Adjusting the Values of the Fundamental Constants

The best values of the fundamental constants can rarely be determined by a direct measurement. Instead, they are usually found at the end of a chain of experimental observations and theoretical relationships.

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In science, comparing theoretical predictions to experimental measurements often requires knowing the value of one or more of the fundamental physical constants or conversion factors, such as the electron mass or the relation between the electron volt and the joule. For these purposes, one looks up and uses the latest values of the necessary constants in a suitable table. The source of the numbers in the table is often not given much thought, however, nor is it critical in most cases.

Further investigation into the source of the numbers reveals that they are determined by a broad range of experimental measurements and theoretical calculations involving many fields of physics and metrology. The best value of even a single constant is likely to be determined by an indirect chain of information based on seemingly unrelated phenomena. For example, the value of the mass of the electron in kilograms is based mainly on the combined information from experiments that involve classical mechanical and electromagnetic measurements, the highest precision optical laser spectroscopy, electrons in a trap, and condensed matter quantum phenomena, together with condensed matter theory and extensive calculations in quantum electrodynamics (QED). This particular chain of information will be examined in more detail below.

Two additional features of the values of the constants are not evident from a table of numbers. First, the numbers form a tightly linked set—very few of the values are independent of the others. In general, a change in a single item of the data on which the constants are based will change many of the values. And second, the numbers in the table are based only on the information available at a particular time. Therefore, the recommended values change over time, but more important, the type of information from which the values are obtained changes as well. For example, in the distant past, the charge of the electron was determined by the classic oil-drop experiment, but that method is no longer competitive. Now the electron charge is determined indirectly from other constants.

The set of constants considered here includes the elementary charge; the masses and magnetic moments of the electron, muon, proton, and neutron; the fine-structure, Planck, Rydberg, Avogadro, Josephson, and von Klitzing constants; various particle mass ratios; and many others. The basic approach to finding a self-consistent set of values of these constants is to identify the critical experiments, determine the theoretical expressions as functions of the fundamental constants that make predictions for the measured quantities, and adjust the values of the constants to achieve the best match between theory and experiment.

The idea of making a systematic study of potentially relevant experimental and theoretical information in order to produce a set of self-consistent values of the constants dates back to Raymond T. Birge, who published such a study in 1929 as the very first article in what is now the *Reviews of Modern Physics*.[1] Since then, there have been many efforts to determine the best values of the constants. In 1969, the Task Group on Fundamental Constants was established by the Committee on Data for Science and Technology (CODATA), which had been founded three years earlier by the International Council of Scientific Unions. The task group's purpose, as stated in the CODATA handbook, is "to periodically provide the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry based on all of the relevant data available at a given point in time." Based in part on the work of NIST staff, three sets of CODATA-recommended values of the constants and conversion factors have been published, one in 1973,[2] one in 1986 and 1987,[3] and the latest in 1999 and 2000.[4] The most recent set is termed the 1998 recommended values, because it is based on the information available as of 31 December 1998. A summary of this adjustment, as well as tables of values of the constants, appears in the Buyers' Guide that accompanied the August 2000 issue of Physics Today. The values of the constants are also available at http://physics.nist.gov/constants on the NIST Physics Laboratory Web site, and a searchable bibliographic database on relevant publications is available at http://physics.nist.gov/constantsbib.

Mass of the electron

One of the recurring themes in the physics behind the fundamental constants is that their values are rarely determined by a direct measurement. A basic physics question illustrates the interplay between the various constants and the indirect relations involved: What is the mass of the electron? Or stated more precisely, How can the single most precise value of the mass of the electron in kilograms be determined from the information available at the time of the 1998 adjustment?

