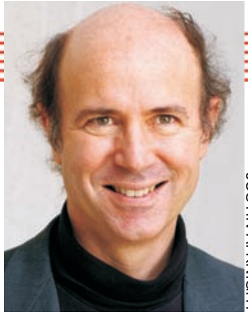

Total Relativity: Mach 2004

Frank Wilczek

In the spirit of the child hero of *The Emperor's New Clothes*, Ernst Mach (1838–1916) cleansed the intellectual atmosphere by making simple observations, obvious in retrospect, that unsettled conventional wisdom. Mach's close critical analysis of the empirical value of physical concepts and his insistence that they must justify their use helped produce the atmosphere in which special and general relativity, and later quantum theory, could be conceived.

Mach's masterpiece is *The Science of Mechanics*.[1] It is fascinating to read, even today, and every physicist ought to have that pleasure. In an annotated narrative, Mach dissects the conceptual innovations and presuppositions that marked the history of the science of motion, from its prescientific roots through the late 19th century. He was especially critical of Newton's concepts of absolute time and space:

> Absolute time can be measured by comparison with no motion; it has therefore neither a practical nor a scientific value; and no one is justified in saying that he knows aught about it. It is an idle metaphysical conception.[1]

Here's what Albert Einstein, in his self-styled "obituary," said about Mach's book:

> Even [James Clerk] Maxwell and [Heinrich] Hertz, who in retrospect appear as those who demolished the faith in mechanics as the final basis of all physical thinking, in their conscious thinking adhered throughout to mechanics as the secured basis of physics. It was Ernst Mach who, in his history of mechanics (Geschichte der Mechanik), shook this dogmatic faith. This book exercised a profound influence upon me in this regard while I was a student. I see Mach's greatness in his incorruptible skepticism and independence.[2]

Special relativity puts all space-time frames that move with respect to one another at constant velocity on an equal footing. It thereby renders moot the notion of a unique "preferred" value for any single object's velocity. Mach's deconstruction of motion, however, went much further. It culminated in a concept of total relativity, Mach's principle, which remains provocative to this day.

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Here is Isaac Newton's original formulation of his concept of absolute space:

> If a bucket, suspended by a long cord, is so often turned about that finally the cord is strongly twisted, then is filled with water, and held at rest together with the water; and afterwards by the acceleration of a second force, it is suddenly set whirling about the contrary way, and continues, while the cord is untwisting itself, for some time in this motion; the surface of the water will at first be level, just as it was before the vessel began to move; but, subsequently, the vessel, by gradually communicating its motion to the water, will make it begin sensibly to rotate, and the water will recede little by little from the middle and rise up at the sides of the vessel, its surface assuming a concave form. (This experiment I have made myself.) . . . when the relative motion of the water had decreased, the rising of the water at the side of the vessel indicated an endeavor to recede from the axis; and this endeavor revealed the real motion of the water.[3]

Mach insisted that the relative motion of bucket and distant stars is responsible for the observed concave surface. In Mach's own words:

> Newton's experiment with the rotating vessel simply informs us, that the relative motion of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the Earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass until they were ultimately several leagues thick.[1]

An ideal unrealized

A remarkable invocation of Mach's principle occurs near the beginning of Einstein's great foundational paper on general relativity:

> In classical mechanics, and no less in the special theory of relativity, there is an inherent epistemological defect which was, perhaps for the first time, clearly pointed out by Ernst Mach. We will elucidate it by the following example: Two fluid bodies of the same size and nature hover freely in space at so great a distance from each other and from all other masses that only those gravitational forces need be taken into account which arise from the interaction of different parts of the same body.
>
> Let either mass, as judged by an observer at rest relative to the other mass, rotate with constant angular velocity about the line joining the masses. This is a verifiable relative motion of the two bodies. Now let us imagine that each of the bodies has been surveyed by means of measuring instruments at rest relatively to itself, and let the surface of S_1 prove to be a sphere, and that of S_2 an ellipsoid of revolution. Thereupon we put the question—What is the reason for this difference between the two bodies? No answer can be admitted as epistemologically satisfactory, unless the reason given is an *observable fact of experience*. . . .
>
> Newtonian mechanics does not give a satisfactory answer to this question. . . .
>
> We have to take it that the general laws of motion, which in particular determine the shapes of S_1 and S_2, must be such that the mechanical behavior of S_1 and S_2 is partly conditioned, in

quite essential respects, by distant masses which we have not included in the system under consideration.[4]

The preceding quotation, part of a lengthy methodological discussion that constitutes a significant fraction of this otherwise terse paper makes it clear, as Einstein acknowledged on many occasions, that as he worked toward constructing general relativity theory his thinking was guided by Mach's principle. Ironically, however, Einstein's general relativity as finally formulated does *not* embody Mach's total relativity principle. If one analyzes the thought-experiment that Einstein outlined using general relativity, the result is just the same as in Newtonian mechanics! Einstein must have realized this, but he does not mention it in the paper.

