letters

Boundary layers, Prandtl's and others

As John D. Anderson Jr pointed out in his excellent article (PHYSICS TODAY, December 2005, page 42), Ludwig Prandtl was a leader in developing the concept of two-dimensional boundary layers, so important in aerodynamics and related fluid problems in which the flow does not change direction with distance from the boundary. However, the equally important and slightly earlier work of Vagn W. Ekman[1] deserves equal exposure and recognition. Ekman was the first to develop the concept of 3D boundary layers, those in which rotation (or curved flow) in a viscous fluid causes a boundary layer with a well-defined depth. The depth of such boundary layers is quite generally $(\nu/\Omega)^{1/2}$, the Ekman depth, where ν is the kinematic viscosity (or an appropriate eddy viscosity) and Ω is some appropriate rotation rate.

Ekman was inspired by reports that icebergs in the Northern Hemisphere generally drifted at an angle to the right of the wind direction, and he sought an explanation in the effect of Earth's rotation. Prior to Ekman's discovery, oceanographers had often assumed that the wind acting on the ocean would produce a surface current in the wind direction; if the wind persisted long enough, they thought, it would lead to a linear decrease of the current from the top of the ocean to the bottom. Ekman discovered that because of the Coriolis force the effect of wind stress would be limited to a boundary layer around 5 to 50 meters deep, depending on wind speed and turbulence. Moreover, for a steady wind stress the net transport in the boundary layer would be exactly 90 degrees to the wind stress and independent of the amount of turbulence, to the right in the Northern Hemisphere and to the left in the Southern.

For the ideal condition of constant viscosity, Ekman found an exact analytical solution for his spiral boundary-layer flow. He also found a second spiral solution for the case in which the mixing coefficient is proportional to the square of the rate of gliding. Curiously, that second solution has a defined finite depth below which the wind has no direct effect. Ekman also conducted laboratory experiments with wind over a rotating tank of water; they clearly showed the predicted effect of the Coriolis force. In an ordinary laboratory case, a laminar Ekman boundary layer is about 1 mm deep. Ekman also recognized that similar turbulent boundary layers occur in the atmosphere and at the bottom of the ocean due to flow over rigid boundaries. Theodore von Karman and U. T. Boedewadt[2] also found analytical solutions to 3D boundary layers: von Karman to the flow due to a rotating disk in a stationary fluid and Boedewadt to the boundary layer beneath a vortex in solid rotation over a stationary boundary. Both of these spiral boundary layers are rather like the Ekman spiral and sometimes are loosely referred to as Ekman layers.

The stability and transition to turbulence in 3D boundary layers is today perhaps of more general application (airfoils, curved pipes, rotating machinery, curved rivers, and so on) than Prandtl's 2D boundary layer. In the geophysical sciences, the wind-driven Ekman transport in the surface Ekman layer is fundamental to all theories of ocean circulation, and in the atmosphere the Ekman spiral and transport toward low pressure are fundamental to theories of hurricanes and all atmospheric vortices.

In past years when I discussed my studies of the Ekman boundary layer with friends in the physics community, the response frequently was "oh yes, the Einstein teacup effect," as though Einstein was the first in this area of study. But when one reviews Einstein's paper[3] it is clear that the notion of a thin boundary layer was absent from his work. So perhaps this note will correct some misimpressions.

References

1. V. W. Ekman, *Archive Math. Astron. Phys.* **2**, 11 (1905).
2. T. von Karman, *Z. Angew. Math. Mech.* **1**, 233 (1921); U. T. Boedewadt, *Z. Angew. Math. Mech.* **20**, 241 (1940).
3. A. Einstein, *Ideas and Opinions*, Crown Publishers, New York (1954), p. 250.

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The concept of the boundary layer is illustrated on a spectacular scale in the circulation of the oceans. While Ludwig Prandtl's boundary layers are regions of slow flow (relative to the boundary), the peculiarities of dynamics on a rotating sphere allow for a viscous boundary layer consisting of an intense, relatively narrow jet at the western edge of the ocean.[1] This is the explanation for major currents such as the Gulf Stream in the Atlantic Ocean and the Kuroshio in the Pacific. Each current is about 100 km wide and 1000 km long, and transports more than 30 million tons of water per second along the coast at speeds about 100 times greater than the average speed outside the jet. Such jets are important ocean features that affect Earth's climate.

Reference

1. See, for example, J. Pedlosky, *Ocean Circulation Theory*, Springer, New York (1996).

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John Anderson's article on Ludwig Prandtl's boundary layer is both interesting and informative. More recently, beginning in 1961, the boundary layer concept has been applied to flow about a type of surface called "continuous."[1] A characteristic of flows over continuous surfaces is that, for any given period, any two solid surface elements exhibit different drag-time histories, as contrasted with finite-surface flows, in which all surface elements exhibit equal drag-time histories. As a result the formation and termination of the boundary layer are not identified with any part of the surface, but are determined by the system's boundaries.

Flows over continuous surfaces constitute a new class of boundary-layer problem. Although the differential equations governing flow around the