Since the kilogram is defined in the International System of Units (SI) to be the mass of the platinum–iridium international prototype of the kilogram housed at the Bureau International des Poids et Mesures (BIPM) near

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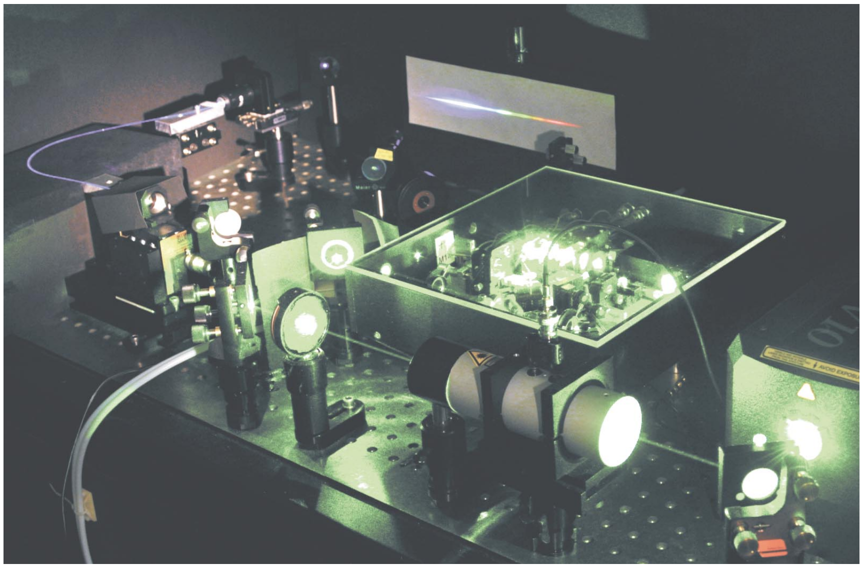


FIGURE 1. PRECISION LASER-SPECTROSCOPY EXPERIMENTS yield the most precise value for the Rydberg constant, which in turn can be used to determine the mass of the electron. This photograph shows an essential element of the current setup used by Theodor Hänsch and colleagues at the Max Planck Institute for Quantum Optics in Garching, Germany, for one such experiment, comparing the ultraviolet 1S–2S hydrogen transition frequency to the microwave frequency that defines the second. (Photo courtesy of R. Holzwarth and T. W. Hänsch.)

Paris, the mass of the electron in kilograms is just the ratio of the electron mass to the mass of that standard kilogram. This means that one end of the chain of experiments and theoretical expressions that leads to the value of the electron mass must involve a comparison to the BIPM kilogram—the only SI unit that is still based on a material object. Yet a quick survey of the available data reveals that there is no direct comparison of the electron and the kilogram, so the value of the ratio is necessarily determined through an indirect route.

The key to the electron mass is, perhaps surprisingly, the definition of the Rydberg constant from atomic spectroscopy: $R_\infty = \alpha^2 m_e c/2h$, which involves the fine-structure constant α, the mass of the electron m_e, the speed of light in vacuum c, and the Planck constant h, all of which we take to be expressed in SI units. The specification of SI units is important here, as it determines the ground rules for answering our question. If we were working with atomic mass units instead of kilograms—that is, if we had asked, What is the mass of the electron in atomic mass units?—the answer could be obtained directly from a mass ratio measurement (and, it turns out, would be much more precise). Evidently, the Rydberg definition gives the electron mass in terms of the other quantities appearing in it: $m_e = 2hR_\infty/\alpha^2 c$. We examine the best determination of each of these quantities in turn.

The speed of light is an exact quantity in the SI as a consequence of the definition of the meter adopted in 1983:

> The meter is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

Note that the speed of light is not given a fixed value directly, but rather the value is fixed as a consequence of this definition of the meter.

The next most precise constant in the expression for the electron mass is the Rydberg constant. As described in box 1, its value is determined primarily by precision laser-spectroscopy measurements on hydrogen and deuterium (figure 1). This field has developed rapidly in the past decade, with measurements achieving an extremely low level of uncertainty. This low uncertainty is possible because the optical frequencies of the transitions can be related directly to the microwave frequency in the 1967 definition of the second:

> The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

In order to interpret the spectroscopy measurements in terms of the Rydberg constant, it is also necessary to have an accurate theoretical expression for the measured transition frequencies. The necessary QED calculations have also been advancing and, when taken together with the results of the experiments, give a value for the Rydberg

