Devices Based on the Fractional Quantum Hall Effect May Fulfill the Promise of Quantum Computing

Recent theoretical work suggests the most arcane variant of quantum computing could become the most practicable.

To grasp the potential power of quantum computing, consider its most basic ingredient, the qubit. Unlike the classical binary bit, the qubit can be on or off or anywhere in between. When qubits are combined, their multiplicity of states balloons to fill a huge Hilbert space in which unitary transformations change myriad states at once.

Quantum mechanics is manifest in small, cold enclaves within the classical macroworld. When heat and other environmental disturbances intrude, they rob a quantum system of its coherence and its ability to compute. Error correction schemes can forestall the loss. But because the schemes work by hiding information among additional qubits, they tax efficiency.

In the face of those limitations, quantum computation based on isolated two-state systems, such as trapped ions, continues to progress. Logic gates and error correction schemes have already been built and run. Still, any computer has to execute long trains of operations. When each qubit's quantum state in each operation must be protected, the chance of decoherence derailing a calculation is high.

In 1997, Alexei Kitaev of the Landau Institute outside Moscow published a revolutionary proposal for fault-tolerant quantum computation.[1] The all-important multiplicity of states resides not in individual particles but in their shared topology. Just as a rubber ring remains a ring if poked or pulled, Kitaev's particle collective retains its coherence if locally perturbed.

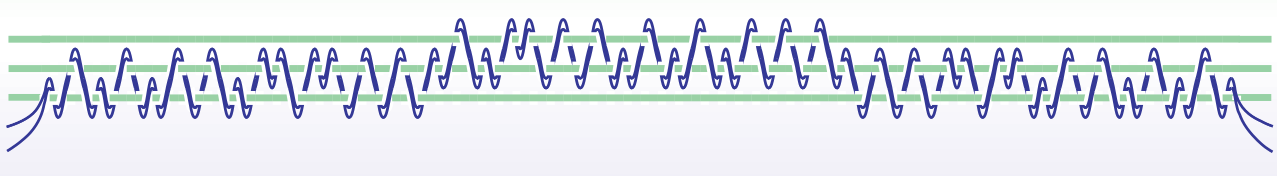

Figure 1. Braiding the world lines of nonabelian quasiparticles around each other results in a sequence of unitary transformations that approximate a logic gate. Here, a conditional NOT (CNOT) gate is realized using two triplets of quasiparticles. One triplet (green) remains in place while two quasiparticles from the other triplet (blue) wind through it. (Adapted from ref. 6.)

Despite its brilliance, the proposal baffled many physicists. The mathematical notation is formidably compact; the collective inhabits an artificial two-dimensional grid; and the Hamiltonian isn't obviously physical. The prospect of ever building a topological quantum computer looked dim.

That pessimism is fading. In April, Sankar Das Sarma of the University of Maryland, Michael Freedman of Microsoft Corp, and Chetan Nayak of UCLA outlined how one might construct a topological logic gate from a familiar material, gallium arsenide.[2]

Turning the outline into a device will be tough, not least because the practicality of the underlying physics is untested. Two new proposals aim to provide the proof.[3,4] The race to compute topologically has begun.

Not your high-school physics

Kitaev proved that topological quantum computing is intrinsically robust. However, the notion of exploiting topology to perform calculations was conceived a decade earlier. In 1988, Freedman, then a mathematics professor at the University of California, San Diego, visited Harvard University. There, he learned about two recent and disparate advances.

Computer scientists had proved that certain problems, like optimizing a traveling salesman's route through his territory, were as equivalently difficult to solve as calculating the Jones polynomial, an invariant of knots and links in three dimensions. Meanwhile, Edward Witten had proved that a certain conformal field theory in two spatial dimensions plus time mathematically resembles the Jones polynomial.

Freedman connected the two proofs and had an epiphany: Rather than trying to solve the Jones polynomial (and, by extension, all the other hard problems), why not simply measure it by manipulating whatever system Witten's quantum field theory applied to? A friend brought him back down to earth. "What Witten thinks of as physics has nothing to do with what you learned in high school," said the physicist. "The stuff probably doesn't exist in the real world."

