

Swift,⁵ provides a striking look at details of the compact-object merger that presumably triggered it. Figure 2 shows BAT's record of gammas detected in the first few minutes. A hard-gamma spike at 0.1 s is followed by softer-gamma emission with a second prompt peak at 1.1 s and a faint late enhancement—similar to the one *HETE* found—centered at about 80 s.

The burst's x-ray afterglow, recorded by *Swift*'s XRT and eventually by *Chandra*, allowed ground-based telescopes to find optical and radio afterglows and confirm that the host was an old elliptical galaxy at $z = 0.26$ with very little star formation.⁶

Surprisingly, the steady fading of the x-ray afterglow after the first two seconds was interrupted by three flare-ups. The first, similar to *HETE*'s unexpected finding, peaked at about one minute, and the last came six hours later. "Such an extended scenario of activity is hard to reconcile with model simulations of a fast, clean merger of two neutron stars," says

Gehrels. Instead, the *Swift* team argues, the episodic flaring may well indicate the stretching, breaking, and piecemeal consumption of a neutron star by a black-hole partner several times its mass.

The supernova scenario explains why long GBRs should be strongly collimated. But compact-object merger models are less clear about collimation of short GBRs.⁷ Evidence from the 24 July burst is contradictory: A break in the slope of the infrared light curve⁶ a day after the burst suggests collimation with an opening angle of about 10° . But the apparent absence of a corresponding break in *Chandra* x-ray data casts doubt on that collimation.

If the short GBRs are generally not collimated, then their energy output in gammas is not very much less than the 10^{51} ergs typical of the long GBRs. Such high energies are not obviously inconsistent with merger models. If, as now seems likely, the short GBRs are indeed caused by mergers of compact objects, their collimation is an

important issue for estimating the rate at which an upgraded LIGO gravitational-wave detector should expect to detect signals from such mergers.

Happily the localization of three short GBRs in just three months holds out the hope that astrophysicists will be able to confront their merger scenarios with many more bursts in the near future. In fact, a very faint short GRB, detected by *Swift* on 13 August, has been tentatively localized to a cluster of galaxies at a redshift of 0.7, much farther away than its three predecessors.

Bertram Schwarzschild

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Order Parameter of the Chiral Potts Model Succumbs at Last to Exact Solution

Exact solutions are prized because they can be used to compare theory with simulation and experiment without ambiguity. Finding them has sometimes proved arduous.

The statistical theory of phase changes in solids and liquids involves formidable mathematical problems." Thus Lars Onsager began his 1944 magnum opus on the two-dimensional Ising model. As if to justify his opening line, he filled the 33 pages that followed with the exposition of a new algebra and the derivation of a crystal's specific heat, partition function, and critical temperature.¹

The Ising model started life in 1925 as a simple one-dimensional quantum mechanical model of ferromagnetism. Each spin in a chain interacts with its two nearest neighbors and aligns with them, or not, depending on the interaction energy, the temperature, and chance.

Onsager generalized the Ising model to two dimensions, but his 1944 paper didn't address what was in a sense the model's original *raison d'être*: a solution for the spontaneous magnetization or its dimensionless equivalent, the order parameter M . He presented a solution without a derivation in 1949. Three years later, C. N. Yang set out to prove it.² After

working six months on the longest calculation of his life, Yang wrote up his analysis and gave, in equation 96, an expression of alluring simplicity:

$$M = (1 - T^2)^{1/8},$$

where T represents a dimensionless temperature.

Now, in an effort that lasted 15 years, Rodney Baxter has solved the order parameter of a further generalization of the Ising model known as the chiral Potts model.³ Baxter retired three years ago from the Australian National University in Canberra. Although the physical and mathematical implications of his solution aren't clear yet, its derivation, says Fred Wu of Northeastern University in Boston, represents "one of the greatest feats in the field of exactly solved models."

Scalar and chiral

In 1952, Cyril Domb asked his graduate student Renfrew Potts to tackle an N -state extension of the Ising model in which like spins are energetically favored. Potts derived a duality relation between the model's

low- and high-temperature behavior, but no one has found a general solution. Only at the critical temperature have the model's properties been calculated.

In 1974, Wu and Y. K. Wang added a further generalization to what became known as the Potts model: a dependence of the interaction energy on direction. The need for such a model became clear a few years later when experimenters began looking at the melting and freezing of atomic monolayers on crystalline surfaces.