Box 1. Transition Frequencies in Hydrogen and Deuterium: The Rydberg Constant

The Rydberg constant R_∞ is determined primarily by comparing theory and experiment for energy levels in hydrogen and deuterium. For example, the equation corresponding to the 1S–2S transition frequency of hydrogen is given approximately by

$$\nu_H(1S_{1/2}\text{–}2S_{1/2}) = \frac{3}{4}R_\infty c\left[1 - \frac{m_e}{m_p} + \frac{11}{48}\alpha^2 - \frac{28}{9}\frac{\alpha^3}{\pi}\ln\alpha^{-2} - \frac{14}{9}\left(\frac{\alpha R_p}{\lambda\!\!\!^{-}_C}\right)^2 + \ldots\right],$$

where m_p is the mass of the proton, R_p is the root-mean-square charge radius of the proton, and $\lambda\!\!\!^{-}_C$ is the Compton wavelength of the electron divided by 2π. The measured value for this transition used in the 1998 adjustment is[6]

$$\nu_H(1S_{1/2}\text{–}2S_{1/2}) = 2\,466\,061\,413\,187.34(84)\ \text{kHz},$$

which has a relative uncertainty of 3.4×10^{-13}.

The above theoretical expression is approximate and indicates only the leading term of each of four corrections. In particular, the four terms beyond the "1" on the right-hand side correspond to contributions from reduced mass, relativistic, radiative, and finite proton size effects, respectively. But as its general form shows, the equation still gives information on the value of the Rydberg constant.

In the 1998 adjustment, 23 transition frequencies or frequency differences in hydrogen or deuterium were included, and the theoretical expressions for the energy levels used in the adjustment were based on many analytic calculations and precise numerical evaluations. The result for the 1998 recommended value for the Rydberg constant is

$$R_\infty = 10\,973\,731.568\,549(83)\ \text{m}^{-1}.$$

Box 2. Anomalous Magnetic Moment of the Electron: The Fine-Structure Constant

The g-factor of the electron, which characterizes the coupling of the electron's spin to a magnetic field, is not equal to the value predicted by the Dirac equation $g_e(\text{Dirac}) = -2$. The deviation from that value is given in terms of the electron magnetic moment anomaly a_e by

$$g_e = -2(1 + a_e).$$

The theoretical expression for a_e can be written as

$$a_e = C_e^{(2)}\left(\frac{\alpha}{\pi}\right) + C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + a_e(\text{had}) + a_e(\text{weak}) + \delta_e\,,$$

where $C_e^{(2)} = 1/2$, the $C_e^{(2i)}$ with $i \geq 2$ are numerical constants, obtained from extensive QED calculations, $a_e(\text{had})$ is a predominantly hadronic vacuum polarization contribution, $a_e(\text{weak})$ is a predominantly electroweak contribution, and δ_e is an additive correction that takes into account the theoretical uncertainty, estimated to be 1.1×10^{-12}.

The anomaly has been measured for the electron and positron,[7] and, assuming CPT invariance holds, we take a weighted average of those values to obtain

$$a_e = 1\,159\,652\,188.3(4.2) \times 10^{-12}.$$

The electron anomalous magnetic moment data provide the most influential information on the value of the fine-structure constant α. Of course, in the final adjustment, all sources of information on α contribute to the 1998 recommended value:

$$\alpha^{-1} = 137.035\,999\,76(50).$$

constant with a relative uncertainty of 7.6×10^{-12}.

The fine-structure constant in the expression for the electron mass is most precisely determined by comparing theory and experiment for the anomalous magnetic moment of the electron, as discussed in box 2. Small corrections to the Dirac-equation value of the magnetic moment of the electron are predicted by QED as a series in powers of the fine-structure constant (figure 2). The electron magnetic moment anomaly has been measured to high accuracy in experiments with electrons in a Penning trap. By equating the measured value to the theoretical expression, one obtains a value for the fine-structure constant that has a relative uncertainty of 3.8×10^{-9}.

