

Citation Statistics from 110 Years of *Physical Review*

Publicly available data reveal long-term systematic features about citation statistics and how papers are referenced. The data also tell fascinating citation histories of individual articles.

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The first article published in the *Physical Review* was received in 1893; the journal's first volume included 6 issues and 24 articles. In the 20th century, the *Physical Review* branched into topical sections and spawned new journals (see figure 1). Today, all articles in the *Physical Review* family of journals (PR) are available online and, as a useful byproduct, all citations in PR articles are electronically available.

The citation data provide a treasure trove of quantitative information. As individuals who write scientific papers, most of us are keenly interested in how often our own work is cited. As dispassionate observers, we can use the citation data to identify influential research, new trends in research, unanticipated connections across fields, and downturns in subfields that are exhausted. A certain pleasure can also be gleaned from the data when they reveal the idiosyncratic features in the citation histories of individual articles.

The investigation of citation statistics has a long history,¹ in which a particularly noteworthy contribution was a 1965 study by Derek John de Solla Price.² In his study, Price built on original models by George Yule and Herbert Simon³ to argue that the distribution in the number of citations to individual publications had a power-law form. Price also noted that well-cited papers continue to be referenced more frequently than less-cited papers, and coined the term cumulative advantage to describe the mechanism that causes a persistently higher rate.⁴ Cumulative advantage means the probability that a publication is cited increases monotonically with the current number of citations. In the framework of currently fashionable evolving network models, the mechanism is called preferential attachment.⁵ Linear preferential attachment models provide

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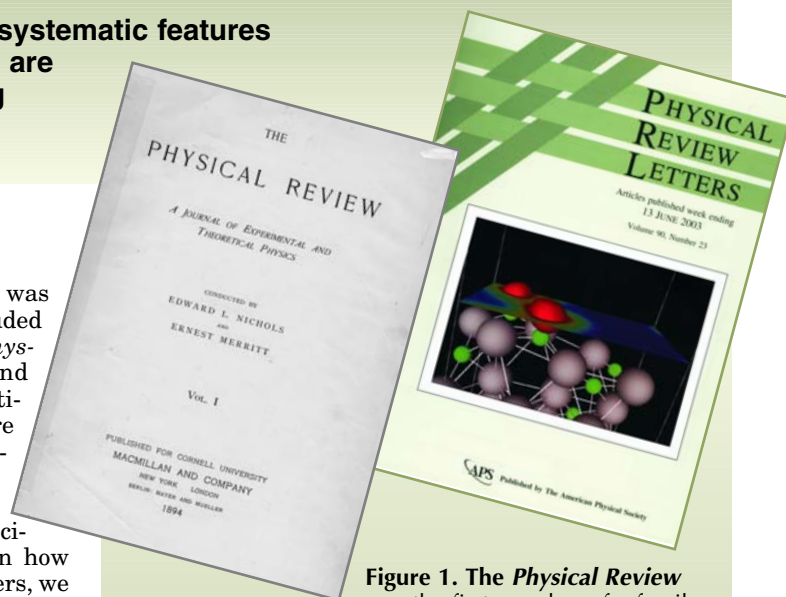


Figure 1. The *Physical Review* was the first member of a family of journals that now includes two series of *Physical Review*, the topical journals *Physical Review A–E*, *Physical Review Letters* (PRL), *Reviews of Modern Physics*, and *Physical Review Special Topics: Accelerators and Beams*. The first issue of *Physical Review* bears a publication date a year later than the receipt of its first article. The data considered in this article extend through 30 June 2003, just after the PRL cover on the right was published.

appealing explanations for the power-law distributions of connections observed in many social systems, natural networks, and manmade networks such as the World Wide Web.⁶ One fundamental motivation for studying citation statistics is to determine whether they exhibit some of the universal features that have been ascribed to prototypical models of growing networks.^{5,7,8}

Before examining the citation data, I offer several caveats. First, the data include only internal citations—that is, citations from PR articles to other PR articles—and are perforce incomplete. For highly cited papers, a previous study found⁹ that total citations typically outnumber internal ones by a factor of 3 to 5, a result that gives a sense of the incompleteness of the PR data. Second, some 5–10% of citations appear to be erroneous,^{9,10} although the recent practice by PR of crosschecking references when manuscripts are submitted has significantly reduced the error

Table 1. *Physical Review* Articles with more than 1000 Citations Through June 2003

