Light's Orbital Angular Momentum

The realization that light beams can have quantized orbital angular momentum in addition to spin angular momentum has led, in recent years, to novel experiments in quantum mechanics and new methods for manipulating microparticles.

Miles Padgett, Johannes Courtial, and Les Allen

Like all wave phenomena, light has mechanical properties. Johannes Kepler suggested that comet tails always point away from the Sun because light carries linear momentum. In 1905, John Poynting developed the theory of electromagnetic radiation pressure and momentum density and, in 1921, Albert Einstein showed that Planck's blackbody law and the motion of molecules in a radiation field could be explained if the linear momentum of a photon is $\hbar\mathbf{k}$. (The wave number $k = 2\pi/\lambda$ and $\hbar = h/2\pi$, where λ is the wavelength and h is Planck's constant.) In modern times, light's linear momentum has been directly exploited for trapping and cooling atoms and molecules.

It was also Poynting who, in 1909, realized that polarized light has angular momentum—spin angular momentum, associated with circular polarization. For a single photon, it has a value of $\pm\hbar$. The idea of light's orbital angular moment came only much later: In 1992, a group at Leiden University in the Netherlands that included one of us (Allen) recognized that light beams with an azimuthal phase dependence of $\exp(-i\ell\phi)$ carries an angular momentum independent of the polarization state.[1] The angle ϕ is the azimuthal coordinate in the beam's cross section, and ℓ can take any integer value, positive or negative. This *orbital* angular momentum, they predicted, would have a value of $L = \ell\hbar$ per photon. Just as with circularly polarized light, the sign of the orbital angular moment indicates its handedness with respect to the beam direction.

For any given ℓ, the beam has ℓ intertwined helical phase fronts, as illustrated in figure 1. A feature of helically phased beams is that the phase singularity on the beam axis dictates zero intensity on the axis. Therefore the cross-sectional intensity pattern of all such beams has an annular character that persists no matter how tightly the beam is focused. The on-axis singularity is a specific instance of phase dislocation, the general literature for which is recent, extensive, and beyond the scope of this article.[2]

The concept of optical orbital angular momentum of light is not altogether new. It is well known that multipolar transitions can produce radiation that carries orbital angular momentum. But such processes are rare and relate, in the visible, to a few "forbidden" atomic and molecular transitions. What's new and exciting is that it is now possible to produce, rather easily, laboratory light beams with quantized orbital angular momentum. These beams can be used to investigate all the analogues of polarized light. For example, one can look for a photon analogue of the spin–orbit coupling of electrons and, quite generally, to search for new optical interactions.

Miles Padgett *and* **Johannes Courtial** *are physicists at Glasgow University in Scotland.* **Les Allen** *holds visiting appointments at the Universities of Glasgow, Strathclyde, and Sussex.*

Optical angular momentum

To a considerable extent, one can understand light's momentum properties without reference to photons. A careful analytic treatment of the electromagnetic field gives the total angular momentum of any light field in terms of a sum of spin and orbital contributions.[1] In free space, the Poynting vector, which gives the direction and magnitude of the momentum flow, is simply the vector product of the electric and magnetic field intensities. For helical phase fronts, the Poynting vector has an azimuthal component, as shown in figure 1. That component produces an orbital angular momentum parallel to the beam axis. Because the momentum circulates about the beam axis, such beams are said to contain an optical vortex.

The most common form of helically phased beam is the so-called Laguerre–Gaussian (LG) laser mode. In general, lasers emit a beam that gradually expands as it propagates. The magnitude and phase of the electric field at different positions in the cross section are described by a mode function. For most laser beams without helical phasing, that function is the product of a Hermite polynomial and a Gaussian. Hermite–Gaussian (HG) modes have several intensity maxima, depending on the order of the polynomials, arrayed in a rectilinear pattern and separated by intensity zeros.

The cylindrical LG modes have an explicit $\exp(-i\ell\phi)$ phase factor. That makes them the natural choice for the description of beams carrying orbital angular momentum. Although LG modes have been produced directly in laser systems,[3] they are more easily produced by the conversion of HG beams.

