Financial Markets: The Most Complex of Many-Body Problems

The Statistical Mechanics of Financial Markets

Johannes Voit
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Reviewed by Robert W. Lourie

Physicists' study of financial markets has deep historical roots, beginning with the now-famous 1900 thesis "Theory of Speculation" by Henri Poincaré's student Louis Bachelier. In modern times, physicists studying financial markets have seized the data-analysis opportunities presented by the availability of large, high-frequency data sets for price movements on many markets. Such analyses are attractive for several reasons: Physics, especially statistical physics, has long been concerned with the appearance of universal features in seemingly unrelated systems; financial markets represent, in some sense, the most complex of many-body problems, as each interacting body is a sentient being; and it is natural to ask whether markets exhibit any statistical structures akin to those in more physical systems. Finally, there is the pragmatic, but hardly unattractive, goal of profiting from insights into market mechanics.

Johannes Voit's *The Statistical Mechanics of Financial Markets* provides an excellent introduction for physicists interested in the statistical properties of financial markets. Appropriately early in the book, the basic financial terms and concepts such as shorts, limit orders, puts, calls, and others are clearly defined. Examples, often with graphs, augment the reader's understanding of what may be a plethora of new terms and ideas.

The random walk, probably the central concept of the book, is discussed in detail. The seminal work of Bachelier is well described and appropriately acknowledged (which other authors are not always careful to do). Bachelier's analysis is applied to the concrete example of a bond future. Connection to Brownian motion (including Albert Einstein's work), and to diffusion processes is then established. Indeed, a general strength of this book is that a corresponding, presumably familiar physical system is analyzed in conjunction with a financial one. The reader is then introduced to more general stochastic processes and geometric Brownian motion, and to the derivation of the log-normal distribution of price moves. The results on geometric Brownian motion are used to obtain the classic result in option pricing, the Black–Scholes equation. These results, and others that Voit presents, hold under somewhat unrealistic, idealized assumptions. While treatments of this kind are entirely appropriate in an introductory text, a somewhat more extensive critique of these assumptions would have been welcome, especially in light of the "long-term capital management" debacle in 1998.

The chapter on Scaling in Financial Data and in Physics may be the most interesting to a cursory reader. Beginning with a brief discussion of the statistical properties of time series (stationarity, correlation, autocorrelation), Voit concentrates on the scaling properties of price movements. He demonstrates that price movements on time scales that vary by several orders of magnitude may be collapsed onto a universal curve by a simple rescaling of the time interval and the observed probability density. His observation leads to the consideration of Levy-stable probability densities, of which the familiar Gaussian is a special case. A key feature of these general Levy distributions is their heavy tails—a considerably higher probability of large (positive or negative) values than in a Gaussian. The financial implication is obvious: If you model the world as Gaussian, you will have a significantly greater probability of striking it rich (or going completely bust!) than your model leads you to believe. The reader interested in a more extensive treatment of scaling and correlations in markets should consult *An Introduction to Econophysics: Correlations and Complexity in Finance* by Rosario M. Mantegna and Eugene Stanley (Cambridge U. Press, 2000).

Voit examines the interesting comparison that can be drawn between the energy cascade over different length scales in turbulent flow and the information cascade over varying time scales in the foreign exchange (FX) market. No other market generates the wealth of high-frequency data that FX does, with its round-the-clock trading of some 10^{12} dollars per day. Voit demonstrates that certain features scale similarly in these two cases. It is hard to know if the similarity is more than accidental. While a microscopic description of turbulent flow is exceedingly ambitious theoretically and computationally, it seems achievable in principle. In contrast, a similar level of description of markets would require a model for each participant, and this is well beyond our grasp.

The Statistical Mechanics of Financial Markets concludes with a somewhat strained analogy between market crashes and earthquakes. The paucity of data on crashes makes the evaluation of the apparent log-periodic precursors to the crash difficult. Although definite physical mechanisms for earthquakes have been identified, considerable disagreement remains on what causes market crashes. Obviously, psychological variables play a centrol role. Of course there would be a great benefit to participants in financial markets if crashes could be reliably predicted. No doubt there will be continuing intense research in this area.

In conclusion, *The Statistical Mechanics of Financial Markets* is an excellent starting point for the physicist interested in the subject. Some of the book's strongest features are its careful definitions, its detailed examples, and the connections it establishes to physical systems. The mathematics are at the level of upper undergraduate statistics and statistical physics, making the book suitable

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for students as well as practicing physicists. A more serious student would need to augment this text with something closer to a traditional approach to time series analysis.