Publication	# cites	Av. age	Title	Author(s)
PR 140, A1133 (1965)	3227	26.7	Self-Consistent Equations Including Exchange and Correlation Effects	W. Kohn, L. J. Sham
PR 136, B864 (1964)	2460	28.7	Inhomogeneous Electron Gas	P. Hohenberg, W. Kohn
PRB 23, 5048 (1981)	2079	14.4	Self-Interaction Correction to Density-Functional Approximations for Many-Electron Systems	J. P. Perdew, A. Zunger
PRL 45, 566 (1980)	1781	15.4	Ground State of the Electron Gas by a Stochastic Method	D. M. Ceperley, B. J. Alder
PR 108, 1175 (1957)	1364	20.2	Theory of Superconductivity	J. Bardeen, L. N. Cooper, J. R. Schrieffer
PRL 19, 1264 (1967)	1306	15.5	A Model of Leptons	S. Weinberg
PRB 12, 3060 (1975)	1259	18.4	Linear Methods in Band Theory	O. K. Anderson
PR 124, 1866 (1961)	1178	28.0	Effects of Configuration Interaction of Intensities and Phase Shifts	U. Fano
RMP 57, 287 (1985)	1055	9.2	Disordered Electronic Systems	P. A. Lee, T. V. Ramakrishnan
RMP 54, 437 (1982)	1045	10.8	Electronic Properties of Two-Dimensional Systems	T. Ando, A. B. Fowler, F. Stern
PRB 13, 5188 (1976)	1023	20.8	Special Points for Brillouin-Zone Integrations	H. J. Monkhorst, J. D. Pack

PR, *Physical Review*; PRB, *Physical Review B*; PRL, *Physical Review Letters*; RMP, *Reviews of Modern Physics*.

rate. Third, papers can be highly cited for many reasons—some substantive and some dubious. Thus the number of citations is merely an approximate proxy for scientific quality.

Citation distribution and attachment rate

The PR citation data cover 353 268 papers and 3 110 839 citations from July 1893 through June 2003. The 329 847 papers with at least 1 citation may be broken down as follows:

- 11 publications with more than 1000 citations
- 79 publications with more than 500 citations
- 237 publications with more than 300 citations
- 2 340 publications with more than 100 citations
- 8 073 publications with more than 50 citations
- 245 459 publications with fewer than 10 citations
- 178 019 publications with fewer than 5 citations
- 84 144 publications with 1 citation.

A somewhat depressing observation is that nearly 70% of all PR articles have been cited fewer than 10 times. (The average number of citations is 8.8.) Also evident is the small number of highly cited publications; table 1 lists the 11 publications with more than 1000 citations.

Citations have grown rapidly with time, a feature that mirrors the growth of the PR family of journals. From 1893 until World War II, the number of annual citations from PR publications doubled approximately every 5.5 years. The number of PR articles published in a given year also doubled every 5.5 years. Following the publication crash of the war years, the number of articles published annually doubled approximately every 15 years.

The citation data naturally raise the question, What is the distribution of citations? That is, what is the probability $P(k)$ that a paper gets cited k times? This question was investigated by Price, who posited the power law $P(k) \propto k^{-\nu}$, with ν positive. A power-law form is exciting for statistical physicists because it implies the absence of a characteristic scale for citations—the influence of a publication may range from useless to earth-shattering. The absence of a characteristic scale in turn implies that citation statistics should exhibit many of the intriguing features associated with phase transitions, which display critical phenomena on all length scales.

Somewhat surprisingly, the probability distribution derived from the more than 3 million PR citations still has significant statistical fluctuations. It proves more useful to study the cumulative distribution $C(k) = \int_k^\infty P(k') dk'$, the probability that a paper is cited at least k times, to reduce these fluctuations.

On a double logarithmic scale, $C(k)$ has a modest negative curvature everywhere. That behavior, illustrated in figure 2, suggests that the distribution decays faster than a power law and is at variance with results of previous, smaller-scale studies that suggested either a power law^{2,11} or a stretched exponential form,¹² $C(k) \propto \exp(-k^\beta)$, with β less than 1. It is intriguing that a good fit over much of the range of the distribution is the log-normal form $C(k) = A \exp\{-b \ln k - c(\ln k)^2\}$. Log-normal forms typically underlie random multiplicative processes. They describe, for example, the distribution of fragment sizes that remain after a rock has been hammered many times.

