Spin and Isospin: Exotic Order in Quantum Hall Ferromagnets

Spin and a peculiar kind of isospin in two-dimensional electron gases can exhibit novel counterintuitive ordering phenomena.

Steven M. Girvin

Quantum mechanics is a strange business, and the quantum physics of strongly correlated many-electron systems can be stranger still. Good examples are the various quantum Hall effects.[1–4] They are among the most remarkable many-body quantum phenomena discovered in the second half of the 20th century, comparable in intellectual import to superconductivity and superfluidity. The quantum Hall effects are an extremely rich set of phenomena with deep and truly fundamental theoretical implications.

The fractional quantum Hall effect has yielded fractional charge, with its attendant spin–statistics peculiarities, as well as phases with unprecedented order parameters. It has beautiful connections to a variety of different topological and conformal field theories more commonly studied as formal models in particle theory. But in the quantum Hall context, each of these theoretical constructs can be made manifest by the twist of an experimental knob. Where else but in condensed-matter physics can an experimenter change the number of flavors of relativistic chiral fermions in a sample, or create a system whose low energy description is a Chern–Simons gauge theory whose fundamental coupling constant (the θ angle) can be set by hand?

The first quantum Hall effect was discovered by Klaus von Klitzing 20 years ago, for which he won the 1985 Nobel Prize in physics. (See PHYSICS TODAY, December 1985, page 17.) Because of recent tremendous technological progress in molecular-beam epitaxy and the fabrication of artificial structures, quantum Hall experimentation continues to bring us striking new discoveries. The early experiments were limited to simple transport measurements that determined energy gaps for charged excitations. Recent advances, however, have given us many new probes—optical, acoustic, microwave, specific heat, tunneling spectroscopy, and NMR—that continue to pose intriguing new puzzles even as they advance our knowledge.

Quantum Hall phenomena

The quantum Hall effect takes place in a two-dimensional electron gas formed in an artificial semiconductor quantum well and subjected to a high magnetic field normal to the plane. In essence, this macroscopic quantum effect is a result of commensuration between the number of electrons N and the number of flux quanta N_Φ in the applied magnetic field. That is to say, the electron population undergoes a series of condensations into new states with highly non-trivial properties whenever the filling factor $\nu \equiv N/N_\Phi$ is an integer or a simple rational fraction.

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Von Klitzing's original observation was, in effect, a sequence of energy gaps yielding (in the limit of zero temperature) electron transport without dissipation—much like a superconductor, but with radically different underlying physics.

The Hall conductivity σ_{xy} in this dissipationless state turns out to be universal. It is given by $\nu e^2/h$ with great precision, irrespective of microscopic or macroscopic details. Therefore, one can exploit this remarkable phenomenon to make a very precise determination of the fine-structure constant and to realize a highly reproducible quantum-mechanical unit of electrical resistance. The quantum Hall effect is now used by standards laboratories around the world to maintain the ohm.

It is an amusing paradox that this ideal behavior occurs only in imperfect samples. That's because disorder produces Anderson localization of quasiparticles, preventing them from contributing to the transport properties. If the laboratory samples were ideal, the effect would go away!

The *integer* quantum Hall effect is due to an excitation gap associated with the filling of discrete kinetic-energy levels (Landau levels) of electrons executing quantized cyclotron orbits in the imposed magnetic field (see figure 1). Coulomb interactions between electrons would seem to be unimportant. When ν is an integer, the chemical potential lies in one of these kinetic energy gaps. The *fractional* quantum Hall effect occurs when one of the Landau levels is fractionally filled. Its physical origins—very different from those of the integer effect—are strong Coulomb correlations that produce a Mott-insulator-like excitation gap.

In some ways, this excitation gap is more like that in a superconductor, because it is not tied to a periodic lattice potential. That permits uniform charge flow of the incompressible electron liquid and hence a quantization of Hall conductivity. The electrons are strongly correlated because all the states in a given Landau level are completely degenerate in kinetic energy. Perturbation theory is therefore useless. But the novel correlation properties of this incompressible electron liquid are captured in a revolutionary wave function proposed by Robert Laughlin, for which he shared the 1998 Nobel Prize in physics with Horst Stormer and Daniel Tsui, who discovered the fractional quantum Hall effect in 1982. (See PHYSICS TODAY, December 1998, page 17.)

