# Dimensional Diversity and New Physics Bring Abbott up to Date

## Flatterland: Like Flatland, Only More So

Ian Stewart
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## The Annotated Flatland: A Romance of Many Dimensions

Edwin A. Abbott, Introduction and Notes by Ian Stewart
*Perseus, Cambridge, Mass., 2002. $30.00 (239 pp.). ISBN 0-7382-0541-9*

*Reviewed by Leonard Parker*

In his 1885 classic, *Flatland* (Roberts Brothers), Edwin A. Abbott was intent on making his three-dimensional readers understand that a fourth dimension may be real despite limitations of human intuition. Abbott wrote about a world peopled by two-dimensional creatures that cannot imagine a third dimension. One of these creatures, A. Square, is visited by a sphere that moves him into the third dimension, where he can view Flatland from “above.” When A. Square returns to Flatland and talks about the reality of the third dimension, he is imprisoned for life.

Today the fourth dimension may be familiar, but not many of us have an intuition for the kinds of spaces encountered in physics and mathematics. Ian Stewart's *Flatterland* begins where Abbott left off. Vikki, a teenage descendant of A. Square (“A” now stands for Albert) finds a copy of the banned book *Flatland* and scans it into her computer before her father destroys the book. Following instructions left by Albert, she summons the Space Hopper, who guides her out of Flatland and introduces her to some beautiful worlds in mathematics and physics.

**Leonard Parker**, *a professor of physics at the University of Wisconsin–Milwaukee, does research in gravitational physics. His current work is on the relationship of quantized fields in curved space-time to the recently observed acceleration of the expansion of the universe.*

A worthy successor to Abbott, Stewart served as the Gresham Professor of Geometry at the University of Warwick, England, in a long line of distinguished scholars that includes Robert Hooke. For furthering the public understanding of science, he received awards from the Royal Society and from the American Association for the Advancement of Science.

*Flatterland* contains many surprising connections and “aha” moments. On one of her first adventures, Vikki finds that unit cubes in $n$ dimensions can be used to understand a class of Richard Hamming's codes for correcting errors in a sequence of binary digits. On another visit, she uses color as a fourth dimension to see how a knot in three dimensions can be untied in a fourth spatial dimension.

Vikki also finds out about self-similar fractals and their fractional dimensions. As the Space Hopper explains, a fractal's dimension is determined by how many copies of the fractal are required to make a larger version of the same fractal. For example, the Sierpinski gasket is constructed by starting with a triangle, cutting the largest possible hole in it that is a similar upside-down triangle, and continuing in this way. Because the first hole has the size and shape of a half-size gasket, you can make a gasket with length-2 sides by joining, corner to corner, 3 gaskets with length-1 sides. The number 3 defines the gasket's fractal dimension as $\log 3 / \log 2$. Similarly, in a plane, you need four length-1 triangles to make a length-2 triangle, so the plane's dimension is $\log 4 / \log 2 = 2$.

It is amazing how clearly Stewart explains normally obscure geometrical concepts. Teachers at any level would gain by studying his pedagogical techniques. A masterly example is his treatment of projective planes, which begins by asking how seven varieties of grapes can be arranged in plots. Each plot has exactly three varieties, any two plots have exactly one variety in common, and any two varieties lie in exactly one common plot. By drawing a graph that shows the grape varieties as points and the plots as connecting segments, Stewart devises a beautifully clear explanation of the 7-point projective plane. He then proceeds to the 13-point projective plane, and to some yet unanswered questions about projective planes.

In a place called Platterland, Vikki encounters non-Euclidean geometry, and in Cat Country, she learns about Schrödinger's poor cat and about an electron's dual wave–particle nature. The narrative slightly missteps here. An electron hits a metal plate, which subsequently emits a photon, a reversal of photon and electron roles in the photoelectric effect. But there follows a good discussion of macroscopic decoherence times as related to Schrödinger's cat. Later in the book, a few other particle-physics inaccuracies occur, but the excellence of the exposition far outweighs those minor flaws.

After visiting the Paradox twins to learn about inertial and accelerated frames in relativity, Vikki enters the amazing domain of the Hawk King, which contains not only black holes and white holes, but wormholes that can serve as time machines. To use a time machine, Vikki and the Space Hopper have to enter a black hole. To emerge, they use a white hole and a time machine that traverse a pair of closed timelike curves! Stewart is at his best in describing closed timelike curves and how to avoid the logical paradoxes their existence appears to imply.

Vikki goes on to learn about big-bang nucleosynthesis and quantum foam. The Space Hopper explains, “That's quantum foam—particles springing into and out of existence, creating space and time along with themselves. Try some, it's quite tasty” (page 282). In teaching Vikki about supersymmetry, the Space Hopper offers an excellent discussion of the viable 10-dimensional superstring theories and why they may be different views of a single 11-dimensional “M-theory.”

One may recall that Abbott's *Flatland* portrays women as line segments and men as polygons of higher status. In Stewart's *Flatterland*, Vikki is a line in Flatland before she goes to other spaces. I won't give away the clever ending, but suffice it to say that Stewart portrays women as more than merely equal to men in ability and social status.

