A Small Puzzle from 1905

In his first relativity paper, Einstein made one erroneous prediction. Although it should have been withdrawn when he generalized the theory to include gravity, the original error has received surprisingly little attention.

Alex Harvey and Engelbert Schucking

The best known of the paradigm-shattering papers published by Albert Einstein in 1905, his annus mirabilis, is the one titled "On the Electrodynamics of Moving Bodies."[1] It was there that the principle of special relativity was set forth. Einstein asserted in this paper that time is relative. That is, if two inertial observers in uniform motion at relative velocity v were equipped with identical clocks, each would judge the other's clock to be running too slowly by a factor $v^2/2c^2$ (excluding higher-order terms).

Einstein immediately applied that result to a pair of identical clocks, one located on the equator and the other at either of the poles. Because of Earth's rotation relative to an almost inertial reference frame, a clock fixed at a point on the equator moves with a speed $v = 0.5$ km/s while a clock at the pole is essentially at rest. Thus, Einstein concluded in this first relativity paper, the equatorial clock is slower than the polar clock by a factor

$$v^2/2c^2 \approx 1.4 \times 10^{-12}.$$

That is, the clock at the equator should lose about a second in 20 000 years relative to the clock at the pole.

Einstein's comment (in the English translation in reference 1) was, "Thence we conclude that a balance clock at the equator must go more slowly, by a very small amount, than a precisely similar clock situated at one of the poles under otherwise identical circumstances."

In 1905 there was no way to check that prediction experimentally. The best timekeepers—astronomical pendulums with an accuracy of about a second per day—were hardly up to the task. (Today we have cesium clocks that are.) It is true that, even then, the interval between meridian transits of a star could be observed and calculated with exquisite accuracy. But there was no way to precisely compare that celestial interval with terrestrial clocks.

Eventually, experimental verification of the time-dilation effect was provided by observation of momentum dependence in the decay lifetimes of muons[2] in 1941 and pions a decade later.[3] Relativistic time dilation had, in fact, been indirectly observed in 1938 by Herbert Ives and G. R. Stilwell, in the transverse Doppler effect in an atomic beam.[4]

Gravitational effects

In 1907, a new consideration drastically altered the situation. With the establishment of the principle of special relativity, Einstein had turned his attention to a new problem: It seemed impossible to construct a viable theory of gravity consistent with the relativity principle. He approached the problem by formulating a "principle of equivalence." The new principle conjectured that a freely falling observer in a gravitational field was equivalent to one at rest in field-free space and that, conversely, an observer being accelerated in field-free space was equivalent to one at rest in a gravitational field (see Einstein's January 1907 letter to Arnold Sommerfeld, PHYSICS TODAY, February 2005, page 14).

The equivalence principle let Einstein study the influence of gravity on physical processes even in the absence of a proper theory of the gravitational field. The principle quickly led him to conclude that the relative rates of clocks were influenced not only by their relative motion, but also by the difference in gravitational potential between their locations.[5] With the advent of the full general theory of relativity in 1916, the effect of gravitational potential on clock rates would become precisely calculable.[6]

Thus, only two years after the 1905 prediction of the time dilation due to differences in motion, it appeared necessary to make adjustments for differences in gravitational potential. Remarkably, the two effects cancel in the special case of equatorial and polar clocks at sea level. The cancellation may be seen as follows: Denote the gravitational potential difference between two locations by $\Delta\Phi$. At Earth's surface, the potential difference between points differing in distance from the planet's center by h is well approximated by gh, where the gravitational acceleration $g \approx 10$ m/s^2 is taken to be constant.

From the equivalence principle, Einstein concluded that the rates of two clocks relatively at rest but at different gravitational potentials should differ fractionally by $\Delta\Phi/c^2$. For example, a clock atop a 30-story building would run faster than a clock in the building's lobby by a part in 10^{-14}. It would gain about one second over the lobby clock in 3 million years. The phenomenon, known as the Einstein gravitational blueshift, was finally demonstrated experimentally in 1959 by Robert Pound and Glen Rebka in a tower at Harvard University.[7] In the weak gravitational fields near Earth, the effect is minuscule. But the global positioning system has to take it into account (see the article by Neil Ashby in PHYSICS TODAY, May 2002, page 41).

To calculate correctly the rate difference between a clock at a pole and one at the equator, one has to take into account both time dilation and gravitational blueshift. Doing that, one finds that these two effects exactly cancel each other if both clocks are at local sea level. One must first evaluate the gravitational potential difference between pole and equator at sea level. Because of Earth's rotation, the surface that defines sea level has a centrifugal bulge at the equator.