Quantum Hall ferromagnetism

At $\nu = 1$ and certain other filling factors, quantum Hall systems exhibit spontaneous magnetic order. This constitutes a very peculiar kind of ferromagnetism: It is itinerant—the electrons are free to move around as in metals

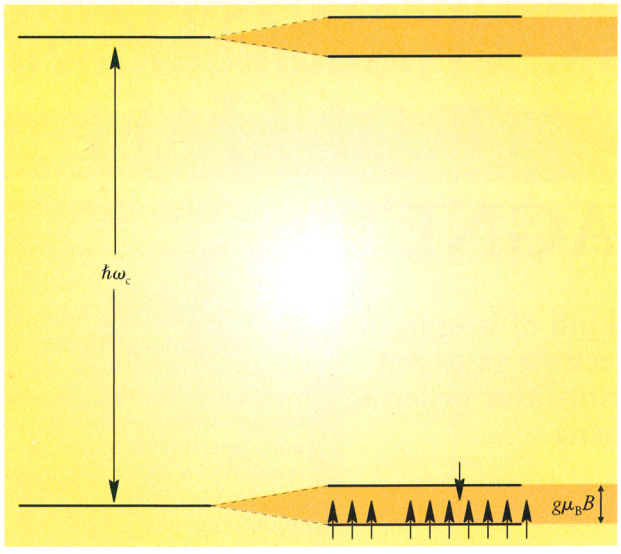


FIGURE 1. LANDAU LEVELS of uniformly spaced kinetic energy of a two-dimensional electron gas in a quantizing magnetic field B whose cyclotron frequency is ω_c. In free space, the Zeeman splitting $g\mu_B B$ equals the Landau level splitting, but in GaAs heterostructures it is nearly two orders of magnitude smaller. At sufficiently low temperature, most of the electron spin orientations are in the lower Zeeman level.

like iron—and yet it exhibits a charge excitation gap that manifests itself by precisely quantized Hall conductivity and the vanishing of the ordinary, dissipative longitudinal conductivity σ_{xx}.

My colleague Allan MacDonald refers to the $\nu = 1$ state as "the world's best understood ferromagnet." The lowest spin state of the lowest Landau level is completely filled and the exact ground state (neglecting small effects from Landau-level mixing) is very simple: It is a single Slater determinant precisely represented by Laughlin's wave function. (See the article by Jainendra Jain in PHYSICS TODAY, April 2000, page 39.) Unlike iron, this ferromagnet is 100% polarized, because the kinetic energy has been frozen into discrete Landau levels and polarizing the electron gas costs no kinetic energy.

For reasons peculiar to the electronic band structure of GaAs, the usual host semiconductor, the external magnetic field couples very strongly to the orbital motion (giving a large Landau level splitting) and very weakly to the spin degrees of freedom (giving an exceptionally small Zeeman gap, as shown in figure 1). Therefore, the spin orientation is not frozen in place, as one might naively expect. The low-energy spin degrees of freedom of this unusual ferromagnet have some rather novel properties that have recently been probed by specific-heat measurements, NMR, and other means.

The simplest excitations out of the ground state are spin waves (magnons), in which the spin orientation undergoes smooth fluctuations in space and time. Because of the unusual circumstance that the ground-state wavefunction is a single, known Slater determinant, the single-magnon excited-state spectrum can also be computed exactly (see figure 2.) One can then use various approximate techniques to predict rather accurately the temperature dependence of the magnetization.[5–7]

One of the interesting features of the physics here is that two dimensions is the lowest dimensionality for which ordering is possible in magnets with Heisenberg (SU_2) symmetry. That is to say, the phase space for spin-wave excitations in two dimensions is large enough so that there is an infrared divergence in the number of excited magnons at any finite temperature. Hence the magnetization, which is 100% at zero temperature, crashes immediately to zero at any finite temperature. In the presence of a small Zeeman coupling, the magnetization begins to drop towards zero (as shown in figure 2b) at a temperature of a few K, characteristic of the Zeeman gap and the spin stiffness.