The development of citations may be characterized by the attachment rate A_k , which gives the likelihood that a paper with k citations will be cited by a new article. To measure the attachment rate, first count the number of times each paper is cited during a specified time range; this gives k . Then, to get A_k , count the number of times each paper with a given k in this time window was cited

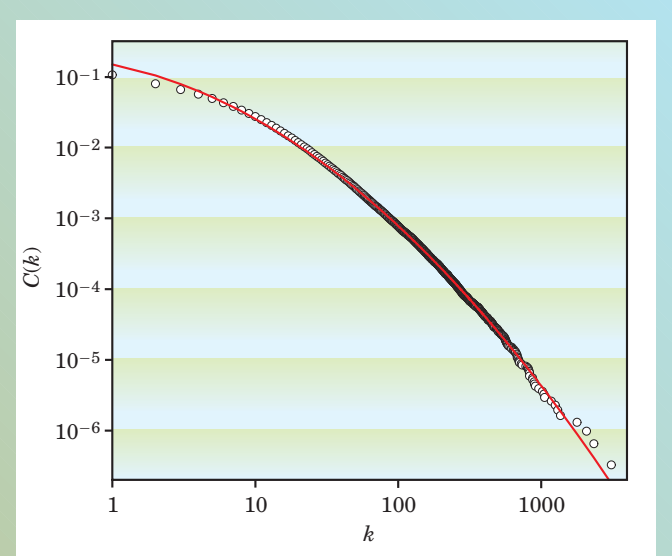


Figure 2. The cumulative citation distribution $C(k)$ versus the number of citations k for all papers published from July 1893 through June 2003 in the *Physical Review* journals. Circles indicate the data. The curve is the log-normal fit $C(k) = A \exp[-b \ln k - c(\ln k)^2]$, with $A = 0.15$, $b = 0.40$, and $c = 0.16$.

in a subsequent window. As shown in figure 3, the data suggest that A_k is a linear function of k , especially for k less than 150, a condition that applies to nearly all PR papers.¹³ Thus linear preferential attachment appears to account for the propagation of citations.

Linear attachment, though, leads to two paradoxes. First, a linear rate implies a power-law citation distribution, but figure 2 indicates that the data are better described by a log-normal form. While a log-normal distribution does arise from the nearly linear attachment rate $A_k = k/(1 + a \ln k)$, with a positive, figure 3 hints that A_k may be growing slightly *faster* than linearly with k . Second, to implement linear preferential attachment consciously, a citer must be aware of the references to every existing paper. That's clearly not realistic. A more realistic process that can lead to linear preferential attachment is the redirection mechanism.^{7,14} In redirection, an author who is writing the reference list for a paper figuratively picks a random paper. Then the author cites either the randomly selected paper (with probability $1 - r$) or one of the references within that paper (with probability r). This purely local process generates the linear form $A_k = k + (1/r - 2)$. Still a mystery is why the myriad of attributes that influences whether a paper gets cited manifests itself as a citation rate that is a nearly linear function of the number of citations.

Age structure

A common adage says that nobody cites classic papers anymore. Is that really true? How long does a paper continue to get cited?

The age of a citation is the difference between the year when a citation occurs and the publication year of the cited paper. Typically, unpopular papers are cited soon after publication, if at all, and then disappear. The converse is also true. For example, the average citation age $\langle a \rangle$ over the entire PR data set is 6.2 years, but articles published before 2000 for which $\langle a \rangle$ is less than 2 years receive, on average, only 3.6 citations. On the other hand, highly cited papers usually continue to be cited for a long time, and vice versa. Papers with more than 100 citations have $\langle a \rangle = 11.7$ years, and the 11 publications with more than 1000 citations have $\langle a \rangle = 18.9$ years. For all PR publications with 500 or fewer citations, the average age grows with the number of citations roughly as $\langle a \rangle = k^\alpha$, with α approximately 0.3.

