

There are probably fewer examples of direct interest to chemists, although the chapter on stochastic processes provides useful prerequisites for electronic structure methods, such as the diffusion Monte Carlo approach. Molecular dynamics is introduced in this chapter in the context of solving the differential equations that correspond to many-body systems obeying Newton's laws.

A particular strength of this book is its coverage of both analytical and numerical methods. Chapter 1 acknowledges that most differential equations do not possess closed-form solutions and may exhibit chaos. Using a numerical tool like *Mathematica* to analyze such situations seems highly appropriate. The author includes definitions and example calculations of Lyapunov exponents to illustrate the effect of chaos. Graphical illustrations of how solutions diverge are used to good effect in this chapter—and also in chapter 2, where the accuracy of Fourier series representations is considered. Dubin's approach should enable students to understand such topics without becoming bogged down in routine but potentially error-prone algebra, and hence obtain physical insight much more rapidly.

Unfortunately, some of the illustrations in the book use features that are not implemented in the older versions of *Mathematica* that I have access to. For example, *Mathematica* 4.1 is required to run all the examples in the book, although most should work with version 4.0. The novelty of audible comparisons between time signals and their Fourier series representations is very entertaining. The coverage of special functions in the context of differential equations is also worthwhile—particularly the discussions of the Dirac delta function.

Although the range of topics covered in the book is not as wide as I had expected from the title, I was pleased to pick up some useful tips. Overall, I recommend *Numerical and Analyti-*

*cal Methods for Scientists and Engineers Using Mathematica* because it provides a wealth of *Mathematica* examples and resources, as well as an insightful way to treat a multitude of differential equations.

**David J. Wales**

*University Chemical Laboratories  
Cambridge, UK*

## The Music of the Primes: Searching to Solve the Greatest Mystery in Mathematics

Marcus du Sautoy  
*HarperCollins, New York, 2003.*  
\$24.95 (352pp.).  
ISBN 0-06-621070-4

## The Riemann Hypothesis: The Greatest Unsolved Problem in Mathematics

Karl Sabbagh  
*Farrar, Straus and Giroux, New York, 2003.* \$25.00 (340 pp.).  
ISBN 0-374-25007-3

## Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics

John Derbyshire  
*Joseph Henry Press, Washington, DC, 2003.* \$27.95 (422 pp.).  
ISBN 0-309-08549-7

The Riemann hypothesis is widely regarded as the most important un-

solved problem in mathematics. Put forward by Bernhard Riemann in 1859, it concerns the positions of the zeros of a certain function, the Riemann zeta function, in the complex plane. The hypothesis is that, apart from some trivial exceptions, all of the zeros of this complex function (which is defined by an infinite series with  $n$ th term  $n^{-z}$ ) lie on a straight line, known as the critical line, corresponding to points with real part  $1/2$ .

Proving this hypothesis is of central importance in mathematics because the Riemann zeta function encodes information about the prime numbers—the atoms of arithmetic. The nontrivial zeros play a pivotal role in an exact formula, first written down by Riemann, for the number of primes less than a given size. Individual zeros determine correlations between the positions of the primes. Finding a proof has been the main goal of most number theorists since Riemann published his hypothesis. In 1900, David Hilbert listed proving the Riemann hypothesis as one of his 23 mathematical challenges for the 20th century. In 2000, the Clay Mathematics Institute listed it as one of its seven Millennium Prize Problems, with \$1 million offered for its solution. Presently, the most we know is that at least 40% of the infinitely many nontrivial zeros satisfy the hypothesis and that it holds true for the first 100 billion of them.

Remarkably, it turns out that striking similarities exist between the Riemann zeros and the quantum energy levels of classically chaotic systems. Michael Berry pointed out, in the mid-1980s, that Riemann's formula relating the zeros to the primes is closely analogous to one discovered by Martin Gutzwiller in 1971 that relates quantum energy levels to unstable classical periodic orbits in the semiclassical limit. In 1973, following a comment by Freeman Dyson, Hugh

# BEAM

## Scanning / Steering / Focussing / Stabilization Systems

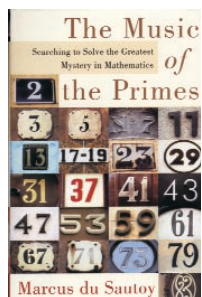
- Piezo Driven for Ultra-Fast Response
- Sub-Microrad Precision
- 1, 2 & 3 Axes and Custom Designs

# PI

Moving the NanoWorld

For more information on PI Active Optics  
Solutions call: 508-832-3456 or go to:

[www.pi.ws/pth](http://www.pi.ws/pth)



spectral statistics in classically chaotic systems are also believed to match the predictions of random-matrix theory. These observations hint at the possibility that the heights of the Riemann zeros might be the energy levels of some quantum chaotic system. Interestingly, if this were true, it would prove the Riemann hypothesis!

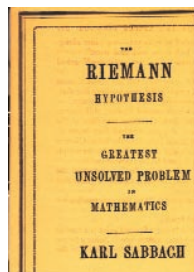
The connections between the Riemann hypothesis and quantum mechanics have generated considerable excitement. But although such connections have inspired several new lines of attack, a proof still continues to be elusive. Nevertheless, these connections have been used to suggest answers to some other long-standing and important problems relating to the zeta function.

