the AAS. All of these detrimental aspects of human technology can potentially limit our ability to make astronomical observations. Light pollution certainly does negatively