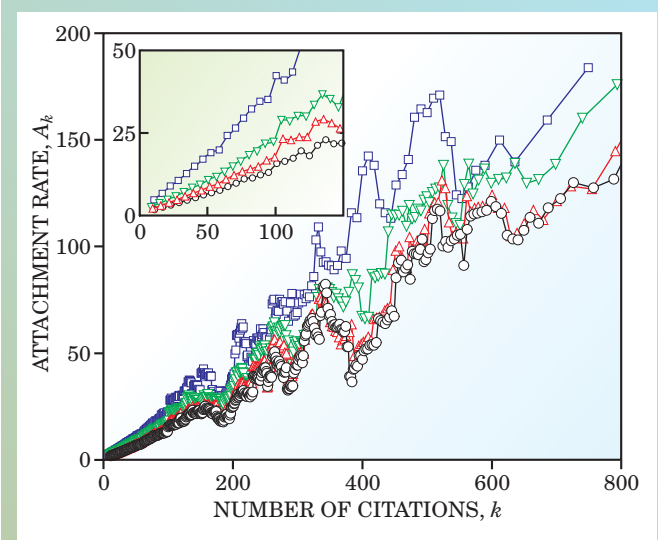


Figure 3. The attachment rate A_k is a nearly linear function of the number of citations k , especially for k less than 150 (inset). The different colors indicate different year ranges for establishing k : 1990–99 (purple), 1980–99 (green), 1970–99 (red), and 1893–1999 (black). The rate A_k is determined from citations in the year 2000. Data have been averaged over a range of $\pm 2.5\%$. Other time ranges for existing and new citations give similar behavior.

The citation age distributions—that is, numbers of citations as a function of age—reveal a fact that is surprising at first sight: The exponential growth of PR articles strongly skews the form of the age distributions! There are, in fact, two distinct age distributions. One is the distribution of *citing* ages, defined as the number of citations of a given age *from* a paper. Remarkably, citing memory is independent of when a paper is published. An author publishing now seems to have the same range of memory as an author who published an article 50 years ago. The citing age distribution roughly decays exponentially with age, except for a sharp decrease in citations during the period of World War II. However, citing an old paper is difficult a priori simply because relatively few old papers exist. As noted by Hideshiro Nakamoto,¹⁵ a more meaningful

Table 2. The 12 Revived Classics, as Defined in the Text, Arranged Chronologically

Publication	# cites	Av. age	Title	Author(s)
PR 40, 749 (1932)	562	55.8	On the Quantum Correction for Thermodynamic Equilibrium	E. Wigner
PR 46, 1002 (1934)	557	51.5	On the Interaction of Electrons in Metals	E. Wigner
PR 47, 777 (1935)	492	59.6	Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?	A. Einstein, B. Podolsky, N. Rosen
PR 56, 340 (1939)	342	49.3	Forces in Molecules	R. P. Feynman
PR 82, 403 (1951)	643	46.4	Interaction Between <i>d</i> -Shells in Transition Metals. II. Ferromagnetic Compounds of Manganese with Perovskite Structure	C. Zener
PR 82, 664 (1951)	663	36.6	On Gauge Invariance and Vacuum Polarization	J. Schwinger
PR 100, 545 (1955)	350	41.9	Neutron Diffraction Study of the Magnetic Properties of the Series of Perovskite-Type Compounds [(1 - <i>x</i>)La, <i>x</i> Ca]MnO ₃	E. O. Wollan, W. C. Koehler
PR 100, 564 (1955)	275	42.0	Theory of the Role of Covalence in the Perovskite-Type Manganites [La, M(II)]MnO ₃	J. B. Goodenough
PR 100, 675 (1955)	461	43.2	Considerations on Double Exchange	P. W. Anderson, H. Hasegawa
PR 109, 1492 (1958)	871	32.0	Absence of Diffusion in Certain Random Lattices	P. W. Anderson
PR 115, 485 (1959)	484	32.4	Significance of Electromagnetic Potentials in the Quantum Theory	Y. Aharonov, D. Bohm
PR 118, 141 (1960)	500	37.1	Effects of Double Exchange in Magnetic Crystals	P.-G. de Gennes

PR, *Physical Review*.