Deflated, Freedman shelved his idea. Fortunately, the stuff does exist—in the bizarre, low-temperature physics of the fractional quantum Hall (FQH) effect.

The quasiparticles in FQH states obey fractional statistics. If you move one quasiparticle around another, it acquires an additional phase factor whose value is neither the $+1$ of a boson nor the -1 of a fermion, but a complex value in between.

Fractional statistics and other FQH properties arise from the unique topology of two-dimensional space. In 2D, particles can't pass above or below each other; they must go around. As they do so, their world lines form braids in the three dimensions of the 2D plane plus time. Figure 1 shows an example.

Among the ingredients in Witten's work on the Jones polynomial were mathematical structures that Gregory Moore and Nathan Seiberg discovered in 1987. In 2D, those struc-

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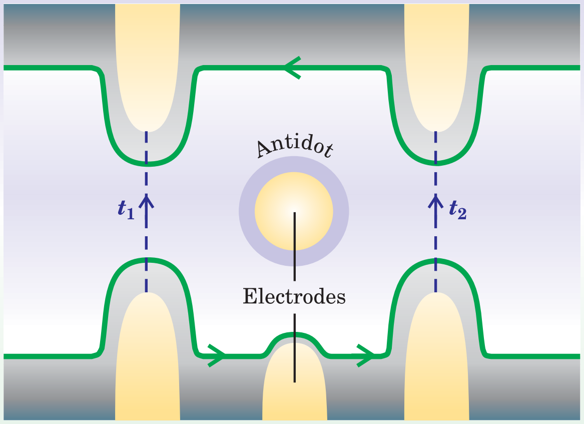

Figure 2. A quantum Hall interferometer could establish an essential requirement of topological quantum computing: that quasiparticles be nonabelian. Two pairs of point contacts make it possible for quasiparticles flowing in edge currents (green) to tunnel across the Hall bar. Because the tunneling is coherent, the two currents t_1 and t_2 interfere with each other. Biasing the lower central electrode or the electrode above the antidot changes the period of the interference pattern if the quasiparticles are nonabelian, but doesn't if they're not. (Adapted from refs. 3 and 4.)

tures describe particles whose statistics are not only fractional but also nonabelian. That is, their unitary transformations don't commute.

Nonabelian statistics would later turn out be an essential feature of topological quantum computing, but in 1987 only a few mathematical physicists explored the concept. Moore knew FQH states were thought to have ordinary, abelian statistics. But, he wondered, could some states be nonabelian? He posed the question to his Yale University colleague Nicholas Read.

Condensed matter theorists can write down an appropriate Hamiltonian for a quantum Hall system. But they can't generally solve it to find the ground state and its excitations. Instead, in an approach pioneered by Robert Laughlin, they divine a wavefunction intuitively and then work backward to see if it works.

Moore and Read found several wavefunctions whose quasiparticle excitations are nonabelian. The simplest and most compelling belongs to a spin-polarized p-wave state now known as the Moore–Read wavefunction.

The nonabelian nature of the Moore–Read state stems from the remarkable collective degeneracy of its quasiparticles. A collective of $2N$ Moore–Read quasiparticles possesses $2^{(N-1)}$ degenerate states. Moving the quasiparticles around each other changes the state of the entire collective in a way that depends only on the topology of the move. The result is one of the building blocks of quantum computation, a unitary transformation in Hilbert space.

The originally discovered FQH states have filling factors whose denominators are odd. But in 1987, Robert Willett and his collaborators found a state with a filling factor of 5/2.

Although the early experiments suggested the spins in the 5/2 state are unpolarized, Martin Greiter, Xiao-Gang Wen, and Frank Wilczek argued in 1991 that the state is described by the Moore–Read wavefunction. Through the mid-1990s, as more detailed calculations and better experiments were performed, the Moore–Read wavefunction gained favor.

In 1996, Freedman read a *Scientific American* article about the FQH effect. The article revived his interest in quantum computing and he began collaborating with Kitaev, who moved from Moscow to Microsoft, and with Nayak, one of the physicists who worked on the 5/2 state with Wilczek. The theoretical connection between FQH and quantum computing was made.