At filling factor $\nu = 1$, spin waves are the lowest energy excitations. But because they do not carry charge, they do not have a large impact on the electrical transport properties. Since the lowest spin state of the lowest Landau level is completely filled at $\nu = 1$, the Pauli exclusion principle tells us that we can add more charge, as illustrated in figure 1, only with reversed spin. In the absence of strong Coulomb interactions, the energy cost of this spin flip is simply the Zeeman energy, which is very small. So one might not expect to see a quantized Hall plateau near $\nu = 1$, because there would be a high density of thermally excited charges. However, the Coulomb interaction exacts a large exchange-energy penalty for having a reversed spin in a ferromagnetic state.[2,7] Thus magnetic order induced by Coulomb interactions turns out to be essential to the integer quantum Hall effect.

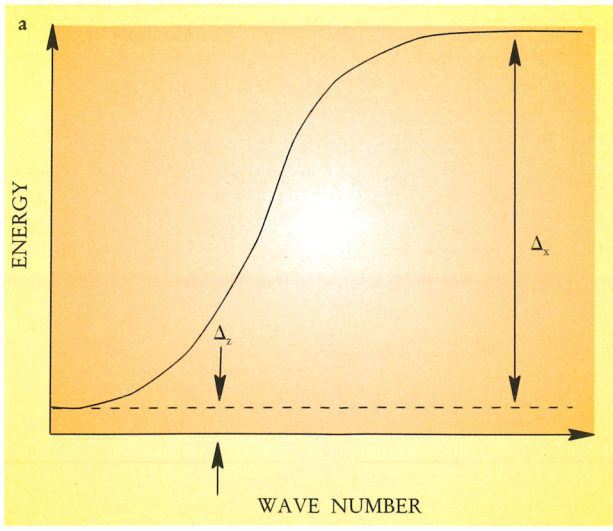
Skyrmions

In 1993, Shivaji Sondhi and collaborators[8] made a notable discovery: Because the exchange energy is large and prefers locally parallel spins, the Zeeman energy being small, it is energetically cheaper to form a topogical spin texture by partially turning over some of the spins. (See the box on page 42.) Such a topological object is called a skyrmion, because of its provenance in the Skyrme model of nuclear physics. Since the system is an itinerant magnet with a quantized Hall conductivity, it turns out that the skyrmion texture accommodates precisely one extra unit of charge. NMR shifts and various optical and transport measurements have confirmed the prediction that each charge added to or removed from the state flips over a handful of spins. (See figure 3.)

In nuclear physics, the Skyrme model imagines the universe in a kind of ferromagnetic state, with a magnetization that is a four-component vector. Thus there are three directions in spin space for fluctuations around the (broken-symmetry) magnetization direction. So one has three different spin waves, representing the three light mesons π^+, π^-, and π^0. The nucleons (the protons, the neutron, and their antiparticles) are taken to be topological defects in this magnetization field. Through the magic of Berry-phase terms in the Lagrangian, these objects are fermions, even though they are excitations of a bosonic order-parameter field.

Essentially the same phenomenon occurs in quantum Hall ferromagnets, the only difference being that the spin waves have a non-relativistic (quadratic) dispersion relation, and the "nucleons" come in only one flavor: the electron and its antiparticle, the hole. Because the quantum Hall ferromagnetic order parameter is a three-component vector, there are only two directions in spin space for fluctuations around the broken-symmetry direction. One might think that this implies that there are two spin wave modes. But, in the nonrelativistic case, it turns out that the two coordinates are canonically conjugate and there is, in fact, only a single ferromagnetic spin wave.