As the temperature drops in those systems, the liquid layer orders itself into domains that either line up with the substrate structure or follow the layer's own ordering. At certain concentrations, equilibrium phase transitions occur between those so-called commensurate and incommensurate phases.

The theorists who tried to understand those transitions, among them Stellan Östlund, David Huse, and Michael Fisher, developed and explored so-called chiral versions of the 2D Potts model in which the interaction between neighboring spins differs depending on whether the neighbors lie on the x - or y -axis of the lattice. As first formulated, the models successfully captured much of the essential

physics, but had too many parameters to be solved generally and exactly.

Mathematically, 2D lattice models resemble 1D spin chains in that nearest-neighbor interactions are represented by so-called transfer matrices. In 1982, Steven Howes, Leo Kadanoff, and Marcel den Nijs investigated the properties of a three-state 1D quantum spin chain whose 2D counterpart is the chiral Potts model. Using a series expansion up to 13 orders, they charted the chain's phase diagram.⁴ To their surprise, along one line in the diagram the expectation value of the magnetization reduced exactly to a value proportional to $(1 - T^2)^{1/9}$.

When Günter von Gehlen and Vladimir Rittenberg saw the result, its simplicity struck them as too peculiar to belong only to the three-state case. In 1985, they analyzed the general N -state chain and found that the two components of its Hamiltonian have certain simplifying and attractive commutations.⁵

Higher-genus

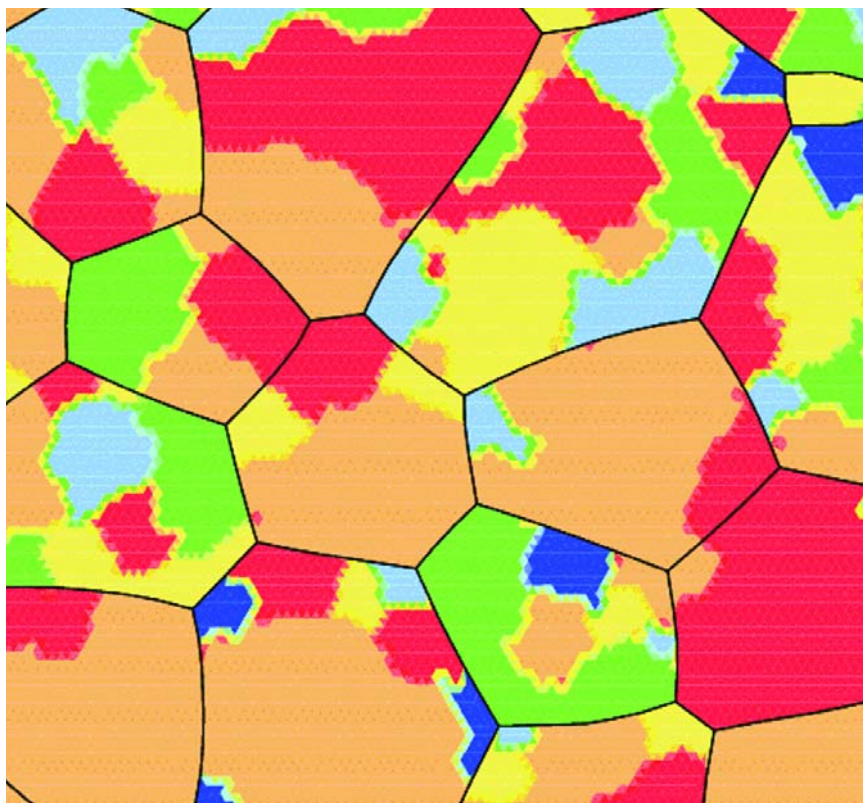
The translation back from quantum spin chains to square lattices took place at SUNY Stony Brook in 1986. Helen Au-Yang, Barry McCoy, and Jacques Perk were trying to account for the values of certain exponents in minimal models of conformal field theory. By chance and diligence, they found a chiral Potts model whose transfer matrices commute not only with each other but also with a related spin-chain Hamiltonian.⁶ They also found a generalization of von Gehlen and Rittenberg's N -state spin chain.

The commutativity is important because, along with other conditions, it implies that the model is integrable and can be solved exactly. But along with the commutativity came a startling and troubling discovery.

In integrable 2D models, it's convenient to parameterize the degree of anisotropy using two so-called spectral variables p and q . Each pair of p and q corresponds to a particular model and a pair of points on a Riemann surface.