Generating the beams

Spin angular momentum depends only on the polarization of the beam, not on its phase. Therefore both HG and LG beams can possess spin angular momentum. Beams carrying spin angular momentum are readily produced by using a quarter-wave plate to convert linearly into circularly polarized light. The Leiden group introduced an analogous trick with cylindrical lenses to transform an HG beam with no angular momentum into a LG beam that carries orbital angular momentum (see figure 2).[4]

Although this conversion process is highly efficient, each LG mode does require a particular initial HG mode. That requirement limits the range of LG modes one can produce. Consequently, the most common method for cre-

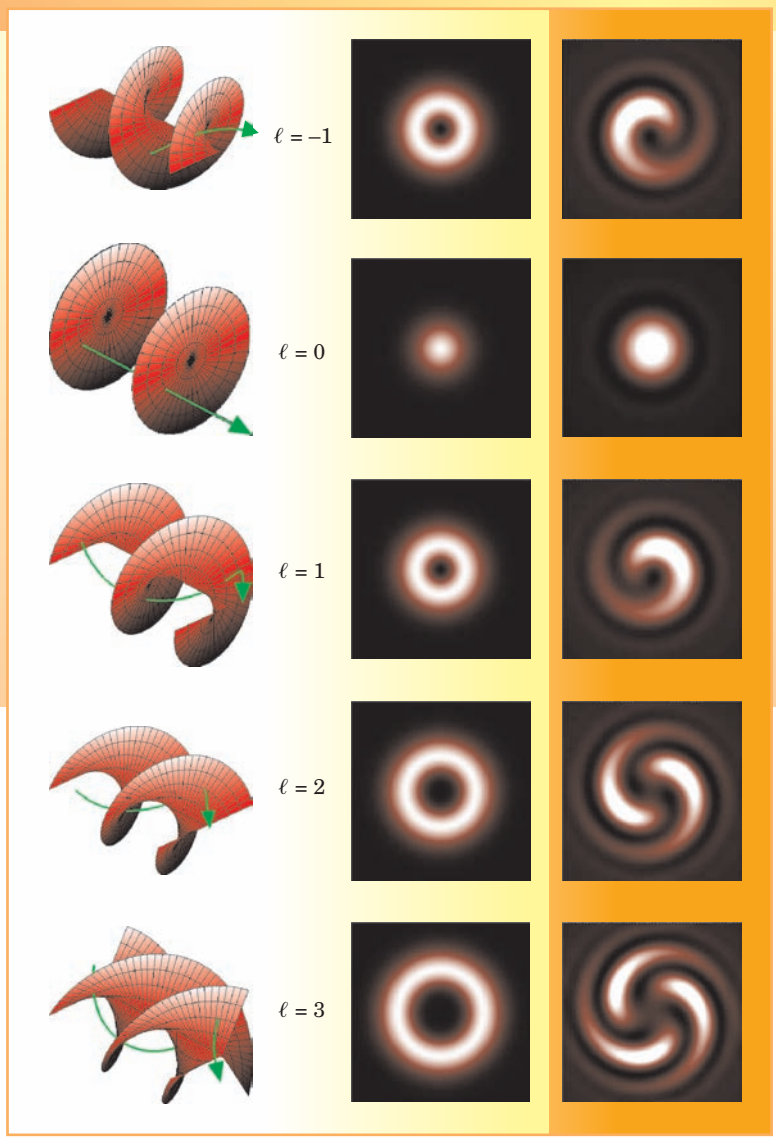


Figure 1. Orbital angular momentum of a light beam, unlike spin angular momentum, is independent of the beam's polarization. It arises from helical phase fronts (left column), at which the Poynting vector (green arrows) is no longer parallel to the beam axis. At any fixed radius within the beam, the Poynting vector follows a spiral trajectory around the axis. Rows are labeled by ℓ, the orbital angular-momentum quantum number. $L = \ell\hbar$ is the beam's orbital angular momentum per photon. For each ℓ, the left column is a schematic snapshot of the beam's instantaneous phase. An instant later, the phase advance is indistinguishable from a small rotation of the beam. By themselves, beams with helical wavefronts have simple annular intensity profiles (center column). But when such a beam is made to interfere with a plane wave, it produces a telltale spiral intensity pattern (right column). The number of spiral arms equals the number ℓ of intertwined helical phase fronts of the helical beam.

ating helical beams has been the use of numerically computed holograms. Such holograms can generate beams with any desired value of orbital angular momentum from the same initial beam (see figure 3). The requisite hologram can be formed by recording, onto photographic film, the interference pattern between a plane wave and the beam one seeks to produce. Illuminating the resulting hologram with another plane wave produces a first-order diffracted beam with the intensity and phase pattern of the desired beam.