Because it costs significant energy (about 30 K) to



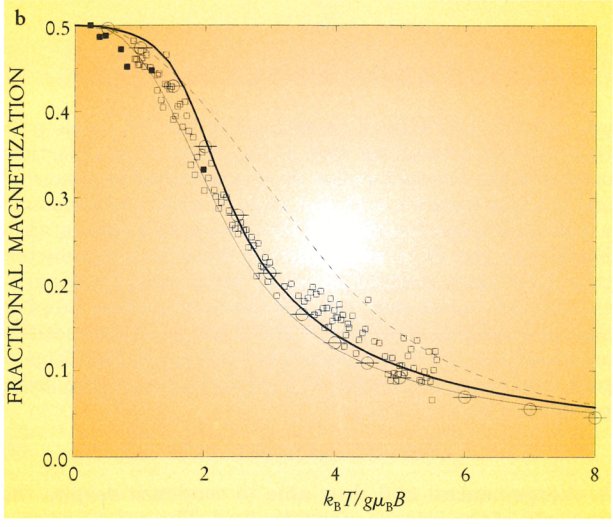


FIGURE 2. SPIN WAVE EXCITATIONS (magnons) from the quantum Hall ground state. **(a)** Dispersion relation for single magnons. At low frequency, the energy grows quadratically with wave number, starting from the Zeeman gap $\Delta_Z \sim 1K$, and saturates at the Coulomb exchange energy $\Delta_X \sim 100K$. **(b)** Temperature dependence of the magnetization at filling factor $\nu = 1$. Temperature is normalized to the Zeeman gap. Squares indicate experimental data.[5] Open circles and curves indicate various theoretical calculations.[6,7]

create a skyrmion or anti-skyrmion, they freeze out and disappear at low temperatures at $\nu = 1$. However, as one moves away from this filling factor, the cheapest way to add or subtract charge is through the formation of a finite density of skyrmions (proportional to $|\nu - 1|$). Thus, away from $\nu = 1$, skyrmions do not freeze out, even at zero temperature. One might ask why skyrmions are not important in ordinary thin-film magnets. Skyrmions can exist there, in principle. But they always freeze out at low temperatures, because they do not carry charge and their density can not be controlled by varying the chemical potential.

Normally we think of manipulating spins by applying magnetic fields. A notable feature of quantum Hall ferromagnets is that, because skyrmions carry charge, one can move spins around by applying electrostatic potentials. For example, a random disorder potential can nucleate skyrmions.

In the presence of skyrmions, the ferromagnetic order is no longer colinear. The skyrmion configuration shown in the box on page 42 is only one of a continuous family of minimum-energy solutions. There exist two "zero modes," corresponding to translation of the skyrmion in real space and uniform rotation in spin space about the axis defined by the Zeeman field. In the presence of many skyrmions, these additional degrees of freedom lead to two totally new classes of low-energy collective excitations—"Goldstone modes" associated with the broken spin rotational and translational symmetry. Unlike ordinary spin waves, these Goldstone modes are not constrained by Larmor's theorem to have a minimum excitation gap given by the Zeeman energy. Indeed at long wavelengths, these excitations can go all the way down to zero frequency. That's because, in semiclassical terms, rotations about the Zeeman axis do not cost any Zeeman energy. In an ordinary ferromagnet, the ground state is invariant under rotations about the Zeeman axis. So the rotation produces no excitation. In a non-colinear system, however, states produced by different rotations are distinguishable from each other. Thus each skyrmion induces a new xy quantum-rotor degree of freedom.[10]

These low-frequency xy spin fluctuations have been indirectly observed through a dramatic enhancement of the nuclear spin-relaxation rate $1/t_1$. Because nuclei precess at frequencies some three orders of magnitude below that of the Zeeman gap, they do not couple effectively to ordinary spin waves in the electron system. So the nuclear relaxation time t_1 can become many minutes, or even hours, at low temperature. But in the presence of skyrmions, t_1 becomes so short ($\sim$ 20 s) that the nuclei come into thermal equilibrium with the lattice through interactions with the electrons in the quantum well. This effect has recently been observed experimentally by Vincent Bayot, Mansour Shayegan and collaborators as a specific-heat enhancement of more than 5 orders of magnitude, due to the entropy of the nuclei[11] (see figure 4).