The remaining constant in the electron mass equation is the Planck constant. The best value of this basic constant of quantum physics is determined by a watt-balance experiment that compares a watt of electrical power to a watt of mechanical power (box 3). A remarkable aspect of this experiment is that the Planck constant, which is the characteristic unit of quantum phenomena, is measured by a two-story-high apparatus, shown in figure 3, that is described by classical mechanics and classical electromagnetic theory. The Planck constant enters through the current and voltage calibrations in the electrical power determination, because they are based on two condensed-matter quantum phenomena, the Josephson effect and the quantum Hall effect. Together with the condensed matter theory of these effects, the calibrations give the electrical power in terms of the Planck constant and accurately known frequencies. On the mechanical side, a standard of mass is used that is ultimately calibrated in terms of the BIPM kilogram. The watt-balance experiment determines the relation of the Planck constant to the kilogram, based on condensed matter physics and mechanical measurements, with a relative uncertainty of 8.7×10^{-8}.

With values for the constants obtained as described, the best value for the electron mass in kilograms follows from its simple equation. The path is indirect and involves a wide range of physics, as indicated in figure 4. While this path gives the most precise value, it is not the only possible path. For example, the fine-structure constant can also be determined from experiments that measure the von Klitzing constant ($R_K = h/e^2 = \mu_0 c/2\alpha$, where μ_0 is exactly defined in the SI) in terms of the impedance of a capacitor whose capacitance in SI units can be calculated. The Planck constant, in combination with the fine-structure constant, can also be obtained from experiments that determine the Josephson constant ($K_J = 2e/h = [8\alpha/\mu_0 ch]^{1/2}$) in terms of a voltage calibrated by a mechanical force. In one such experiment, a voltage was applied between a horizontal electrode plate and a pool of mercury below it, and the attractive force that lifts the mercury was determined. The connection to the kilogram is through an independent measurement of the density of mercury.

The 1998 recommended value for the electron mass, based on all the available information, is $m_e = 9.109\,381\,88(72) \times 10^{-31}$ kg. (The number in parentheses is the one-standard-deviation uncertainty in the last two digits of the value.)

The high precision of the watt-balance apparatus suggests a possible new definition of the kilogram. This last

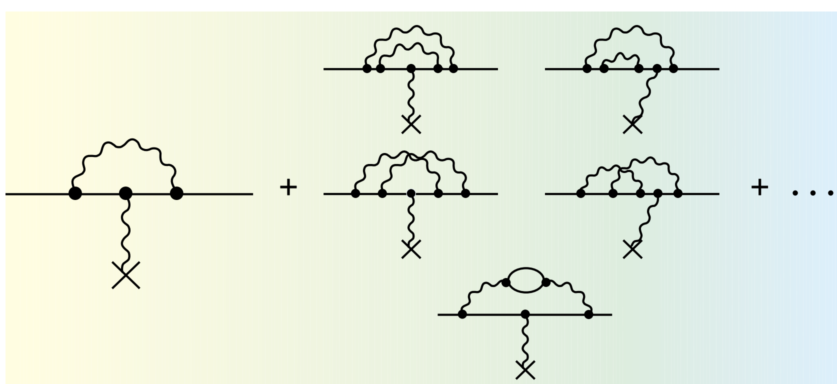


FIGURE 2. QUANTUM ELECTRODYNAMICS (QED) plays an increasingly important role in relating experimental results to fundamental physical constants. Shown here are the Feynman diagrams for the lowest-order contribution to the anomalous magnetic moment of the electron (left), first calculated by Julian Schwinger in 1948, and for the next-lowest-order contributions (right). In the diagrams, the straight lines represent the spacetime trajectory of the electron, the wavy lines represent photons that propagate the electromagnetic interaction, and the × represents the external magnetic field that interacts with the electron's magnetic moment. Comparison of theoretical calculations to the actual measurement of the electron's anomalous magnetic moment yields a precise value for the fine-structure constant α.