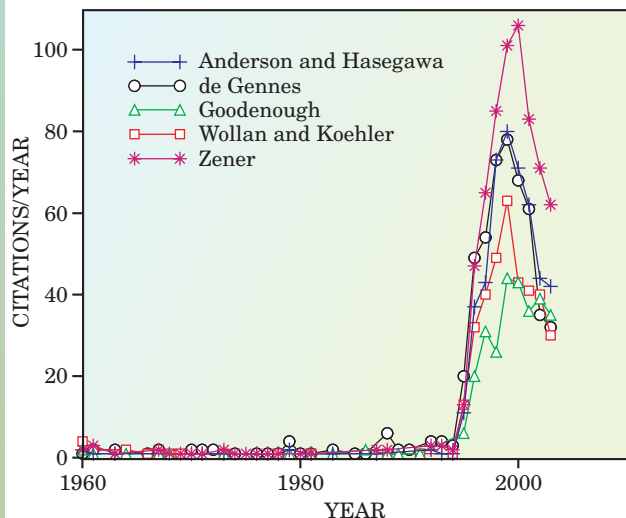


Figure 4. Five revived classics of relevance to colossal magnetoresistance have similar citation histories. Table 2 gives more information about these papers.

fine a revived classic as a nonreview PR article, published before 1961, that has received more than 250 citations and has a ratio of the average citation age to the age of the paper greater than 0.7. Thus, revived classics are well-cited old papers with the bulk of their citations occurring long after publication. Only the 12 papers listed in table 2 fit these criteria.

The clustered citation histories of the five articles plotted in figure 4 are particularly striking. Those articles, published between 1951 and 1960 (with three in the same issue of *Physical Review*), investigated the double exchange mechanism in Perovskite manganites, the mechanism responsible for the phenomenon of colossal magnetoresistance. That topic became popular in the 1990s because of the confluence of new synthesis and measurement techniques in thin-film transition-metal oxides, the sheer magnitude of the effect, and the clever use of the term “colossal.” The citation burst more than 40 years after the publication of the five articles is unique in the history of the PR journals.

The other seven papers in table 2 have different claims to fame. Eugene Wigner’s 1932 paper had 115 citations before 1980 and an additional 447 through June 2003. Similarly, the Albert Einstein, Boris Podolsky, and Nathan Rosen (EPR) paper had 36 citations before 1980 and 456 more through June 2003. With average citation ages of 55.8 and 59.6, respectively, those are the longest-lived articles with more than 35 citations in the PR family. Those papers, as well the one by Yakir Aharonov and David Bohm, owe their renewed popularity to the upsurge of interest in quantum information phenomena. Wigner’s 1934 paper deals with correlations in an electron gas, a problem of enduring interest in condensed matter physics. Julian Schwinger’s work is a classic contribution to quantum electrodynamics. The 1958 publication by Philip Anderson helped launch the field of localization. And Richard Feynman’s paper presented a widely applicable method for calculating molecular forces. Feynman’s article is noteworthy because it is cited in all PR journals except the accelerators and beams special topics journal.

Publications that announce discoveries often receive a citation spike when the contribution becomes recognized. I arbitrarily define a discovery paper as having more than 500 citations and a ratio of average citation age to publication age of less than 0.4; I exclude articles published in *Reviews of Modern Physics* and compilations by the Particle Data Group. Table 3 lists the 11 such discovery papers; all were published between 1962 and 1991. A trend in that

citing age distribution is obtained by rescaling the distribution by the total number of articles in each citing year. So, if one is interested in journal citations from 2005, the number of cited papers that are, say, four years old should be scaled by the total number of papers published in 2001. The rescaling has a dramatic effect: The nearly exponential citing age distribution is transformed into a power law! An analogous skewing due to the rapid growth of PR articles also occurs in the distribution of *cited* ages, that is, the number of citations of a given age *to* an article.

Individual citation histories

The citation histories of well-cited publications are diverse and quite different from the collective citation history of all PR articles. The varied histories roughly fall into classes that include revived classic works or “sleeping beauties,”¹⁶ major discoveries, and hot publications. It’s fun to review examples of each class.

Sometimes a publication will become in vogue after a long dormancy—a revival of an old classic. I arbitrarily de-

Table 3. The 11 Discovery Papers, as Defined in the Text, Arranged Chronologically