The holographic approach can take advantage of the high-quality spatial light modulators (SLMs) that have recently become available. These pixelated liquid-crystal devices take the place of the photographic film. Furthermore, numerically calculated holographic patterns can be displayed on an SLM. These devices produce reconfigurable, computer-controlled holograms that allow a simple laser beam to be converted into an exotic beam with almost any desired phase and amplitude structure. And the beam pattern can be changed many times per second to meet experimental requirements. Figure 3 shows how a comparatively simple "forked" holographic pattern can transform the plane-wave output of a conventional laser into a pair of LG beams carrying orbital angular momentum.[5] In recent years, SLMs have been used in applications as diverse as adaptive optics, real-time holography, and optical tweezing.

Unlike spin angular momentum, which has only two independent states corresponding to left- and right-handed circular polarization, orbital angular momentum has an unlimited number of possible states, corresponding to all integer values of ℓ. Although the link between spin angular momentum and circular polarization is clear, the link between orbital angular momentum and other ways of describing the beam is less obvious. It's tempting, for example, to directly associate the orbital component to the ℓ-value of an optical vortex; but that's wrong. Because the center of the vortex is a position of zero optical intensity, it carries neither linear nor angular momentum. Instead, the angular momentum is associated with regions of high intensity, which for an LG mode is a bright annular ring.

That association is well illustrated by a recent experiment by Lluis Torner and coworkers at the University of Catalonia in Barcelona, Spain.[6] They showed that, after the beam passes through the focus of a cylindrical lens, the azimuthal component of the linear momentum near the vortex center is reversed, but the total orbital angular momentum of the beam remains unchanged. The reversal of the vortex is simply image inversion in geometrical optics; it has no implications for orbital angular momentum.

Orbital angular momentum arises whenever a beam's phase fronts are not perpendicular to the propagation direction. In the approximation of geometric optics, one would say that the light rays that make up the beam are skewed with respect to its axis. Simplistic as it is, this skewed-ray model predicts the correct result in most experimental situations.

Measuring the angular momentum of a light beam is not easy. The first demonstration of the transfer of spin angular momentum from a light beam was carried out in 1936 by Richard Beth at Princeton University.[7] The experiment was extremely demanding. A suspended quarter-wave plate took angular momentum from a circularly polarized beam. The plate's macroscopic size and corresponding high moment of inertia, however, meant

