but presents enormous challenges. At most, there would be just one electron per atom in half of one full plane available for conduction.

Conductivity aside, the demonstration of a weakly stabilized Fermi level offers hope to those interested in making reliable SiC ohmic contact with other molecules or devices. The presence of a metallic component in an intrinsically insulating substrate like SiC might facilitate charge transport from SiC to an external metal contact.

Soukiassian and Chabal are also trying to determine whether the dangling bonds really fall on just one side of the trough (as rendered in figure 1b) or whether a random pattern is more likely. In any case, at some range of temperatures, they expect hydrogen to hop between the third-layer dimers in a way that would smear out the 1D line of electronic states. That effect—hydrogen hopping—could lead to interesting physics itself and provide a basis for studying 1D behavior.

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The Force Need Not Be With You: Curvature Begets Motion

Cyclic deformations can alter the free-fall motion of a composite body as it moves through curved spacetime.

Classical mechanics, like geometry, can yield surprising results in a non-Euclidean space. One such result, recently discussed in the journal *Science* by MIT planetary scientist Jack Wisdom, concerns the motion of a composite object as it undergoes a cyclic series of changes in body shape.[1] Consider, for example, a person making the precise, regular motions of an Olympic swimmer. In empty Euclidean (flat) space, the swimmer's center of mass wouldn't move; water in the pool, of course, provides the external reaction force that propels real swimmers. The curved-space surprise is that a swimmer executing an appropriate cycle of internal changes can move, even without external forces.

The ability to swim in a curved space suggests that swimming should be possible in four-dimensional curved spacetime as well. But results that are true for spatial surfaces aren't always true for spacetime. In his *Science* article, Wisdom considered a particular spacetime, called Schwarzschild space, that describes spacetime outside a spherically symmetric gravitational source. (A black hole is the most famous example.) He found that a composite object can indeed swim in Schwarzschild space. "It's a beautiful idea," says astrophysicist Edmund Bertschinger of MIT, "that opens a new avenue into rigid-body dynamics in general relativity."

Swimming on a sphere

As a prelude to his more complicated analyses, Wisdom discussed a simple system for which the mathematics behind center-of-mass translation can be pictorially displayed. Figure 1 illustrates two small disks undergoing a four-step cyclic process on the surface of a sphere. At the beginning of the cycle, the two disks are at the equator of the sphere, one right on top of the other; the system has zero angular momentum. In the first step, some massless coupling mechanism rotates each disk by the same angle but in opposite senses, consistent with angular momentum conservation. In the second step, each disk is translated along a meridian through an equal angle; one moves north, one moves south. That step also conserves angular momentum. In the third step, the initial disk rotations are undone. While the disks return to their unrotated states, they each have some angular momentum, and, as the figure shows, the two angular momenta do not cancel. To conserve angular momentum, each disk must move westward along its latitude line. In the final step, the disks are brought back to the equator along a meridian. They end up west of where they began, but otherwise the system has not changed.

Wisdom showed that if the radius r of each disk is small compared to that of the sphere, R, and if the angles (θ and φ respectively) by which each disk is rotated and moved along a meridian are also small, then the net angular translation ψ of the disks along the equator is

$$\psi = (1/2)(r/R)^2\theta\varphi.$$

The translation involves only geometric

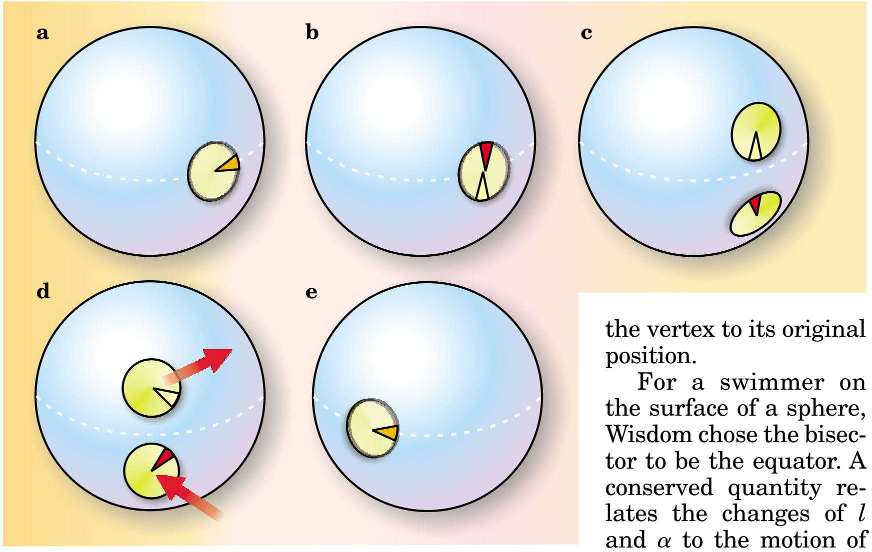

Figure 1. Small disks on the surface of a sphere move their center of mass by means of a cyclic series of four steps. **(a)** Two disks are initially located on the equator. **(b)** They rotate an equal angle but in opposite senses. **(c)** After the rotation is completed, the disks move equal amounts along a meridian, one north, the other south. **(d)** The rotations of step (b) are undone. These rotations generate angular momenta (red arrows) whose sum does not vanish. In order that angular momentum be conserved, the two disks must move westward. **(e)** The translations of step (c) are undone. The net result of the cycle is that the disks move westward along the equator.