remaining material definition of an SI unit could be eliminated by redefining the kilogram[5] to be the mass corresponding to a specified frequency ν_k, according to the Einstein and Planck relations $E = mc^2$ and $E = h\nu$. With such a definition, we would have $h = (1\ \text{kg})\ c^2/\nu_k$, which expresses the Planck constant as an exact quantity. This exact value for the Planck constant would be the consequence of the definition of the kilogram, just as the exact value of the speed of light is a consequence of the definition of the meter. Thus, with the Planck constant exactly defined, the watt-balance apparatus would be a precision scale that could measure the mass of objects in terms of the newly defined kilogram. Also, with this definition, the mass of the electron in kilograms would have about an order-of-magnitude smaller uncertainty.

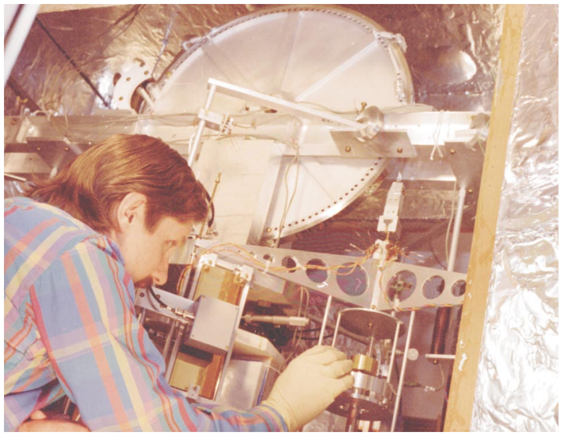


FIGURE 3. WATT-BALANCE EXPERIMENTS, which compare mechanical power to electrical power, produce the best value for the Planck constant h. In NIST's two-story-tall watt balance, the gravitational force on a mass standard is balanced by the magnetic force on a current-carrying coil in one phase of the experiment. **(a)** Richard Steiner positions the mass standard at the top of the watt balance. **(b)** Edwin Williams checks the superconducting magnet used to generate the radial magnetic field at the lower level of the watt-balance experiment.

Adjusting the constants

The example of the electron mass illustrates how the information that leads to the values of the constants can be indirect and how different paths provide redundant constraints on their values. To obtain the best values, it is necessary to take all of this information into account simultaneously; to do this in a consistent manner, a least-squares approach is used to determine the values of the constants.

In the approach of the 1998 adjustment, the information is divided into three categories: input data, observational equations, and adjusted constants. The input data are results of measurements that provide the best constraints on the values of the constants. The observational equations are theoretical expressions that give values of the quantities in the input data category as functions of the adjusted constants. The adjusted constants are a suitably chosen set of fundamental constants that are determined by the adjustment. The adjustment's role is to find the values that best reproduce the input data by means of the theoretical expressions.

Examples of input data are the measured value of the anomalous magnetic moment of the electron, measured values of transition frequencies in hydrogen, ratios of magnetic moments of various particles, ratios of masses of various atoms, neutron diffraction data, and silicon lattice-spacing data. The guiding principle of gathering input data is that the values represent actual measured quantities. In particular, this means not using data that has been analyzed with information based on old values of the fundamental constants. Since these values will be updated by the adjustment, they must be removed from the analysis so that, in effect, the final values of the constants are used instead. For example, if an experiment reports the value of an x-ray wavelength based on Bragg diffraction by a silicon crystal, the input datum is not taken

Box 3. Watt-Balance Experiment: The Planck Constant

The Planck constant h can be measured by comparing a watt of mechanical power expressed in terms of the meter, kilogram, and second to a watt of electrical power expressed in terms of the Josephson constant $K_J = 2e/h$ (which relates frequency and voltage through the Josephson effect) and the von Klitzing constant $R_K = h/e^2$ (which has units of resistance and originates in the integer quantum Hall effect) in the combination