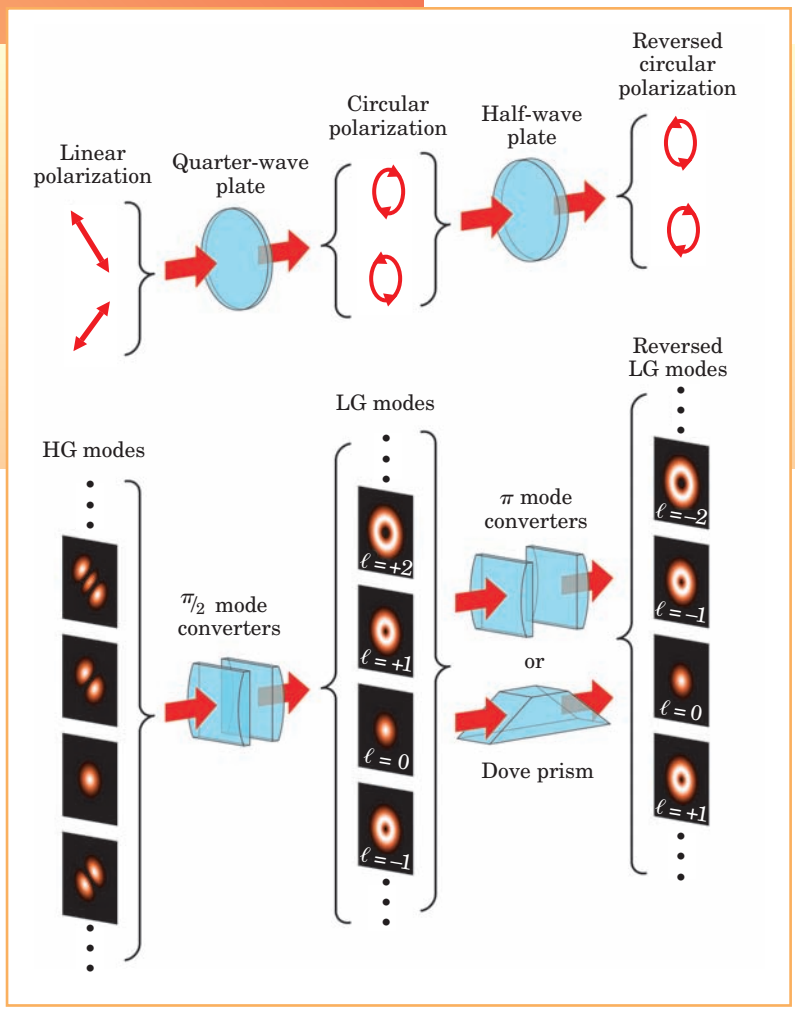


Figure 2. A pair of cylindrical lenses can serve as a converter that transforms a Hermite–Gaussian (HG) mode into a Laguerre–Gaussian (LG) mode carrying orbital angular momentum, much as a quarter-wave plate converts linearly polarized light to circular polarization. Because the same lens pair works for any HG mode, it can produce a wide range of LG modes. Increasing the separation between the cylindrical lenses can reverse the handedness of the LG mode, just as a half-wave plate reverses circular polarization. In practice, LG reversal is more conveniently achieved with an image inverter such as a Dove prism.

that its resultant rotation was tiny.

The analogous experiment for transferring orbital angular momentum to a suspended cylindrical lens has proved too difficult. Instead, a number of groups have examined angular-momentum transfer to microscopic particles held by optical tweezers.[8] Optical tweezers rely on the strong intensity gradient at the tight focus of a laser beam. At such a focus, any small, lightweight dielectric particle experiences a gradient force sufficient to attract it to the axis. So the particle is held in place without mechanical suspension.

In 1995, Halina Rubinsztein-Dunlop and coworkers at the University of Queensland in Brisbane, Australia, used an unpolarized, helically phased laser beam to impart orbital angular momentum to a small ceramic particle held by optical tweezers.[9] In 1998, the Brisbane group repeated the experiment,[10] this time imparting spin angular momentum from a polarized beam to a birefringent particle—a microscopic encore to Beth's 1936 experiment.

The previous year, two of us (Padgett and Allen) performed an experiment with Neil Simpson that demonstrated the use of a circularly polarized *and* helically phased beam as an optical wrench (in Britain we call it an optical spanner).[11] We showed, for $\ell = 1$, that when the spin and orbital angular-momentum components of the beam had the same sense, they combined to induce a rapid rotation of a small transparent particle. But when they were of opposite sense, summing to zero angular momentum, the particle did not rotate. This later experiment demonstrated that the orbital angular momentum associated with $\ell=1$ is mechanically equivalent to the angular momentum $\hbar$ associated with photon spin.

Although spin and orbital angular momentum are equivalent in many ways, they have, in general, different interaction properties. Because the orbital angular momentum of a light beam arises from the inclination of its phase fronts, its interactions with particles away from the axis are most easily understood in terms of the azimuthal component of the beam's linear momentum.

At the microscopic level, such interactions have been observed with polarized helical beams acting also as optical tweezers (see figure 4). In several experiments, a small transparent particle was confined away from the axis in the beam's annular ring of light.[12] The particle's tangential recoil due to the helical phase fronts caused it to orbit around the beam axis. At the same time, the beam's spin angular momentum caused the particle to rotate on its own axis.

At the level of individual atoms, things are somewhat different. Spin angular momentum per photon is a constant at every position within the beam. For example, a Zeeman transition can be excited by circularly polarized light of the appropriate frequency anywhere an atom finds itself within the beam's cross section. By contrast, orbital angular momentum is not transferred in the same way to atoms positioned at different radial distance from the beam axis. It is the Poynting vector's tangential component that makes the atom orbit the beam axis.

The detailed behavior of the atom is modified by frequency shifts, relaxation times, and the atom's degree of excitation. In 1994, such effects were analytically examined for atoms in LG beams by Allen, Mohamed Babiker, and coworkers.[11] Their calculation showed that, in addition to the torque around the axis, the atom experiences a shift in transition frequency. Both effects are proportional to the beam's orbital angular momentum.