High mobility

One reason the 5/2 state was missed in the first FQH experiments is its fragility. Like a superconductor, the 5/2 state has an energy gap between the ground state and the next highest state. If too many quasiparticles become thermally excited, they cross the gap and destroy the FQH state.

To observe the 5/2 state, Willett had to run his experiment at 20 mK—hardly a practical temperature for computation. But it's possible to raise the operating temperature by making the gap wider. Doing so depends on increasing the purity of the material.

Impurities trap charge carriers, thereby frustrating their mobility and, with it, their ability to cohere in a collective state. In his 1987 experiment, Willett used a GaAs sample made by Loren Pfeiffer of Bell Labs. Its mobility was 1.3×10^6 cm^2 V^{-1} s^{-1}. Now, Pfeiffer can make samples with mobilities 24 times higher, widening the temperature of the gap to the still low, but less troublesome, 200 mK.

Das Sarma knew of Pfeiffer's prowess and progress with GaAs. When he began collaborating with Freedman, Kitaev, and Nayak, he told them a device made from the semiconductor was feasible.

Das Sarma, Freedman, and Nayak sketched out the basic components of a device that could act as a NOT gate. But the evidence that the wavefunction found by Moore and Read describes the 5/2 state observed by Willett remains mostly numerical. Proving that the quasiparticles in the 5/2 state are indeed nonabelian is therefore a necessary first step.

Two independent proposals to do that have just been posted on the arXiv server. One proposal is by Ady Stern of the Weizmann Institute in Rehovot, Israel, and Harvard's Bertrand Halperin.[3] The other is by Kitaev, who is now at Caltech, and his Caltech colleagues Parsa Bonderson and Kiril Shtengel.[4]

In their basic elements, the NOT gate and the two simpler proposals all resemble the point-contact quantum Hall interferometer that a group from Harvard, MIT, and UCLA proposed in 1997.[5] Figure 2 shows Stern and Halperin's version of the interferometer. It consists of a Hall bar with several electrodes and an antidot—that is, a small patch where the carrier concentration is depleted. Conditions are such that the electron fluid in the middle of the bar (dashed area) is deep within the 5/2 state (white area). Around the edges (green arrows), quasiparticles flow in a chiral edge current.

Two pairs of point contacts pinch the edge current and bring the top and bottom branches close enough that tunneling occurs across the bar. The tunneling rates, t_1 and t_2, lower the net current along the edges, thereby increasing resistance, which is what one measures.

As the quasiparticles circulate in the central zone, they acquire an additional phase Ω. Because of the system's coherence, the tunneling contribution to the resistance is proportional to $|t_1 + e^{i2\pi\Omega}t_2|^2$. How the nature and number of the quasiparticles influence Ω is the key to the experiment.

The Ω depends on the number of flux quanta inside the quasiparticle's path. That number can be controlled by biasing an electrode on the lower edge, the effect of which is to force the circulating quasiparticle to follow a path that encompasses fewer flux quanta. As a consequence, the electrical resistance oscillates with the variation of that electrode's voltage.

Alternatively, the number of flux quanta enclosed by the path can be altered by varying the magnetic field. The effect of that variation is more complicated, however, as it also changes the bulk filling factor and, with it, the number of quasiparticles in the central zone. Biasing the antidot can also change the number of enclosed quasiparticles.

According to Stern and Halperin's analysis, if the quasiparticles are nonabelian, the period of the oscillation will depend on whether the number of enclosed quasiparticles is odd or even. This surprising odd–even behavior comes from the quasiparticles' shared degeneracy that ordinary, abelian quasiparticles lack.

Das Sarma, Freedman, and Nayak's NOT gate is not much more complicated. It contains an additional antidot and an additional pair of tunneling electrodes.

Universal computation

Even before topological quantum computing looked as though it might be feasible, Kitaev's paper inspired theorists to explore its properties. In 2000, Freedman and Kitaev, working with Michael Larsen and Zhengang Wang of Indiana University, proved that topological and qubit-based quantum computers are equivalent or, rather, that each can faithfully simulate the other.