Isospin Ordering in Bilayer Systems

Ordinary spin is not the only internal degree of freedom that can spontaneously become ordered. It is now possible to make a pair of identical electron gases in quantum wells separated by a distance (~10 nm) comparable to the electron spacing within a single quantum well. Under these conditions, one can expect strong interlayer correlations and new types of ordering phenomena associated with the layer degree of freedom.[12] The many-body physics of two-layer systems can also be found in wide single-well systems with the two (nearly degenerate) lowest quantum-well subband states playing the role of a pseudospin degree of freedom.[13]

One of the peculiarities of quantum mechanics is that, even in the absence of tunneling between the layers, the electrons can be in a coherent state in which their layer index is uncertain. To understand the implications of this, we can define a pseudospin, which we also call "isospin," after the abstract spin Heisenberg introduced to distinguish neutrons from protons. In our case, the isospin is up if the electron is in the first layer and down if it is in the second. Spontaneous interlayer coherence corresponds to pseudospin magnetization lying in the xy plane, corresponding to a coherent mixture of pseudospin up and down.

If the total filling factor for the two layers is $\nu = 1$, the

Skyrmions and Topological Quantum Numbers

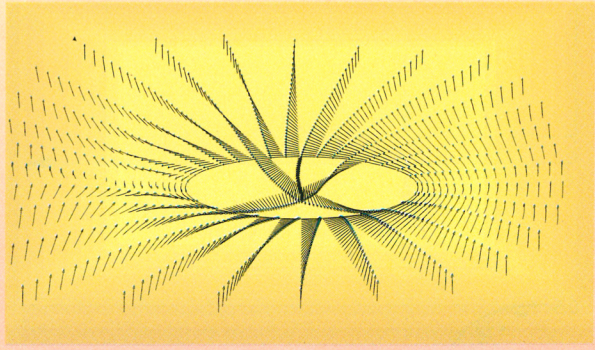
In this illustration of skyrmion spin texture in a quantum Hall ferromagnet, note that the spins are all up at infinity but down at the origin. At intermediate distances, they have a vortex-like configuration. Because of the quantized Hall conductivity, skyrmions carry extra charge. Although this extra charge is distributed throughout the core region, its total value is quantized. In fact, the skyrmion charge is directly proportional to the "topological charge" of the magnetization order-parameter field $\mathbf{m}(\mathbf{r})$, and is given by the remarkable formula

$$Q = \frac{h}{e}\sigma_{xy}\int d^2r \frac{1}{8\pi}\varepsilon^{\mu\lambda}\varepsilon_{abc}m^a\partial_\mu m^b\partial_\lambda m^c.$$

where σ_{xy} is the Hall conductivity. The epsilons are the fully antisymmetric tensors of second and third rank.

The physics behind this equation is the following: An electron traveling through a region will have its spin aligned with the local magnetization direction by the exchange field. Thus its spin direction will vary as the electron moves through the spin texture. If the spin direction is twisting in two directions at once (as required by the two spatial derivatives in the equation), the electron acquires a path-dependent Berry phase, much as if it were traveling through some additional magnetic flux. Adding flux draws in or expels charge proportional to the amount of this flux.

This same picture was used by Laughlin to derive the fractional charge of the quasiparticles in the case where the Hall conductivity σ_{xy} is described by a fractional quantum number. At filling factor $\nu = 1$, the Hall conductivity $\sigma_{xy} = e^2/h$ and the skyrmion binds exactly one extra electron (or hole). Therefore it must be a fermion.



Coulomb exchange energy will strongly favor this magnetic order, just as it does for real spins. That's because the spatial part of the fermionic wavefunction must vanish if two electrons with the same pseudospin orientation approach each other. (In contrast to the previous sections, we assume here that the real spins have been frozen into a ferromagnetic state and can be ignored.)