Spin–orbit coupling of electrons determines much of the atom's energy-level structure. In an attempt to see if such a coupling can also exist for photons, Allen and company have shown that a light beam exerts a dissipative azimuthal force on atoms proportional to the product of the beam's spin and orbital angular momentum.[13] That force is very small—of the same order as terms invariably neglected in atom-trapping theory. But the prediction that this tiny force should be reversed when the beam's spin changes handedness was a novel one.

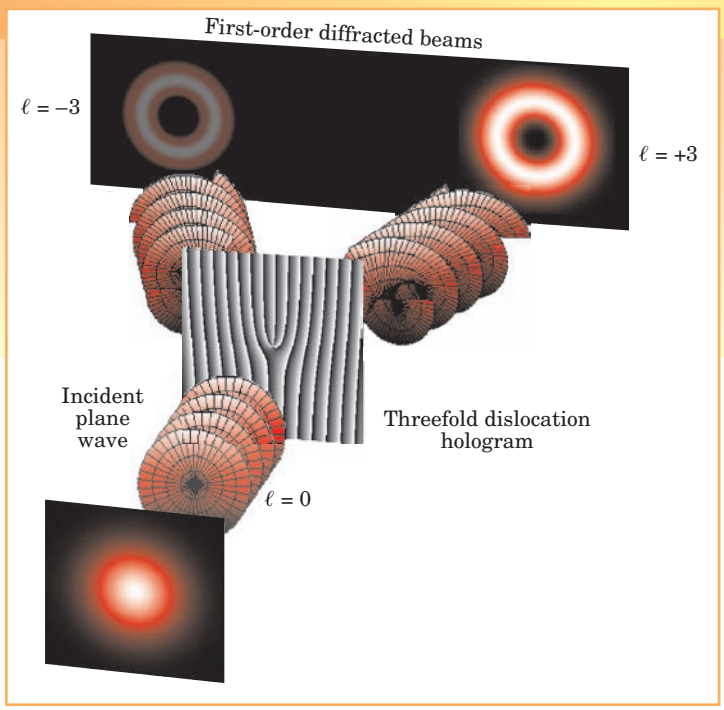


Figure 3. A hologram can produce a light beam with helical phase fronts and associated orbital angular momentum $\ell\hbar$ per photon. The appropriate hologram can be calculated, or it can be generated from the interference pattern between the desired beam form and a plane wave. The resulting holographic pattern resembles a diffraction grating, but it has an ℓ-pronged dislocation at the beam axis. (The one shown here has three prongs.) When the hologram is illuminated with a plane wave, the first-order diffracted beams have the desired helical phase fronts.

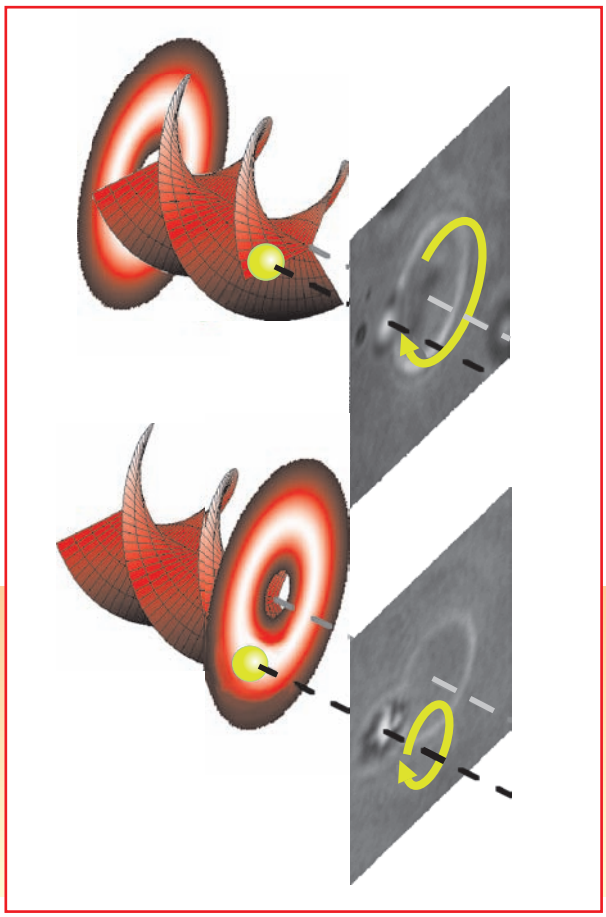
Rotational frequency shifts

In 1979, Bruce Garetz and Stephen Arnold at the Polytechnic Institute in New York observed that a circularly polarized light beam is frequency shifted if it is steadily rotated about its own axis.[14] Classically, this shift is simply what one would observe if a clock were laid face up at the center of a rotating turntable. Looking down on the clock's face, one seems to see the second hand sweeping at the wrong angular speed—the sum of the clockwork motion and the turntable's rotation.