Three books have recently been published that explain the background and history of the Riemann hypothesis and are aimed at a general audience. Of the three, Marcus du Sautoy's *The Music of the Primes: Searching to Solve the Greatest Mystery in Mathematics* is the most straightforward to review. Du Sautoy is a mathematician at the University of Oxford in England. His book explains the basic mathematical ideas at a level accessible to readers who have no more than a high-school-level mathematics background. It also contains a wealth of marvelous mathematical anecdotes. Du Sautoy clearly has a passion and talent for communicating with nonexpert audiences. His style will also appeal to researchers in other areas of science and mathematics.

Karl Sabbagh is a journalist, author, and television producer who specializes in covering scientific subjects. His book *The Riemann Hypothesis: The Greatest Unsolved Problem in Mathematics* is the least mathematical of the three. It focuses more on so-

Montgomery conjectured that correlations between the heights of the zeros on the scale of their mean spacing are statistically the same as those of the eigenvalues of large random Hermitian matrices. And quantum

biological (or anthropological) issues: What drives researchers to work on a problem that has defeated so many of the most brilliant mathematical minds for nearly 150 years? Some people have devoted a significant fraction of their creative lives to the Riemann hypothesis without, as far as we know, bringing us any closer to a solution. Sabbagh treats this human side of the story in a thoughtful and interesting way. In particular, he concentrates on Louis de Branges de Bourcia, a modern-day mathematician who has already proved one famous conjecture due to Ludwig Bieberbach in 1916. De Branges de Bourcia has repeatedly claimed to have proved the Riemann hypothesis, but his claims have never been accepted by the wider mathematics community. (As it happens, the mathematics community was slow to accept his proof of the Bieberbach conjecture.) Sabbagh paints a sympathetic but ultimately tragic picture of this important mathematician who now feels isolated from and ignored by other researchers in the field.



John Derbyshire has a background in mathematics but now works as a novelist and journalist. His book *Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics* is the most unusual.

The odd-numbered chapters contain mathematical material, and the even-numbered ones contain historical and background information. Derbyshire's book includes significantly more mathematics than the others, but it is still accessible to anyone comfortable with simple mathematical formulas. The author's goal is to explain Riemann's connection between the primes and the zeros of the zeta function, as well as other important 20th-century mathematical developments. For Derbyshire to have pulled his objective off so successfully is a remarkable achievement.

It would be invidious to compare these books directly because each has different aims. For professional scientists or mathematicians who want to know about the history of the Riemann hypothesis and the mathematics associated with it, du Sautoy's book might be the most appropriate. It also contains more information about the links between the hypothesis and quantum mechanics than the other two books. For nonscientists who want to gain an insight into research culture and the way researchers think, Sabbagh's book is likely to be interesting. And finally, for

readers who seek a deeper level of understanding of the hypothesis and more biographical details about Riemann, Derbyshire's remarkable book is, in my view, a gem.

**Jonathan P. Keating**  
University of Bristol  
Bristol, England

## New Books

### Materials Science

**Silicon Carbide: Materials, Processing, and Devices.** Z. C. Feng, J. H. Zhao, eds. *Optoelectronic Properties of Semiconductors and Superlattices* 20. Taylor & Francis, New York, 2004. \$160.00 (389 pp.). ISBN 1-59169-023-4

### Miscellaneous

**2004 Graduate Programs in Physics, Astronomy, and Related Fields.** American Institute of Physics. AIP, Melville, NY, 2003. \$62.00 *paper* (852 pp.). ISBN 0-7354-0123-3

**A Dictionary of Physics.** 4th edition. A. Isaacs, ed. Oxford U. Press, New York, 2003 [2000, reissued]. \$16.95 *paper* (546 pp.). ISBN 0-19-860759-8

**Proceedings of the International Symposium on Frontiers of Science: In Celebration of the 80th Birthday of C. N. Yang.** H.-T. Nieh, ed. Proc. symp., Beijing, China, June 2002. World Scientific, River Edge, NJ, 2003. \$88.00, \$46.00 *paper* (549 pp.). ISBN 981-238-407-3, ISBN 981-288-414-6 *paper*

**Sahara: The Extraordinary History of the World's Largest Desert.** M. de Villiers, S. Hirtle. Walker, New York, 2003 [2002, reissued]. \$14.00 *paper* (326 pp.). ISBN 0-8027-7678-7

**The Science of Superheroes.** L. H. Gresh, R. Weinberg. Wiley, Hoboken, NJ, 2002. \$24.95, \$15.95 *paper* (200 pp.). ISBN 0-471-02460-0, ISBN 0-471-46882-7 *paper*

### Nonlinear Science and Chaos

**Nonlinear Gravitodynamics: The Lense-Thirring Effect, a Documentary Introduction to Current Research.** R. Ruffini, C. Sigismondi, eds. World Scientific, River Edge, NJ, 2003. \$68.00 (509 pp.). ISBN 981-238-347-6

**Nonlinear Science: Emergence and Dynamics of Coherent Structures.** 2nd ed. A. Scott. *Oxford Texts in Applied and Engineering Mathematics* 8. Oxford U. Press, New York, 2003 [1999]. \$69.50 (480 pp.). ISBN 0-19-852852-3

**Thinking in Complexity: The Computational Dynamics of Matter, Mind, and Mankind.** 4th ed. K. Mainzer. Springer-Verlag, New York, 2004 [1997]. \$49.95 (456 pp.). ISBN 3-540-00239-1

### Nuclear Physics

**Nuclear Dynamics: From Quarks to Nuclei.** M. T. Peña, A. Stadler, A. M. Eiró,