For real spins, the Coulomb interaction is spin invariant. For pseudospins, we must take into account the fact that intralayer repulsion is slightly stronger than interlayer repulsion. If the pseudospin were to become ordered in the z direction, all of the electrons would be in one single layer, resulting in a large capacitive charging energy. That would lead to an "easy plane" anisotropy in which the pseudospin ferromagnetic order prefers to lie in the xy plane.

When the charging energy is not severe, a good approximation to the xy ordered state is

$$|\Psi\rangle = \prod_k (c^\dagger_{k\uparrow} + e^{i\varphi}c^\dagger_{k\downarrow})|0\rangle, \qquad (1)$$

where each $c^\dagger$ is the creation operator (acting on the vacuum state $|0\rangle$) for a given pseudospin in the kth single-particle spatial orbital. In this state, every single-particle orbital in the lowest Landau level is occupied by precisely one electron (hence $\nu = 1$). But each of these electrons is in a coherent superposition of the two pseudospin states. Much like the BCS wavefunction for a superconductor, this state has a definite phase φ, but an indefinite particle number. In our case, it is not the total particle number that is indefinite, but rather the particle-number difference between the two layers.[14] In contrast to the Cooper-pair field order parameter of a superconductor, the order parameter here

$$\Psi(\mathbf{r}) \equiv \langle\psi^\dagger_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\rangle \sim e^{i\varphi(\mathbf{r})} \qquad (2)$$

is charge-neutral and thus able to condense despite the presence of the intense magnetic field. The order parameter at each point $\mathbf{r}$ is the expectation value of the spin-raising operator at that point. Because each electron is in a coherent superposition of states in different layers, one can destroy an electron in one layer and create an electron in the other, without leaving the ground state. In a certain sense, the coherent state is like an excitonic insulator with a particle and hole bound together—with the important difference that we do not know which layer each is in. This neutral object can travel through the magnetic field without suffering a classical Lorentz force or any Aharanov–Bohm phase shift.

In the absence of tunneling between the layers, the electrons have no way of determining the phase angle φ. Therefore, the energy must be independent of its global

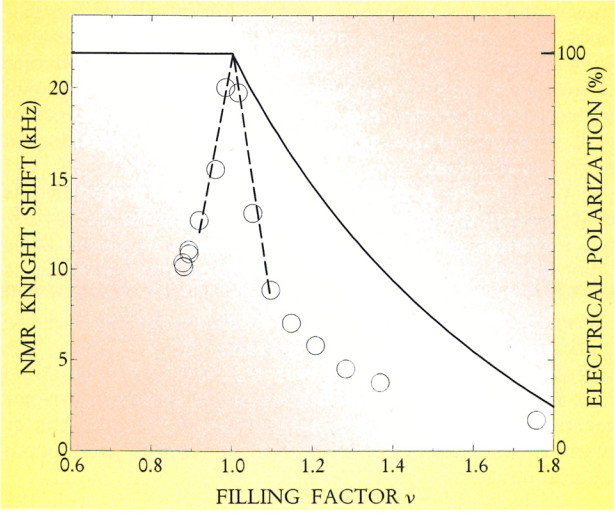


FIGURE 3. MEASURED NMR SHIFT yields electron spin polarization as a function of filling factor near $\nu = 1$. This "Knight shift" is the change in nuclear precession frequency due to hyperfine coupling to the electron spin density. Circles are data from ref. 9. The steep fall-off on both sides of the 100% polarization peak at $\nu = 1$ indicates that typically 4 spins flip over for each charge added (or subtracted). The observed symmetry around the peak is due to the particle–hole symmetry between skyrmions and antiskyrmions. By contrast, the solid line is the prediction for non-interacting electrons.