Publication	# cites	Av. age	Title	Author(s)
PR 125, 1067 (1962)	587	7.0	Symmetries of Baryons and Mesons	M. Gell-Mann
PR 182, 1190 (1969)	563	13.8	Nucleon-Nucleus Optical-Model Parameters, $A > 40$, $E < 50$ MeV	F. D. Becchetti Jr, G. W. Greenlees
PRD 2, 1285 (1970)	738	11.2	Weak Interactions with Lepton-Hadron Symmetry	S. L. Glashow, J. Iliopoulos, L. Maiani
PRL 32, 438 (1974)	545	11.1	Unity of All Elementary-Particle Forces	H. Georgi, S. L. Glashow
PRD 10, 2445 (1974)	577	11.9	Confinement of Quarks	K. G. Wilson
PRD 12, 147 (1975)	501	10.7	Hadron Masses in a Gauge Theory	A. De Rújula, H. Georgi, S. L. Glashow
PRL 53, 1951 (1984)	559	7.9	Metallic Phase with Long-Range Orientational Order and No Translational Symmetry	D. Shechtman, I. Belch, D. Gratias, J. W. Cahn
PRA 33, 1141 (1986)	501	6.4	Fractal Measures and Their Singularities: The Characterization of Strange Sets	T. C. Halsey et al.
PRL 58, 908 (1987)	625	1.9	Superconductivity at 93 K in a New Mixed-Phase Yb-Ba-Cu-O Compound System at Ambient Pressure	M. K. Wu et al.
PRL 58, 2794 (1987)	525	4.8	Theory of High- T_c Superconductivity of Oxides	V. J. Emery
PRB 43, 130 (1991)	677	5.2	Thermal Fluctuations, Quenched Disorder, Phase Transitions, and Transport in Type-II Superconductors	D. S. Fisher, M. P. A. Fisher, D. A. Huse

PR, *Physical Review*; PRA, *Physical Review A*; PRB, *Physical Review B*; PRD, *Physical Review D*; PRL, *Physical Review Letters*.

Table 4. The 10 Hot Papers, as Defined in the Text, Arranged Chronologically

Publication	# cites	Av. age	Title	Author(s)
PR 40 , 749 (1932)	562	55.8	On the Quantum Correction for Thermodynamic Equilibrium	E. Wigner
PR 47 , 777 (1935)	492	59.6	Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?	A. Einstein, B. Podolsky, N. Rosen
PR 109 , 1492 (1958)	871	32.0	Absence of Diffusion in Certain Random Lattices	P. W. Anderson
PR 136 , B864 (1964)	2460	28.7	Inhomogeneous Electron Gas	P. Hohenberg, W. Kohn
PR 140 , A1133 (1965)	3227	26.6	Self-Consistent Equations Including Exchange and Correlation Effects	W. Kohn, L. J. Sham
PRB 13 , 5188 (1976)	1023	20.8	Special Points for Brillouin-Zone Integrations	H. J. Monkhorst, J. D. Pack
PRL 48 , 1425 (1982)	829	15.1	Efficacious Form for Model Pseudopotentials	L. Kleinman, D. M. Bylander
PRB 41 , 7892 (1990)	691	9.7	Soft Self-Consistent Pseudopotentials in a Generalized Eigenvalue Formalism	D. Vanderbilt
PRB 45 , 13244 (1992)	394	8.1	Accurate and Simple Analytic Representation of the Electron-Gas Correlation Energy	J. P. Perdew, Y. Wang
PRL 70 , 1895 (1993)	495	7.4	Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels	C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W. K. Wootters

PR, *Physical Review*; PRB, *Physical Review B*; PRL, *Physical Review Letters*.

group of papers is the shift from elementary-particle physics (the six articles published before 1976) to condensed matter physics (the five articles published after 1983). The earlier discovery papers reflected major developments in elementary-particle physics, including $SU(3)$ symmetry, the prediction of charm, and grand unified theories. The condensed matter articles are on quasicrystals, multifractals, and high-temperature superconductivity. If the citation threshold is relaxed to 300, an additional seven papers fit the discovery criteria. All of those are concerned with high-temperature superconductivity, and all but one appear during the golden age of the field, 1987–89.

It is not clear whether the shift in the field of discovery publications stems from a sea change in research direction or from prosaic concerns. The past 15 years have seen a major upsurge in quantum condensed matter physics that perhaps stems from the discovery of high-temperature superconductivity. But recent elementary-particle physics discoveries may be underrepresented in PR because many CERN-based discoveries have been published in journals outside the PR family.