Circularly polarized light behaves in just the same way. The rotation of the beam on its axis slightly speeds up or slows down the much faster optical rotation of the electric field vector. The mechanical rotation can be achieved with a rotating half-wave plate. This rotational frequency shift is, in some sense, an angular Doppler shift. But it is not the same as the Doppler shift associated with rotating bodies viewed from the side, which is simply a manifestation of the usual translational Doppler effect in which the rotation of an object produces a linear-velocity component along the line of sight. In fact, the angular Doppler shift is maximal when the line of sight is the rotation axis—precisely the direction in which the linear Doppler shift vanishes.

In 1998, our Glasgow group managed to observe the rotational frequency shift with a linearly polarized millimeter-wave beam that carried orbital angular momentum; millimeter-wave beams are more forgiving than shorter wavelengths of inevitable misalignments in such experiments.[11] The key to understanding the frequency shift in a helical beam is to recognize that the time evolution of a helical phase front is indistinguishable from rotation about the beam axis. So, a single rotation of the beam advances or retards its phase by ℓ cycles.

In a follow-up experiment later that year, with a beam that had both circular polarization and a helical phase front, we showed that spin and orbital angular momentum combine to give a rotational frequency shift proportional to the *total* angular momentum. That result is a generalization of the frequency shift predicted by Iwo and Zofia Bialynicki-Birula at Warsaw University in 1997 for the transition frequency of a rotating atom.[15]



Interaction with nonlinear crystals

The high power density achievable with focused laser beams has made nonlinear optics a common phenomenon in the optics laboratory. In much the same way that large input voltages cause audio amplifiers to distort, yielding

Figure 4. Transferring angular momentum from light to matter can make the matter rotate. When a circularly polarized laser beam with helical phase fronts traps a micron-sized dielectric particle (yellow ball) in an annular ring of light around the beam axis, one can observe the transfer of both orbital and spin angular momentum.[12] The trapping is a form of optical tweezing accomplished, without mechanical constraints, by the ring's intensity gradient. The orbital angular momentum transferred to the particle makes it orbit around the beam axis (top). The spin angular momentum, on the other hand, sets the particle spinning on its own axis (bottom).

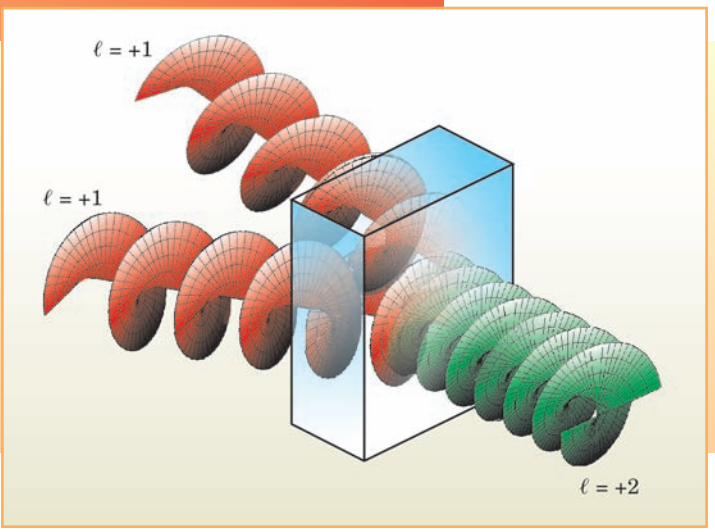


Figure 5. Second-harmonic generation with helical phase-front light beams in a nonlinear crystal conserves orbital angular momentum as well as linear momentum and energy within the light field. The two input beams are from the same source, and the output frequency is double that of the input. The process also doubles $\ell\hbar$, the orbital angular momentum per photon. Nonlinear effects in a dielectric crystal can also produce a reverse three-photon interaction that results in frequency down-conversion. A single high-energy photon becomes two photons of lower energy. That phenomenon also conserves energy, linear momentum, and orbital angular momentum within the light field.

output that contains extraneous frequencies, intense light beams distort the dielectric response of many optical crystals. This distortion, or nonlinearity, causes the light emitted by the crystal to include frequencies that were not in the optical input.

