

Black-Hole Physics in an Electromagnetic Waveguide

By rapidly changing the dielectric constant in part of the guide, one can create mode-trapping horizons and an analogue of Hawking radiation.

A salmon and a photon share an important property. A salmon swimming upstream to spawn is unable to reach its destination if the current is too strong. Now suppose the current in the river increases in the downstream direction. There could then be a sharp divide where the current's speed equals the speed at which a salmon can swim. A salmon placed on the upstream side of the divide will spawn, but a salmon on the other side will inevitably be swept farther downstream.

Just as the river with varying current can have a sharp boundary that is of crucial importance for the salmon, so too a black hole, with its varying gravitational field, has a horizon that separates two distinct possibilities for the behavior of light. A photon that is farther from the black hole than the horizon can escape to infinity. Once inside the horizon, though, light (and anything else) is invariably sucked into the black-hole singularity.

William Unruh, a physicist at the University of British Columbia, used to think of salmon analogies as amusing illustrations to incorporate into colloquia. But by 1980 he began to take such analogies seriously. He recognized that the equations of motion for some long-wavelength sound waves in a flowing medium are identical to the equations that describe a scalar field in a curved space-time. In particular, horizons for sound propagation could be present in what Unruh called dumb holes.

Unruh went a step further and asked what happens if one quantizes sound in a dumb hole.¹ The remarkable answer was that just as a black hole emits a thermal spectrum of photons called Hawking radiation (see the box on page 20), the acoustic dumb hole radiates phonons.

Inspired by Unruh's ideas, several investigators proposed acoustic analogues of black holes involving such systems

as liquid helium and Bose-Einstein condensates (BECs).² In principle, those analogues can be used to experimentally verify the phenomenon of Hawking radiation, which has never actually been observed. In addition, analogue experiments may shed light on a technical difficulty, called the trans-Planckian problem, that is associated with Hawking radiation.

All the proposed analogue systems based on an underlying fluid, however, present significant, perhaps insurmountable, problems to the experimenter. In a paper soon to be published in *Physical Review Letters*, Ralf Schützhold of the Dresden University of Technology in Germany teamed up with Unruh to propose a different kind of black-hole analogue whose main element is an electromagnetic waveguide.³ Their proposal also poses experimental challenges, but it avoids problems that plague the older, fluid-based models.

The long and the short of it

The Schützhold-Unruh waveguide, as illustrated in panel a of the figure, com-

prises a large number of cells, each of length Δx and each including a dielectric capacitor and an inductor. To create a horizon for electromagnetic waveguide modes, the two theorists suggest sweeping a laser beam rapidly along the guide, say from left to right, so as to excite the material in the capacitors and increase its dielectric constant. That increase means the speed of light in the guide will be slower to the left of the laser spot than it is to the right, as shown in panel b of the figure. If the laser spot moves at a rate intermediate between the light speeds in the two sides of the guide, then the location of the spot is a horizon for mode propagation. An observer moving with the horizon can see modes cross the horizon from the right, but any mode excited to the left of the horizon must propagate to the left, just as the unfortunate salmon in a strong current is inevitably swept downstream.

The mathematics behind the preceding description is the wave equation for the vector potential in the guide. That equation, for wavelengths much larger than the cell length, is the same as one that describes the propagation of photons in a curved space-time with a horizon. The parameter that determines the Hawking temperature of the radiation emitted in the guide is the gradient of the waveguide light speed as one crosses the horizon.

The modular structure of the waveguide is invisible to the long-wavelength modes, but one might expect short-wavelength modes to be affected by the guide's structure. Contemplation of short-wavelength modes in the earlier acoustic analogues has yielded a surprising result that may shed light on the trans-Planckian problem.

Imagine tracing a photon of Hawking radiation back in time to its origin near the black-hole horizon. As the photon approaches the horizon, it is blueshifted: The wavelength decreases. Indeed, the famously strange properties of the black-hole metric imply that the wavelength should decrease to

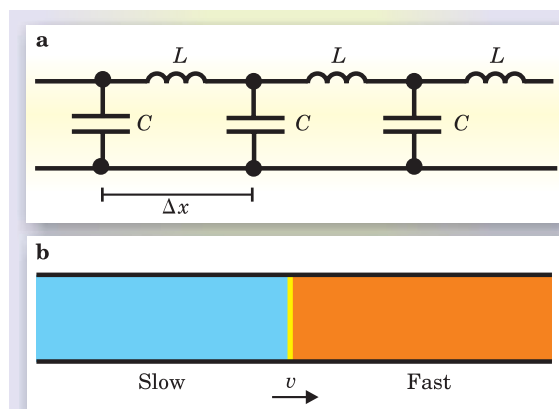


Figure 1. A waveguide with repeating units of capacitors and inductors (a) can display behavior analogous to that of a black hole. (b) A laser beam (yellow) scanning with speed v excites the capacitor material and divides the guide into two regions with different speeds of light. If v lies between the two light speeds, the boundary separating the slow and fast regions acts as a horizon and the guide emits Hawking radiation.

arbitrarily small sizes near the horizon. Thus, Stephen Hawking's derivation of the eponymous radiation phenomenon appears to depend in an essential way on physics at trans-Planckian lengths well below the Planck scale of 10^{-35} m, obtained by appropriately combining $\hbar$, c , and G . But at that scale, quantum-gravity effects should be important and the assumptions used by Hawking shouldn't apply.