Nominal sea level is a surface on which the sum of the gravitational potential and the centrifugal pseudopotential, that is $\Phi - v^2/2$, is constant, v being the rotational velocity of any point fixed on the surface. Therefore, $\Delta\Phi = \Phi_E - \Phi_P$, the difference between equatorial and polar gravitational potential, equals $v^2/2$, where v is now the rotational velocity at the equator. Thus the gravitational blueshift of a clock at sea level on the equator precisely can-

Alex Harvey *is a professor emeritus of physics at Queens College of the City University of New York and a visiting scholar at New York University.* **Engelbert Schucking** *is a professor of physics at New York University.*

Albert Einstein in 1911 in Brussels at the First Solvay Conference, standing second from right. Seated (l–r) are Walther Nernst, Marcel Brillouin, Ernest Solvay, Hendrik Lorentz, Emil Warburg, Jean Perrin, Wilhelm Wien, Marie Curie, and Henri Poincaré. Standing (l–r) are R. Goldschmidt, Max Planck, Heinrich Rubens, Arnold Sommerfeld, Frederick Lindemann, Marcel De Broglie, Martin Knudsen, Fritz Hasenöhrl, G. Hostelet, Eduard Herzen, James Jeans, Ernest Rutherford, Heike Kamerlingh Onnes, Einstein, and Paul Langevin. (Photo by Benjamin Couprie, courtesy of AIP Emilio Segrè Visual Archives.)

cels the special-relativistic time dilation due to its velocity.

The same result can also be obtained from general relativity. One goes to a noninertial frame corotating with Earth, in which both clocks at rest. Therefore the corotating frame shows no time-dilation difference between them. Furthermore, there is no gravitational blueshift difference because both sea-level clocks are at the same gravitational potential, $gh - v^2/2$, in the rotating frame. That is, general relativity treats gravitational and inertial forces as equivalent.

A belated footnote

The problem with the erroneous prediction in the 1905 paper went completely unnoticed until 1913, when a collection of papers pertaining to relativity was published.[8] Edited by Otto Blumenthal, a mathematician who knew Einstein, the collection contained, among other things, the 1905 Einstein paper that introduced special relativity.[1] Blumenthal indicated in the preface that the project was initiated at the suggestion of Arnold Sommerfeld and made possible by "the friendly cooperation of Messrs. H. A. Lorentz and Einstein."

Einstein's 1905 paper appears in Blumenthal's 1913 anthology with five newly added footnotes. One is of particular interest here. A footnote was appended to the sentence quoted in the third paragraph of this article. Its English translation (page 50 of the Dover reprint in reference 1) is: "Not a pendulum clock which is physically a system to which the earth belongs. This case had to be excluded." Why this footnote was added is not clear. It is not correct, nor does it address the issue of the correctness of the original prediction.

The matter might have rested there had not a question been raised as to the authorship of the footnotes. Arthur Miller, in his extensive monograph on Einstein's paper, attributes them to Sommerfeld.[9] He gives no basis for this attribution, and a good case for the source being Einstein himself has been made by Ian McCausland.[10] Miller does not discuss the incorrectness of the prediction.

It seems strange that, despite intense scrutiny of the groundbreaking 1905 paper, no historian of science has ever noted, much less discussed, the incorrectness of its prediction of a rate difference between equatorial and polar clocks. There is no mention, even, in the exhaustive discussion in the collected Einstein papers.[11] There is no record of Einstein's having corrected the erroneous 1905 prediction. In 1907 he suggested that time dilation could be tested by observing the transverse Doppler effect in the light emitted by canal rays—that is, positive ions passing through "canals" in the cathodes of gas discharge tubes.[12]