Another development concerns the FQH state at a filling factor of 12/5. The state was observed for the first time last year by Jian-Sheng Xia of the University of Florida and his collaborators, but its properties were anticipated earlier. In a 1999 paper, Read and Edward Rezayi of the California State University in Los Angeles identified the Moore–Read state as the second in a series of states. The third member, at a filling factor of 12/5, has nonabelian quasiparticles.

Xia observed the 12/5 state at a temperature of 9 mK, which, from the practical point of view, makes the state less attractive than the 5/2 state. However, to theorists, the 12/5 state would make a better topological quantum computer. No matter how one winds quasiparticles around each other in the 5/2 state, the Hilbert space isn't dense enough to yield even the minimum number—two—of the logic gates needed for computation.

That's not the case for quasiparticles in the 12/5 state. Indeed, in a recent paper, Nicholas Bonesteel, Layla Hormozi, and Georgios Zikos of Florida State University and Steven Simon of Lucent Technologies' Bell Labs provide a recipe for constructing logical operations by manipulating triplets of quasiparticles.[6] Figure 1 shows their conditional NOT gate.

However, because of its much wider gap, the 5/2 state will most likely be the first to be manipulated in the lab. Freedman and Kitaev are investigating ways to compensate for the state's computational shortcomings by modifying device architecture.

Back in 1993, when he was a graduate student at Princeton University, Nayak chose to work on the quantum Hall effect for his thesis. "I just thought it was an incredibly cool, beautiful subject," he recalls. "The idea it could be useful beyond a good measure of the fine-structure constant didn't cross my mind."

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Trapping Ions in Pairs Extends the Reach of Ultraprecise Optical Spectroscopy

Thanks to techniques borrowed from quantum computation, once-unsuitable ions can now be used for atomic clocks.

In standards labs around the world, physicists are building and testing the next generation of atomic clocks. Like their cesium-based forebears, the new clocks keep time by locking onto atomic resonances. To deliver high accuracy, a resonance must be sharp, but it must also be stable.

Because high frequency brings high stability, clockmakers seek optical transitions. And because the environment undermines stability, they work with single atoms or ions isolated in traps.

Spectrally speaking, the singly charged aluminum cation looks ideal for making an atomic clock. One of its hyperfine transitions ($^1S_0 \rightarrow {}^3P_0$), has a Q of 2×10^{17} and barely wavers under the influence of stray electric and magnetic fields that leak from lab equipment.

But aluminum has an unfortunate drawback. Unlike the current favorite ions of atomic clockmakers—strontium, ytterbium, and mercury—aluminum lacks a convenient transition for removing kinetic energy. If the ion remains too restless after being isolated in its trap, its motion shifts and smears the clock transition's superlative sharpness.

Now, David Wineland and his collaborators at NIST's campus in Boulder, Colorado, have demonstrated an ultraprecise method of frequency determination that doesn't require a fortuitous coincidence of clock and cooling transitions in the same species. Instead, the NIST group picks two different ion species.[1] One ion provides the clock transition, while the other provides the cooling transition. Thanks to the ions' Coulomb coupling, the cooling ion not only removes excess energy from both ions, but also acquires then divulges the probability amplitudes of the clock ion's quantum state. From those amplitudes, the clock transition's frequency is derived.

The NIST team is already running an atomic clock based on aluminum and beryllium ion pairs, but the method works for other combinations and has other applications. With an anticipated precision of 1 part in 10^{18}, the method can potentially validate the most exacting calculations of quantum electrodynamics, measure the nuclear charge radius of short-lived isotopes, and test if nature's fundamental constants vary in time.

Motional modes

Piet Schmidt, who is now at the University of Innsbruck in Austria, Till Rosenband of NIST, and Christopher Langer, a graduate student at the University of Colorado, set up and ran the first demonstration of the paired-ion method. For the experiment, which took place early this year, they paired $^{27}Al^+$ with $^9Be^+$. The aluminum ion's prime clock transition $^1S_0 \rightarrow {}^3P_0$ is somewhat difficult to work with. To test their method, the NIST group chose instead a different transition, $^1S_0 \rightarrow {}^3P_1$.

When trapped together, the two