$$K_J^2 R_K = \frac{4}{h}.$$

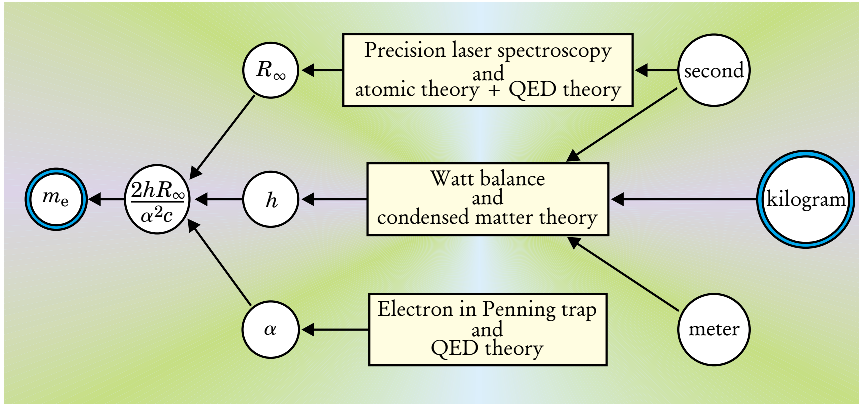
The apparatus that makes the comparison is called a watt balance.[8] The basic principle of the watt balance is illustrated by one of its implementations.[9] A horizontal coil of wire is suspended in a radial magnetic field. The current in the coil needed to support the weight of a mass standard is measured in one phase of the experiment. In the second phase, the strength of the field is determined by slowly moving the coil vertically and measuring the induced voltage. The current I and mass m in the first phase and the velocity v and induced voltage U in the second phase are related by

$$mgv = IU = Af_1f_2h,$$

where g is the local acceleration of free fall, which is accurately measured with an absolute gravimeter. Since the voltage U and the voltage and resistance that determine I are calibrated in terms of the Josephson and von Klitzing constants, the constant A is, in principle, exactly known, and f_1 and f_2 are the accurately known frequencies applied to the Josephson junctions in the two phases of the experiment. This equation gives h in terms of quantities directly measured in the experiment. The 1998 recommended value of the Planck constant, which is determined primarily by the watt-balance experiment, is

$$h = 6.626\,068\,76(52) \times 10^{-34}\ \text{J s}.$$

FIGURE 4. DETERMINATION of the most accurate value of the mass of the electron in kilograms relies on experiments and theories from varied fields of physics.



to be the reported wavelength, but rather the ratio of the x-ray wavelength to the silicon crystal's lattice spacing, which is the actual measured quantity. The lattice spacing is one of the constants in the adjustment, and its value will be optimized based on all the information that influences it, including the wavelength–lattice-spacing ratio.

Observational equations can range from a completely trivial statement that a quantity equals itself to a complex set of formulas that encompass decades of work on detailed QED calculations. In the former case, if a constant is measured directly, then the measured value is formally compared to the corresponding adjusted constant. At the other extreme, the transition frequencies in hydrogen and deuterium or the ground-state hyperfine splitting of muonium are described by observational equations that are quite involved and include the results of numerous contributions, from the early days of quantum mechanics to very recent advances.

Built into the selection of the observational equations are assumptions about what constitutes the correct theory on which to base the values of the fundamental constants. The established theory is taken to be ordinary quantum mechanics and its successive generalizations—QED, electroweak theory, and the Standard Model of particle physics—as well as the basic equations of the Josephson effect and the integer quantum Hall effect from condensed matter theory. Not all of this theory is beyond question at the level of accuracy needed, but it is necessary to make some assumptions in order to assign values to the constants. Since there is redundancy in the evaluation of the constants, the consistency of values derived from different theoretical relations is a check on the consistency of the theory, and there is no convincing evidence from the 1998 adjustment that any theoretical assumptions should be discarded.

The fundamental constants whose values are adjusted to fit the data form a relatively small subset of all of the 1998 CODATA recommended values. The adjusted set includes the Rydberg constant, the fine-structure constant, the Planck constant, various particle masses in atomic mass units, lattice spacings of various specific silicon crystal samples, and the molar gas constant. Other fundamental constants, such as the electron mass in kilograms and energy conversion factors, are derived from the subset based on exact theoretical relations. There is no unique choice for the constants that are included in the directly adjusted set, but they must form an independent set—no member of the subset may be related to others by a theoretical identity. Covariances between the values of the adjusted constants are taken into account in calculating the values of the derived constants, so the adjusted constants are not necessarily more precise than the derived constants.