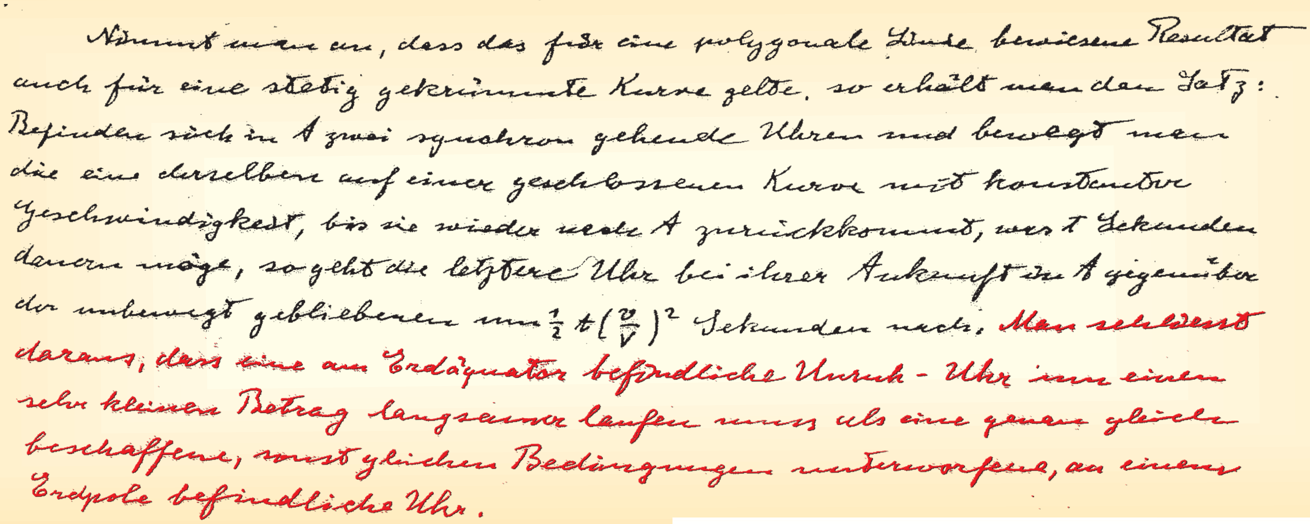
Some history

The 1905 paper has been studied by numerous historians of science, and this year the world celebrates its centenary. Nonetheless, we have found only two references to the error in the many commentaries on the paper. In April 1982 there appeared a *New York Times* op-ed article by Jeremy Bernstein entitled "Accepting Scientific Ideas."[13] The article dealt with the criteria for distinguishing "crank" scientific papers from the real thing. The criterion at issue was a paper's capability of making accurate predictions. With uncanny perspicacity, Bernstein picked out the one prediction from Einstein's annus mirabilis that was wrong—the prediction that a clock at the equator would run slower than an identical clock at either pole.

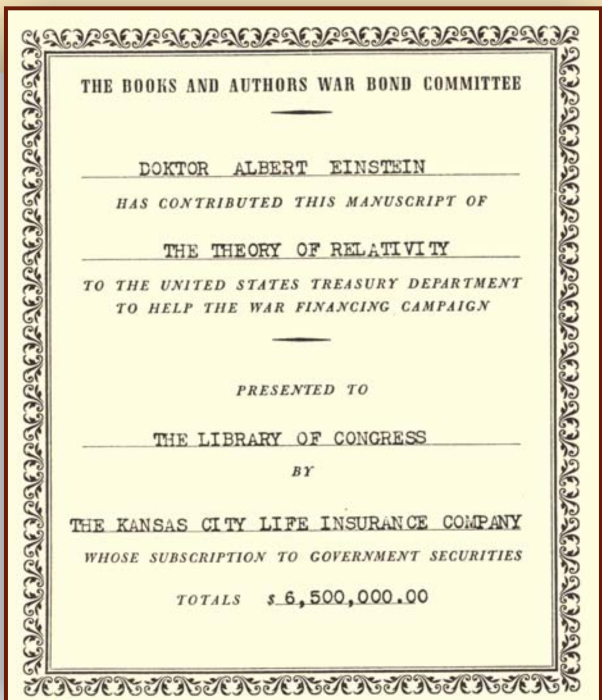
In a magazine article nine years later, Bernstein gave an accurate discussion of the cancellation between the time dilation and gravitational blueshift effects at sea level as if he were repeating something that was widely known among physicists.[14] In a sense, it was. A discussion of the relevant physics had appeared in 1975 in a book of problems in relativity and gravitation by Alan Lightman and three other students of Kip Thorne.[15]

Coda

During a World War II bond drive, Einstein was asked if he would donate the 1905 relativity manuscript to be auctioned


Nimmt man an, dass das für eine polygonale Linie bewiesene Resultat auch für eine stetig gekrümmte Kurve gelte, so erhält man den Satz: Befinden sich in A zwei synchron gehende Uhren und bewegt man die eine derselben auf einer geschlossenen Kurve mit konstanter Geschwindigkeit, bis sie wieder nach A zurückkommt, was t Sekunden dauern möge, so geht die letztere Uhr bei ihrer Ankunft in A gegenüber der unbewegt gebliebenen um $\frac{1}{2}t\left(\frac{v}{V}\right)^2$ Sekunden nach. Man schliesst daraus, dass eine am Erdäquator befindliche Unruh-Uhr um einen sehr kleinen Betrag langsamer laufen muss als eine genau gleich beschaffene, sonst gleichen Bedingungen unterworfene, an einem Erdpole befindliche Uhr.

Excerpt from Einstein's 1943 handwritten copy of his first 1905 relativity paper.[1] Einstein wrote the copy for a US war-bond drive. As the inset shows, the holograph fetched $6.5 million at the drive's auction. The sentence highlighted in red, translated in the third paragraph of this article, erroneously predicts that a clock at the equator would run slower than a clock at the North Pole. Einstein realized as early as 1907 that clock rates increase with gravitational potential. That effect cancels the time dilation of the equatorial clock. Nonetheless, he neither omitted the sentence in the 1943 copy nor added a corrective footnote.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off. He responded that he had discarded the manuscript after the paper was published, but he would write a new manuscript for the auction. It was purchased by the Kansas City Life Insurance Company for $6.5 million and presented to the Library of Congress.

In his Einstein biography, Abraham Pais repeats a story told to him by Einstein's secretary Helen Dukas.[16] In November 1943, she was dictating the paper to Einstein so that he could rewrite it by hand for the bond drive. At one point, he balked at what she had just read out and said, "I could have said that more simply." Nonetheless, he copied the original 1905 paper faithfully. As the reproduced excerpt above shows, he even left in the original erroneous prediction, unadorned by any corrective footnote.

We are indebted to the manuscript division of the Library of Congress for a copy of the Einstein holograph and the information concerning its auction. [Also see page 16 for Schucking's tale of a 1929 postcard from Einstein to his son Hans Albert.]

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