The most efficient nonlinear processes involve interaction among three waves. One either combines two incident waves to form a third, in a process known as frequency up-conversion, or one performs frequency down-conversion by splitting a single incoming wave into two. In both cases, conservation of energy at the photon level dictates that the largest of the three frequencies is the sum of the two lesser frequencies.

Conservation of momentum also plays a role. When the interacting beams are plane waves, all the Poynting vectors are collinear, and momentum conservation dictates a relationship among the three refractive indices. The requisite balancing of refractive indices is called phase matching. Remember that the Poynting vector in helically phased beams describes a spiral trajectory around the beam axis. One might think that this spiraling modifies the phase-matching conditions; but that's not the case.

Consider, for example, second-harmonic generation in which a single beam, split in two, constitutes both inputs, and the output beam is doubled in frequency. The orbital angular momentum is also doubled (see figure 5). In the photon picture, this means that two photons combine to form one photon of twice the energy, twice the linear momentum, and twice the orbital angular momentum. Contrast this conservation of orbital angular momentum within the light fields against what happens to the photons' spin. There has to be a transfer of spin angular momentum to the frequency-doubling crystal, because a single photon cannot have a spin angular momentum of $2\hbar$.

For down-conversion, energy conservation allows the initial beam to be split into two input beams with any combination of frequencies, so long as their sum equals the frequency of the initial beam. The phase-matching condition is what controls that frequency-splitting ratio. But there is nothing like the phase-matching restriction to constrain the division of the initial orbital angular momentum between the two input beams. In principle, any combination of orbital angular momenta that conserves the initial beam's orbital angular momentum is allowed. In fact, each of the split beams is in a mixture of orbital angular momentum states with a well-defined mean value. But as we shall see, the peculiarly quantum mechanical relationship between the beams can only be observed at the single-photon level.

Any number of fields can interact nonlinearly within the constraints of energy and momentum conservation. But efficiency imposes practical limits. Take, for example, four-wave mixing in which three light waves combine to produce a fourth. One such process is phase conjugation, which occurs when a nonlinear material is optically excited by two pump beams so that an additional signal beam creates a beam traveling in the opposite direction with opposite phase. It's a kind of mirror.

In a recent phase-conjugation experiment at Pernambuco Federal University in Recife, Brazil, Sergio Barreiro and Jose Wellington Tabosa used cold cesium atoms as the nonlinear material.[16] They reported that if the signal beam carries orbital angular momentum, its phase information and angular momentum is transferred first to the atoms and then, after a deexcitation time, to the "reflected" beam. Such a mechanism might one day be exploited for memory storage in multidimensional information processing.

Quantum effects

The first truly quantum-mechanical experiment using light beams with orbital angular momentum was reported in 2001 by Anton Zeilinger's group at the University of Vienna.[17] In a down-conversion experiment, the group demonstrated that the conservation of orbital angular momentum applied individually to each pair of emitted photons. This demonstration extended to orbital angular momentum the analogous tests of quantum mechanics carried out by Alain Aspect's group in the early 1980s. They had shown that the spin angular momentum of down-converted photon pairs was an entangled quantum state. The spin of a photon is defined by two states. But, as we've seen, the number of possible orbital angular-momentum states is unlimited. That difference presents the prospect of a deeper examination of quantum entanglement between photon states than has previously been possible.

The degree of a beam's polarization is readily measured with a polarizer. But orbital angular momentum, because of the arbitrarily large number of quantum states, is more difficult to quantify. One can check for a specific value of ℓ by means of the same holograms one uses to create helical phase fronts. But that method doesn't permit the ℓ of individual photons to be measured unambiguously.