The 1998 adjustment

The latest adjustment of the fundamental constants provides a new set of recommended values. The uncertainties of the new values are in most cases about 1/5 to 1/12, and in some cases 1/160, of the uncertainties of the corresponding previously recommended values. However, as box 4 describes, the new recommended value of the Newtonian constant of gravitation has a larger uncertainty than the earlier value.

A striking aspect of the latest adjustment is the extent to which the recommended values depend on QED theory. This increase is coupled to the increase in precision in the constants, as both experiments and theory related to QED

Box 4. Gravitational Attraction Experiments: The Newtonian Constant of Gravitation

One of the oldest fundamental constants is the Newtonian constant of gravitation G, which gives the strength of the gravitational force of attraction between any two objects according to the familiar formula

$$F = G\frac{m_1 m_2}{r^2},$$

where m_1 and m_2 are the masses of the two objects and r is the distance between them (assumed to be large compared to their extent). In effect measured by Henry Cavendish in 1798, this constant has shown resistance to improvement over the years. In fact, it is the only constant whose recommended value in the 1998 adjustment has a larger uncertainty than its recommended value in the prior 1986 adjustment. The reason for the increased uncertainty is that after the 1986 value was recommended, a new, highly credible experiment reported a value for G that disagreed significantly with the recommended value.[10] Furthermore, a small, but previously unknown, anharmonicity was found in the suspension of torsion balances, such as the one used in the experiment on which the 1986 value was based. These facts suggested that the gravity experiments were not understood as well as was believed. Thus, the 1986 value was retained as the 1998 recommended value, but its uncertainty was increased by about a factor of 12 to recognize these issues and to alert users to the problem. As a result of these considerations, the 1998 recommended value is

$$G = 6.673(10) \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}.$$

Recently, a precise result of a new experiment that is in relatively good agreement (within two standard deviations) with the 1986 recommended value has been reported[11] (see PHYSICS TODAY, July 2000, page 21).

improve. The relation of resistance calibration standards to the theoretical calculation of higher-order Feynman diagrams illustrates the broad impact of QED theory. Over the past decade, most standards laboratories, including NIST, have shifted to resistance standards based on the quantum Hall effect rather than on banks of standard resistors. According to the theory of the integer quantum Hall effect, the associated von Klitzing constant is a simple and exact function of the fine-structure constant. As described in box 2, the value of the fine-structure constant is most strongly determined by comparison of theory and experiment for the electron anomalous magnetic moment, and the theoretical expression, in turn, is determined by high-order QED calculations. In addition to this influence on standards for the ohm, if the kilogram were defined so that the Planck constant was an exact constant, then the Josephson constant would also depend on exact constants together with the fine-structure constant. As a result, high-accuracy voltage standards, which are now usually based on the Josephson effect, would also be limited only by QED experiment and theory.

The remarkable reduction in uncertainty that has been possible for the fundamental constants over the past decade is accompanied by a liability. Many of the values in the 1998 adjustment rely predominantly on a single item of data either from experiment or theory, or both as in the case of the fine-structure constant. As a result, one or more such items of data may possibly be found to be in error by subsequent investigations, in which case the values of the constants would change. We view this as a necessary risk, since the alternative would be to enlarge the uncertainties of the values of the constants to be "certain" that they are correct. This alternative would provide values that are considerably less precise and still not necessarily correct. More important, such an approach would not make full use of the information provided by the most advanced and accurate experimental and theoretical results.

In view of the steady progress in the field of fundamental constants, the values are bound to become outdated soon after they are recommended and published. Indeed, new information relevant to the values of the constants has already become available since the 1998 adjustment. To keep the values up-to-date to the extent possible, CODATA expects to provide new recommended values at four-year intervals in the future.

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