In analogue models, one might expect that Hawking modes, when traced back to the horizon, would be blueshifted so as to sense the system's microstructure. Thus, the argument for analogue Hawking radiation, which does not take that structure into account, would be suspect. While thinking about that problem for acoustic analogues, the University of Maryland's Theodore Jacobson recognized that the speed of sound is not truly independent of wavelength, though deviations become apparent only for modes with wavelengths near the atomic scale. However, as Jacobson, Unruh, and others discovered, the wavelength dependence of the sound speed profoundly changes the behavior of acoustic Hawking radiation when it is traced back toward a dumb-hole horizon.⁴

In the so-called subluminal case, which applies to the waveguide and to some acoustic analogues, a mode traced back in time turns around and heads back outward when it is blueshifted to a wavelength comparable to the atomic spacing or waveguide cell size: Arbitrarily small wavelengths do not arise extremely close to the horizon. Even so, the structure of the analogue would seem to have a role to play in any radiation that might be generated.

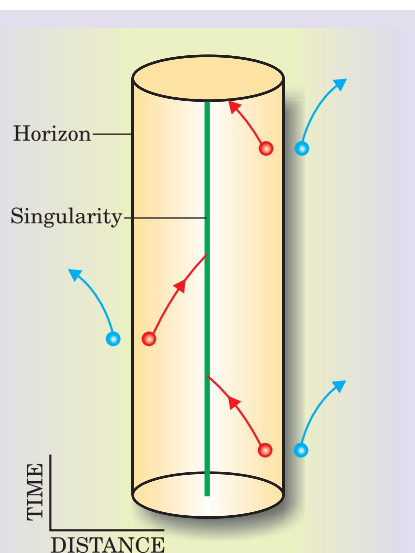
The structure, however, is immaterial. Despite possibly complicated dynamics, the short-wavelength modes see the approach of the horizon as a slow, adiabatic process; thus, short-wavelength modes originally in their ground state will remain so as the horizon approaches. And the condition of being in the ground state near the horizon leads to Hawking radiation.

The waveguide proposed by Schützhold and Unruh can help test the picture just sketched. But what does a waveguide test have to do with real spacetime? Different physicists answer that question differently. Unruh says that if the radiation emitted by the waveguide is truly independent of structural details, then trans-Planckian physics—and therefore any assumptions that Hawking made about it—is irrelevant to the generation of Hawking radiation.

Hawking Radiation

In 1974, Stephen Hawking announced that black holes do not live forever. Instead, they emit a thermal spectrum of radiation at a temperature—the Hawking temperature—determined by the mass M of the black hole and fundamental constants: $T = \hbar c^3 / 8\pi k_B MG$, where $\hbar$ is Planck's constant, c is the speed of light, k_B is Boltzmann's constant, and G is Newton's constant. For a solar-mass black hole, the Hawking temperature is roughly 10^{-7} K, many orders of magnitude below the 3 K temperature of the cosmic microwave background.

One can imagine Hawking radiation as arising from vacuum fluctuations that produce photon pairs with one member of the pair on each side of the black hole's horizon. As illustrated here, the photons outside the horizon can escape the black hole's gravitational pull and carry energy away. Because energy is conserved, photons within the horizon have negative energy. That negative energy decreases the total mass of the black hole, which shrinks a bit.



Matt Visser (Victoria University of Wellington, New Zealand) notes that some physicists believe the universe has structure at the Planck scale that would break Lorentz invariance. Observations made with structured systems such as a waveguide, Visser says, might provide hints about how large-scale Lorentz-invariant physics connects to a non-invariant regime.

If you build it

A waveguide black-hole analogue has several advantages over analogues based on fluids. Some advantages derive from the solid-state nature of the guide; fluid analogues must contend with a number of potentially serious fluid instabilities. The high speed of light in the waveguide—much greater than the speed of sound in helium or BECs—yields other advantages: The waveguide generates a much higher Hawking temperature than fluid analogues; Schützhold and Unruh estimate the Hawking temperature for their waveguide to be 10–100 mK. To detect radiation in that range, the guide would need to be cooled to 10 mK or so, which is technically feasible. One would also need detectors with very little noise to detect radiation in the millikelvin range, but such detectors already exist.

Still, the analogue suggested by Schützhold and Unruh presents daunting challenges to the experimenter who contemplates actually building it. To accumulate measurable amounts of radiation, the waveguide would have to be very long; as a

practical matter it may need to be rolled up in a spiral. A laser spot would have to pass rapidly and accurately over the waveguide to sequentially illuminate the component cells and excite the dielectric material within. And that dielectric may not be easily accessible to the laser: It is contained in a conducting guide, and the guide itself must be maintained at a low temperature. Furthermore, although Schützhold and Unruh offer a theoretical proof of concept for a material whose dielectric constant significantly increases when it is excited, they do not suggest any specific material. An additional worry is that as the laser modifies the dielectric, it would create waveguide modes that mask the Hawking radiation.

Unruh grants that building a working waveguide analogue would be challenging but thinks that if experimenters devote themselves to the effort, they should be able to pull it off. "I've always been amazed at what my experimental colleagues can do," he notes. "Things which to me seem absurd, to them seem merely difficult."

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References

1. W. G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981).
2. For a review of gravitational analogues, see C. Barcelo, S. Liberati, M. Visser, <http://arXiv.org/abs/gr-qc/0505065>.
3. R. Schützhold, W. Unruh, *Phys. Rev. Lett.* (in press).
4. See, for example, T. Jacobson, *Prog. Theor. Phys. Supp.* **136**, 1 (1999).