Last year, however, our group devised a method for using interferometers to sort single photons into their different orbital angular-momentum states.[11] Our interferometer had a beam rotator in one arm. That introduced an ℓ-dependent phase shift that allowed the photons to be sorted into states of odd and even ℓ. Subsequent interferometer stages then permitted further sorting into specific

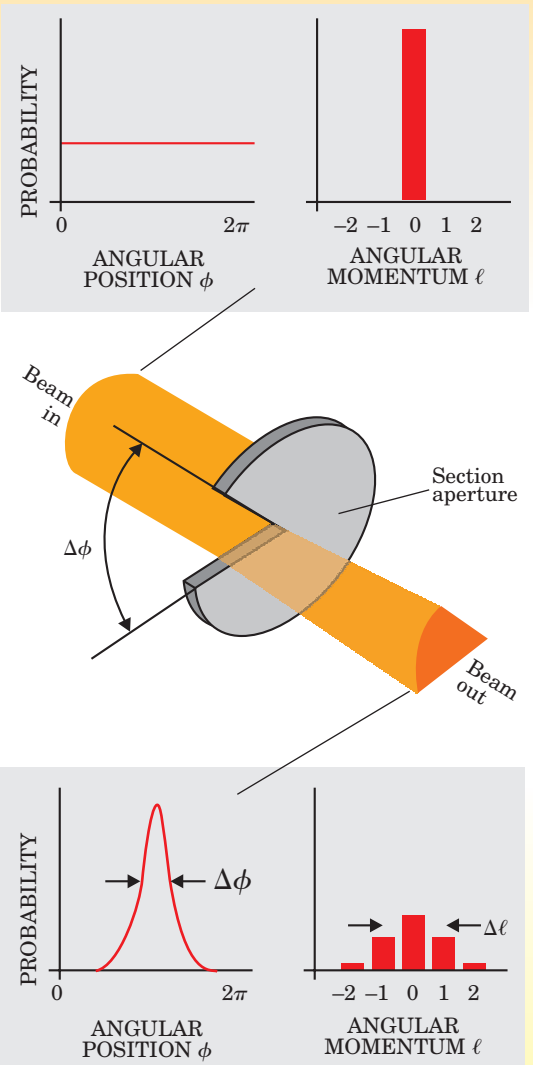


Figure 6. Heisenberg's uncertainty principle is manifested in a recent experiment at Glasgow University with a light beam that initially carries no orbital angular momentum. The range $\Delta\phi$ of azimuthal angular positions for a photon in a cross section of the beam is defined by a sector aperture. Upstream of the aperture, the beam is in an $\ell = 0$ eigenstate of orbital angular momentum (upper panel). But the uncertainty principle dictates that the restriction in ϕ introduce a spread (lower panel) in the orbital angular momentum $L = \ell\hbar$ per photon. For narrow apertures, the relationship is $\Delta\phi\Delta L \geq \hbar/2$.

orbital angular-momentum states.

An essential aspect of the quantum world is the fundamental limit that Heisenberg's uncertainty principle sets on the accuracy with which one can simultaneously know the values of a pair of conjugate variables. Angular position and orbital angular momentum are such a pair. In work not yet published, Steve Barnett and our group have recently demonstrated that this aspect of the Heisenberg principle can actually introduce orbital angular momentum—albeit with an expectation value of zero—into a light beam that starts out with none (see figure 6).

The experiment defines the angular position of a photon in the beam's cross section by making it pass through a V-shaped sector aperture whose apex is coincident with the beam axis. We have shown that, when a beam in an eigenstate of orbital angular momentum $L = \ell\hbar$ per photon passes through such an aperture, it acquires a distribution of orbital angular-momentum states. For narrow apertures, we showed that the product of aperture width, $\Delta\phi$, and angular momentum spread, ΔL, is bounded by $\hbar/2$, as quantum mechanics tells us it must be.

The exploration and exploitation of light with orbital angular momentum are still in early days. There's much to be learned about the interaction of such light with atoms. Exciting applications with rotating optical micromachines are in prospect. For quantum communication and information processing, the expansion of the Hilbert space of angular-momentum states offers opportunities for new approaches to encryption and data storage.

Martin Harwit has proposed that the orbital angular momentum of light from celestial sources might provide a new window on the cosmos.[18] For example, the observation of orbital angular momentum in light scattered by black holes could be very instructive. And we could search for signals from extraterrestrials who might be availing themselves of the high information density made possible by orbital angular momentum. At the birth of modern astronomy, Kepler studied the role of light's linear momentum. Perhaps astronomers will soon find something equally valuable in its orbital angular momentum.

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