In *The Annotated Flatland*, Stewart augments *Flatland* with an introduction and extensive notes, both to explain the historical context and to update and expand selected topics. The notes are wonderfully clear and succinct. For example, in slightly more than half a page, a note on $\pi$ summarizes Archimedes's use of polygons on a circle to bracket the value of $\pi$, mentions the 1761 proof that $\pi$ is irrational and the 1882 proof that $\pi$ is transcendental, and defines irrational and transcendental numbers. A longer note discusses Euclidean construction of regular polygons (those familiar Flatland shapes), and even mentions Gauss's proof of when such constructions are possible. In a note on cellular automata, Stewart observes that if Flatlanders had cellular-automata brains, their intelligence would not be limited by their dimensionality. I found the notes in *The Annotated Flatland* to be gripping and enjoyable.

*Flatterland* and *The Annotated Flatland* are instructive, stimulating, and great fun. I highly recommend both books to physicists and students who enjoy having fun with mathematics.

## Physics and the Art of Dance: Understanding Movement

Kenneth Laws
*Oxford U. Press, New York, 2002. $39.95 (236 pp.).*
*ISBN 0-19-514482-1*

What do Maxwell's equations and Fokine's *Les Sylphides* have in common? Both are towering and seminal achievements—and almost completely unfamiliar outside their respective areas. Both have beauty that is generally inaccessible without knowledge of their language and tradition. Each is couched in a precise vocabulary. Few people outside the fields know that vocabulary, and even fewer are familiar with the vocabularies of both.

Kenneth Laws is evidently one of those few. He has written a most unusual book, *Physics and the Art of Dance: Understanding Movement*, to bridge the gap between the two worlds. After teaching college-level physics for more than a dozen years at Dickinson College in Carlisle, Pennsylvania, Laws became interested in classical ballet and in the application of physics principles to it. The preface gives an intriguing hint of how strange the experience must have been: In his acknowledgements, Laws thanks the artistic director of the Central Pennsylvania Youth Ballet, who "has accepted me in classes in which the next oldest dancers were sometimes fifty years younger than I!" (preface, p. XV).

Laws has been teaching ballet classes since the 1980s, wrote two previous books about physics applied to dance, and wrote an article on the subject for PHYSICS TODAY (February, 1985, page 24). *Physics and the Art of Dance* is an extended and enhanced addition to the earlier works. Addressed to dancers, the book is a detailed and practical exposition of the mechanics of certain basic and complicated movements of classical ballet. The physics in the book is at the first-year college level, but it is interesting to see the principles of mechanics applied in such detail to management of the human body.

The book is organized in logical order around problems of motion, from simple retention of balance to more complicated movements. All positions are given their proper French terminology, and details are scrupulously accurate. The first section discusses such concepts as equilibrium and center of gravity, and explains how they apply to the dancer's body. The author suggests adjustments the dancer can make to retain or regain balance by applying these concepts. After dealing with balance, he turns to motion: horizontal motion, vertical jumps, the role of acceleration, the use of arm motion to affect the height of a jump, and illusions caused by the configuration of the body while it is in the air. One section deals with the elasticity and friction of floors.

In a chapter that clearly explains torque and angular momentum in a pirouette, the book progresses to more complex movement involving turns. I find entertaining the very fact that "torque" and "pirouette" appear in one sentence, and it may convey some of the flavor of this book. Subsequent chapters address turns in the air, and then the more complicated problems of dancing with a partner.

The writing is lucid, friendly, and to the point. Laws obviously knows his ballet as well as his physics. Each chapter begins with a "challenge problem" similar in style to those in introductory science textbooks—but here, rather then finding "what causes your hair to stand on end before a storm," we find such questions as how to do a 16-turn pirouette at constant tempo (p. 63). The text then shows how such problems may be solved by use of physical principles. The accompanying photographs clarify the text and accent details of the movement. The nonprofessional reader will gain from the photographs a heightened awareness of the structure of ballet positions, a sense of the beauty of "line," and a sense of the attempt the dancer must make to achieve a specific body shape.

Following the text are 11 appendices that give a more detailed examination of the physics. They begin with an outline of the principles of mechanics and follow up with quantitative analyses of various problems of movement. The appendices can be quite entertaining, as in a table that shows the time evolution of tilt from the vertical for tall and short toppling dancers.

This book will surely have immense value to dancers. Its value to nondancers is less obvious, and lies mainly in the insight it gives into the classical ballet, which in its own way is as precise and formal as mathematics. The methodical approach of the book may make ballet more accessible to the scientist. The detailed analysis of ballet movements will enhance the reader's understanding and appreciation of ballet performance. And I found it fascinating to see physical principles applied in such practical detail to positioning the human body in the beautiful and abstract movement of the ballet.

The book ends with an appeal for bridges between science and dance. "We know that the basic physical principles are sound and do indeed apply to the human body. The challenge is to make an understanding of this framework not a mere abstraction but *useful* in a way that contributes to improvement in dance technique and to an appreciation of the beauty of dance." Indeed the book fulfills that aim.

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## Envisioning Science: The Design and Craft of the Science Image

Felice Frankel
*MIT Press, Cambridge, Mass., 2002. $55.00 (328 pp.).*
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The mechanics of continuous media—fluids, deformable solids, and fractures—was long considered a natural part of theoretical physics. Well known and widely used were the multivolume courses of theoretical physics by Max Planck, Arnold Sommerfeld, Lev Davidovich Landau and Evgenii Mikhailovich Lifschitz; each of those courses contained one or two volumes on continuum mechanics. Nowadays, continuum mechanics is not extensively taught as a part of fundamental physics. Because continuum mechanics