An Introduction to Particle Accelerators

E. J. N. Wilson
Oxford U. Press, New York, 2001. $90.00, $45.00 paper (252 pp.). ISBN 0-19-852054-9, ISBN 0-19-850829-8 paper

In this short, descriptive "textbook" Edmund Wilson has written what he calls *An Introduction to Particle Accelerators*. The book, he explains, sets out to remedy "the lack of a simple introduction which reveals the physical principles . . . and which best matches the needs of a graduate engineer or physicist confronting the subject for the first time." He has not written a book of that description. But that should not deter casual readers from curling up with this paperback. From it they will learn a bit about the accelerators around the world, their technologies, and the physical principles used to create them.

The three parts of Wilson's book unfold into 14 chapters that touch briefly on essentially every aspect of particle accelerators, from history to future possibilities. The extensive table of contents and the space allocated to each topic reveal the character of the book—a travelogue survey of accelerator physics, technology, and applications.

After a brief history, *An Introduction to Particle Accelerators* eases into technical discussions of the transverse focusing of particle beams. Wilson discusses longitudinal dynamics, and then returns to transverse dynamics with imperfections and nonlinearities. A special section on electron beam dynamics and synchrotron radiation is followed by a quick stop at instabilities. In chapter 10 we finally arrive at acceleration in a particle accelerator, with a detour to radio-frequency (RF) technology. The tour winds down with a discussion of applications of accelerators and future research. An introduction for graduate study should rather cover half the material at twice the depth—or all the material at twice the length.

The book is written in a folksy style; Wilson places his hand on the reader's shoulder as he gives his tour. For example, "[A] particle oscillates in this focusing system like a small sphere rolling down a slightly inclined gutter with constant speed. . . ." Along this tour we are presented with equations, graphs, and pictures that serve predominantly as decorations for the text. The physics of particle beams is not so much developed as it is stated with assurance. From time to time Wilson falls into an abbreviated development of the analysis of particle motion in an accelerator system, but he usually apologizes—for treating betatron motion, for example, "in a rather rigorous way." Some readers, Wilson explains, "might find the following sections too confusing if we carry through all the terms from the rigorous theory into a study of imperfections, . . ." Wilson takes care of the reader, leading him gently through some of the complexities of the real accelerator world—imperfections and all.

The history of particle accelerators is being written every day and many of the early practitioners are alive and well. These, living historians might have versions that differ somewhat from Wilson's. The 1958 paper by Ernest Courant and Hartland Snyder (*Annals of Physics* **3**, 1, 1958), for example, was pivotal for its development of a powerful mathematical theory that could be applied to the practical design of all of today's modern accelerators and storage rings; Wilson might have emphasized this more (actually, the entire physics community should emphasize this more). He also distorts the Nicholas Christofilos story. Christofilos did not become a colleague of Courant's until after his (Christofilos's) contributions had been acknowledged and he was hired at Brookhaven in 1953.

Wilson's constant referral to the Courant–Snyder matrix and the Courant–Snyder beta function as the Twiss parameters and Twiss matrix is an incorrect attribution that permeates the field. Some years ago Frank Cole contacted Richard Twiss, who didn't understand why the parameters were named for him.

Finally, the student of physics should be somewhat careful regarding a confusion in the book about Liouville's Theorem, which expresses the incompressibility of phase space volume in any Hamiltonian system. One gets the impression that the invariance of the emittance is a consequence of this principle alone. Actually, the invariance also requires the linearity of Hamilton's equations; it is a dynamical consequence of their solution.

An Introduction to Particle Accelerators is probably not the right book for the graduate student in engineering or physics who is planning a career in the field. However, it is an easily accessible descriptive walk through the physics and technologies of particle accelerators. As such it could be a useful read for scientists who find that their research depends heavily on one of the many different types of accelerators in use around the world.

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Statistical Mechanics of Learning

A. Engel, C. Van den Broeck
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In recent years physicists with an interest in statistical mechanics, in their search for interesting problems, have strayed increasingly far from their traditional home area of actual physical systems. One distant field in which they have had a significant impact is learning theory. *Statistical Mechanics of Learning*, by Andreas Engel of the University of Magdeburg and Chris Van den Broeck of the Limburg University Center, summarizes the results that have been achieved. The authors have themselves been in the thick of this action, and they give an exceptionally lucid account not only of what we have learned but also of how the calculations are done.

Learning theory has a long history, dominated in its development by statisticians, computer scientists, and mathematical psychologists. The field's core problem is essentially one of statistical inference. The following simple example illustrates the main points: Suppose we have a function—a rule, or input–output relation—that is implemented by some machine. (I use "machine" in a very general sense; it could be an animal or some other natural phenomenon. All that is necessary is that its output depend on its input.) We do not know in detail how the machine actually works; all we can do is observe and measure its response to some set of inputs. The general question is, then: What can we infer about the function on the basis of this example set of input–output pairs? If we try to make a machine of our own based on these examples, how well can we expect it to imitate the original machine?

Like the mathematicians before them, statistical physicists naturally focus on some simple model systems for which one can hope to calculate something nontrivial. The ones most