quantities and is independent of how rapidly the cycle is executed. The disk motion is reminiscent of other motions governed by "geometric phase," as discussed by Edward Purcell[2] and by Alfred Shapere and Frank Wilczek.[3]

Conserved quantities

In the absence of external forces, cyclic processes in flat space cannot lead to center-of-mass motion. Nonetheless, a description of a swimmer unsuccessfully trying to move in flat space gives a flavor of Wisdom's more complicated analyses of motions in curved spaces and in the spacetime outside a black hole.

Figure 2 shows a simple two-legged swimmer comprising a "vertex" mass separated by a distance l from two equal "leg" masses. The half-opening angle at the vertex is α, and the x axis bisects the swimmer. The four steps of the ultimately unsuccessful swimming cycle are to increase l, decrease α, decrease l to its original value, and increase α to its original angle.

Conservation of momentum—that the center of mass is fixed for a swimmer initially at rest—allows one to relate the motion of the vertex to the change in the swimmer's leg length and angle. During the first two steps of the cycle, the vertex moves backward. The last two steps of the cycle return the vertex to its original position.

For a swimmer on the surface of a sphere, Wisdom chose the bisector to be the equator. A conserved quantity relates the changes of l and α to the motion of the vertex along the equator. The conserved quantity, though, is not simply a sum of three coordinate velocities multiplied by mass, as it is in flat space. Rather, it is a sum of three terms of the form $m \sin^2\theta\, d\psi/dt$—the terms include mass m as well as both polar (θ) and azimuthal (ψ) coordinates.

As the swimming cycle progresses, the vertex remains on the equator, but the polar coordinates of the leg masses change. For this reason, the backward motion of the vertex during the first two steps of the cycle is not exactly compensated by the forward motion of the vertex during the final two steps.

Swimming in spacetime

To demonstrate swimming in a real curved spacetime—Schwarzschild space—Wisdom considered a tripod-like composite with a vertex mass and three equal leg masses. All leg masses were the same length l from the vertex along geodesic struts. Each strut was displaced by a common angle α from an axis that points radially from the mass that defines the Schwarzschild space to the vertex of the tripod. The tripod attempts to "swim against the current" in that it falls toward the gravitating mass while undergoing its swimming cycle.

By relating the motion of the swimmer's vertex to changes in l and α, Wisdom concluded that the swimming action is successful—the tripod swimmer moves in the radial direction relative to its free-fall motion toward the attracting mass. If the swimmer is small compared to the local effective radius of curvature R, swims with modest changes in l and α, and if its

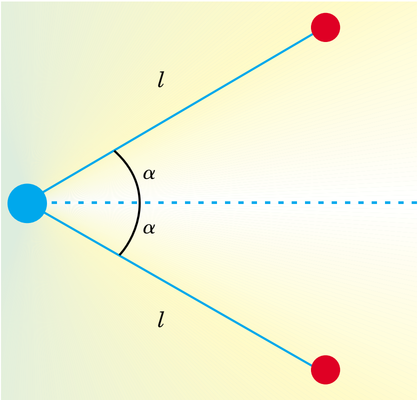

Figure 2. A simple swimmer may be parameterized by the equal lengths l of its two legs, and the half-opening angle α between a leg and the symmetry axis. Conservation of momentum guarantees that such a swimmer executing cyclic changes in l and α can't move in flat space. But it can move on curved manifolds.

masses all move slowly compared to light, the radial displacement achieved in a swimming cycle will scale as $(l/R)^2$, as in the disks-and-sphere example. That displacement is small enough that swimming is not a useful propulsion mechanism.

For example, consider a swimmer having roughly equal leg and vertex masses, extending its struts by a meter, and located just above Earth's surface. The swimming cycle would yield a radial displacement of a minuscule 10^{-23} meters, distinctly less than the distance the swimmer would fall toward Earth. On the other hand, a swimmer in a circular orbit about a spherical mass might be able to actually increase its orbital radius by an appropriate swimming cycle. Wisdom hopes to explore that conjecture.

In some stories that describe the adventures of Baron von Munchausen, the legendary prevaricator claims to have pulled himself out of the mud by tugging on his bootstraps. That's impossible—even for a baron in a curved space. But, says Wisdom, von Munchausen might have liberated himself by tugging on his bootstraps and kicking his heels.

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