Riemann surfaces are classified by genus. Surfaces of genus 0 have no handles or holes and the equations that relate p and q are trigonometric. For genus 1, the equations are elliptic. Though complicated, elliptic equations are tractable thanks to Niels Henrik Abel, Carl Jacobi, and their fellow algebraic geometers of the past century and a half.

The Stony Brook researchers found their integrable three-state chiral Potts model had a genus of 10. At that time, string theorists were struggling



Potts models, both scalar and chiral, have been used to study melting and crystallization on surfaces. In this simulation, the colors represent six different orientations of subgrains that form within grains (outlined in black). Another application of Potts models appears on the cover. (Courtesy of Paul Bons, University of Tübingen.)

with equations of genus 2. "For even looking at genus 10, we were ridiculed," Perk recalls.

In 1987, Baxter invited Au-Yang and Perk to visit ANU. By then, Baxter had exactly solved a host of lattice models of increasing complexity and generality. He, Au-Yang, and Perk set to work on chiral Potts and derived elegant and compact expressions for the model's interaction energies.⁷

Using those energies, McCoy, Perk, and Tang, together with Giuseppe Albertini studied the integrable N -state chiral Potts model. Like Howes, Kadanoff, and den Nijs, they derived a series expansion.⁸ In the words of their paper, they found that "all the available evidence supports the conjecture that $M = (1 - T^2)^\beta$." Here, $\beta = n(N - n)/2N^2$ and n is any integer from 1 to $N - 1$. Baxter took up the challenge of proving the conjecture.

The quest begins

Despite the problem's high genus, Baxter first applied the battery of elliptic methods that had worked for problems of lower genus. But even his most sophisticated method, corner transfer matrices, failed.

The first hint of a way out of the

impasse came in 1993. Michio Jimbo, Tetsuji Miwa, and Atsushi Nakayashiki of Kyoto University in Japan developed a new method⁹ and applied it to an Ising generalization that Baxter had worked on in the 1970s called the eight-vertex model. When Baxter tried to solve the chiral Potts order parameter with the Kyoto method, his analysis stopped short of a solution, but ended in a set of general functional equations. Going from those general equations to a specific proof took 11 years.

The main obstacle was analyticity. By working with a large, finite lattice, Onsager could take the thermodynamic limit of, say, the heat capacity by setting the number of spins to infinity right at the end of the calculation. In the Kyoto and other methods, one takes an infinite limit in the beginning. Analyticity is less secure.

Baxter's breakthrough came over two days in December 2004. It's difficult to explain, but boils down to recognizing which topological manipulations and algebraic simplifications would preserve the symmetry of the problem while at the same time establishing its analyticity.

When McCoy wrestled with prov-

ing his own conjecture, he would oscillate between two extremes: Is the machinery of 150 years of algebraic geometry useful or is it not? "The clear answer that Baxter has given us," he says, "is that algebraic geometry is unnecessary."

What other answers does the solution point to? ANU's Murray Bachelor points out that the exactly solved models of the 1960s and 70s prompted the discovery of quantum groups, which in turn led to new insights into low-dimensional topology, representation theory, and knot invariants. "We can only begin to speculate on which new areas these latest developments will inspire," he says.

One area could be string theory. In the four dimensions our universe appears to possess, the string equations are genus 2 and higher. Theorists avoid the attendant mathematical complexity by working in 10 dimensions, where they can solve equations perturbatively. Baxter's coup shows that at least one genus-10 problem can be solved without having to use a genus-10 method.

Max Dresden, the physicist and historian of science, once observed:

Models in statistical mechanics are like études in music. They could be finger exercises to improve one's technique, or they could be like Chopin's études and acquire a life and importance (and charm) of their own.

Whatever the eventual importance and charm of the chiral Potts model, Baxter's fingers, after 15 years spent on its order parameter, are thoroughly exercised.

Charles Day

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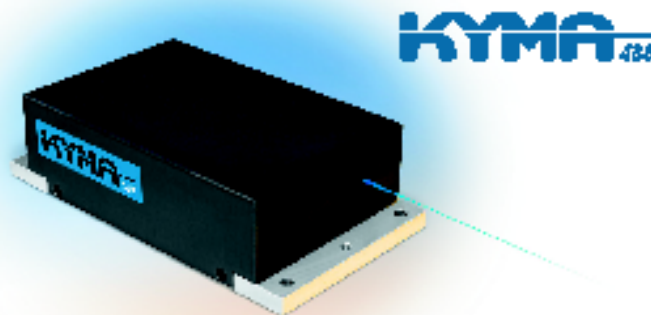
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