Though Mach's principle is not an automatic consequence of the equations of general relativity, Einstein attempted in later work to impose it as a criterion to select out acceptable (that is, "epistemologically satisfactory") solutions. To rule out the troublesome behavior realized in his thought-experiment with two isolated bodies, he postulated that in reality there is no such entity as an isolated body! The universe must be spatially closed, with no boundary, and on large scales uniformly filled with matter. Although those ideas have been extremely stimulating for cosmology, it is (to say the least) not clear that they are true. Modern inflationary cosmology, for example, is agnostic regarding ultimate closure and definitely suggests nonuniformity on the largest scales. That suggestion may not be the last word on the subject, but it's become harder than ever to regard Einstein's cosmological implementation of Mach's principle as anything but an ad hoc patch.

Mechanistic variations on the theme, more in line with Mach's original idea that the distant stars cause inertia, appear in the literature. The Lense–Thirring effect demonstrates "frame-dragging" by a rotating shell: Inertial frames inside the shell rotate, in the same sense as the shell, relative to the inertial frames at infinity. Dennis Sciama has argued that a gravitational analog of electromagnetic induction would react against relative acceleration, and could act at such large distances as to provide the effect of inertia.

The perspective of symmetry

But these quasi-cosmological implementations of Mach's principle (which, when examined in detail, both have serious technical problems), like Einstein's, raise questions of symmetry. If inertia depends on the distribution of matter at large distances, why, in a lumpy universe, should inertia be accurately isotropic? Why, in an expanding universe, should it be constant? To derive these properties from the distant stars would appear to require fine adjustment of their influence. To put it bluntly, it leaves us at the mercy of astrology.

Einstein's principle of equivalence, in the form it is actually realized in general relativity, appears to represent a deeper insight. It states that in any small region of spacetime it is possible to find systems of coordinates—inertial frames—in which the laws of special relativity are valid. (And it states further that a gravitational field is equivalent, over a small region, to the use of a frame that is accelerated with respect to the local inertial frames.) This principle can be phrased as a symmetry principle: local Lorentz invariance.

Mach's principle, or total relativity, goes beyond the principle of equivalence. Total relativity can also be stated as a symmetry principle. It instructs us that in the primary equations (in other words, before their solution reveals the crucial influence of distant bodies!) we should put *all* motions on an equal footing, not just those that correspond to constant relative velocity. It claims that the choice of coordinates is entirely a matter of convention and requires that we remove all intrinsic structure from spacetime. On that basis any choice of coordinates should be on equal footing, since the labels implementing the coordinates could be undergoing arbitrary motions. But in general relativity, spacetime is not without structure, and it is not true that all coordinate systems are equally good (notwithstanding contrary statements that pervade the literature—starting, as we've seen, with Einstein's original paper). General relativity includes a metric field, which tells us how to assign numerical measures to intervals of time and space. It's convenient to choose frames in which the metric field takes its simplest possible form, because in such frames the laws of physics assume their simplest form.

Posing the issue, Einstein *versus* Mach, as a question of symmetry brings it within a circle of ideas that are central to modern fundamental physics. In the standard electroweak model, we have a Higgs field that breaks local gauge symmetries of the primary equations; in quantum chromodynamics, we have a quark–antiquark condensate that breaks both those symmetries and others; and in unification schemes, generalizations of the symmetry-breaking idea are used freely.

The perspective of symmetry naturally suggests questions that could prove fruitful in the future of physics. It invites us to contemplate the possibility of primary theories enjoying larger symmetries than are realized in the equivalence principle of general relativity. Mach's principle, from this perspective, is the hypothesis that a larger, primary theory should include total relativity—that is, physical equivalence among all different coordinate systems. (A different generalization appears in Kaluza–Klein theory and its modern descendants: In the process of compactifying extra dimensions, the higher-dimensional equivalence principle is broken down to the smaller symmetry of its 3 + 1 dimensional version.) This primary symmetry of the equations must, of course, be badly broken in the particular solution that describes the world we observe. Nevertheless, its conceptual influence would be felt through restrictions it imposes on the equations of the physics. As we struggle with the problem of the cosmological "constant," constructive suggestions for augmenting the equivalence principle could prove most welcome.

Mach's austere empiricism is a disinfectant that, taken too far, can induce sterility. Mach himself never accepted special relativity. He also denounced atomism and harassed his great contemporary Ludwig Boltzmann over it.[5] In private correspondence (quoted in reference 5), Einstein wrote that Mach's approach to science "cannot give birth to anything living, it can only exterminate vermin." Yet in this sharp statement, I believe Einstein meant to be judicious. Exterminating vermin is a necessary and sometimes challenging task, even if it is not so transcendent as giving birth. In the world of ideas, as opposed to the world of events, we can choose what to retain. The good that men do lives after them, the evil is oft interred with their bones.

References

1. E. Mach, *The Science of Mechanics: A Critical Historical Account of its Development*, Open Court, La Salle, IL (1893).
2. A. Einstein, in *Albert Einstein: Philosopher–Scientist*, P. Schilpp, ed., Harper and Row, New York (1949), p. 1.
3. I. Newton, *Mathematical Principles of Natural Philosophy*, I. B. Cohen, A. Whitman, trans., U. of Calif. Press, Berkeley (1999).
4. A. Einstein, *Annalen der Physik* **49**, 769 (1916); Eng. trans. in *The Principle of Relativity*, Dover, New York (1952), p. 111.
5. D. Lindley, *Boltzmann's Atom: The Great Debate That Launched a Revolution in Physics*, Free Press, New York (2001). ■