A number of classic, highly cited publications have noteworthy citation histories. Figure 5 illustrates some of these histories. Citations to “Theory of Superconductivity” (*Physical Review*, volume 108, page 1175, 1957) by John Bardeen, Leon Cooper, and Robert Schrieffer (BCS) closely track the activity in superconductivity; the paper received its fewest citations in 1985, the year before the discovery of high-temperature superconductivity. The BCS paper is the earliest paper with more than 1000 citations in the PR family. Steven Weinberg’s paper (W) “A Model of Leptons,” on the electroweak theory (*Physical Review Letters*, volume 19, page 1264, 1967), has a broad citation peak followed by a relatively slow decay, as befits this seminal paper’s long-term influence. On the other hand, the average citation age for the 1974 publications that announced the discovery of the J/ψ particle (*Physical Review Letters*, volume 33, pages 1404 and 1406, 1974), is less than 3 years!

An unusual example is “Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions” (*Physical Review Letters*, volume 42, page 673, 1979), by Elihu Abrahams, Anderson, Don Licciardello, and T. V. Ramakrishnan (the so-called gang of four; G4). Since publication, the G4 paper has been cited 30–60 times annually, a striking testament to its long-term impact. The paper with the most citations in all PR journals is “Self-Consistent Equations Including Exchange and Correlation Effects” (*Physical Review*, volume 140, page A1133, 1965), by Walter Kohn and Lu Sham (KS). Amazingly, citations

to this publication have been steadily increasing for nearly 40 years.

The KS paper is also an example of what may be called a hot paper, defined as a nonreview paper with 350 or more citations, an average ratio of citation age to publication age greater than two-thirds, and a citation rate increasing with time. Ten papers, listed in table 4, fit these criteria. The 1932 Wigner and 1935 EPR articles, both more than 70 years old, and the two most-cited PR papers of all time, KS and the 1964 article by Pierre Hohenberg and Kohn, are all hot. Astounding!

Of the remaining six hot papers, five are in quantum condensed matter physics. Three of them build on the formalism introduced in the seminal articles by Hohenberg and Kohn and by Kohn and Sham. Another, Anderson’s 1958 localization paper, can be viewed both as hot and as the revival of a classic. The newest hot article, by Charles Bennett and coauthors, is concerned with quantum information theory, a research area that has recently become fashionable and that has also led to the sharp increase in citations to Wigner’s 1932 paper and to the 1935 EPR paper.

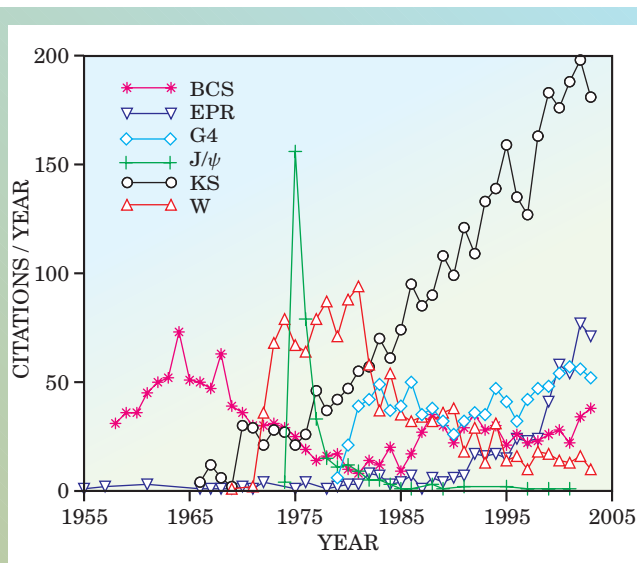


Figure 5. Six classic, highly-cited publications have varied citation histories. The abbreviations are defined in the text.

MAGNETS

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A unique window

A small number of physicists have played a remarkably large role in top-cited PR publications. Two individuals have coauthored five papers from among the top 100 cited PR articles:⁹ Kohn, who occupies positions 1, 2, 24, 96, and 100, and Anderson, with positions 9, 19, 20, 35, and 41. Wigner appears four times (4, 8, 53, and 55), and Lars Onsager (16, 64, and 68) and John Slater (12, 27, and 40) each appear three times.

The PR citation data provide a unique window for studying the development of citations, and the work I have described can be extended and applied in many ways. For example, constructing a graphical representation of the entire dynamically changing citation network should be revealing. Such a graph could show how fields develop and could expose unexpected connections between disparate areas. A practical, if more controversial, use of citation data would be to construct retrospective journals that include only highly cited papers. Such journals would provide a welcome reduction in the total literature volume, because only 30% of all articles have more than 10 citations and a mere 2.3% have more than 50 citations. A repository for all papers would still be necessary, though, as sleeping beauties do emerge long after publication.

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