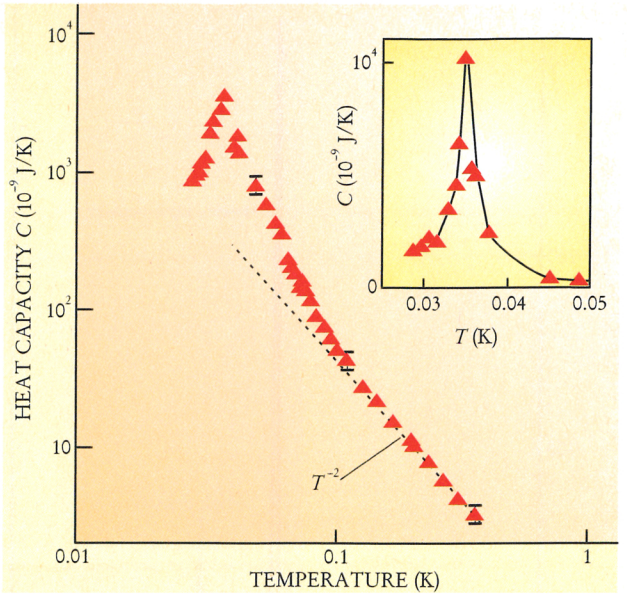


FIGURE 4. SPECIFIC HEAT is greatly enhanced by the presence of skyrmions. They dramatically shorten the nuclear spin-lattice relaxation time, thus bringing the nuclei into thermal equilibrium. Dashed line is a calculation of the contribution of a model that assumes all nuclei in the quantum well contribute to the specific heat. At low temperatures, nuclei in the insulating barriers just outside the well raise the specific heat beyond this prediction.[11] The sharpness of this additional peak (inset linear plot) is not well understood.

value. The exchange energy can, however, depend on spatial gradients of φ. The leading term in a gradient expansion is therefore

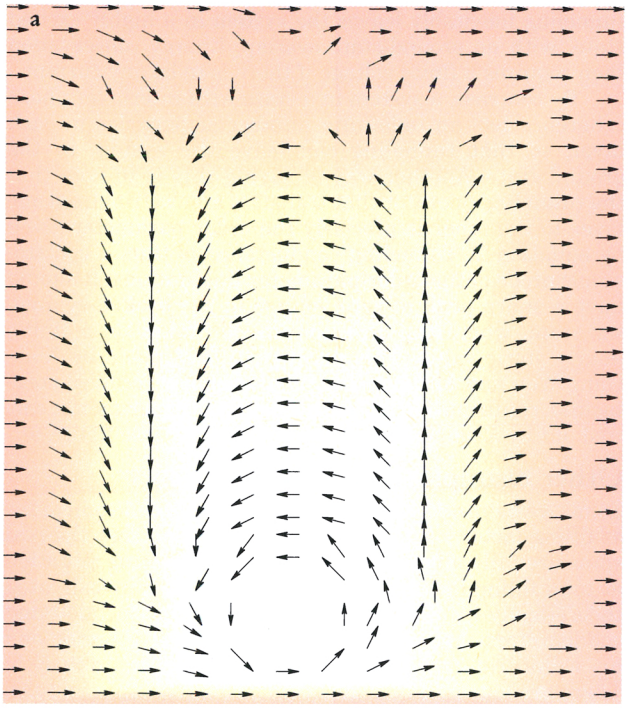
$$U = \frac{1}{2}\rho_s \int d^2r |\nabla\varphi|^2, \qquad (3)$$

where the pseudospin stiffness ρ_s has a typical value of about half a kelvin. (In general, spin stiffness is a measure of the energy cost of twisting spins out of perfect alignment.) Given the xy symmetry of this model, we anticipate that the system will undergo a Kosterlitz–Thouless phase transition at a temperature on the order of ρ_s.

This phase transition occurs when topological defects (vortices) in the phase field become unbound as a result of entropy gain, even though their interaction potential grows logarithmically with distance. In a superconducting film, such logarithmic interaction among vortices is due to the kinetic energy of supercurrents circulating around the vortices. But here there is no kinetic energy, and the energy cost is instead due to the loss of Coulomb exchange energy when there is a phase gradient. The "charge" conjugate to the order-parameter phase φ is the z component of the pseudospin, which is the charge difference between the layers. Therefore the supercurrent $\mathbf{J} = \rho_s\nabla\varphi$ corresponds to *oppositely* directed charge currents in the two layers.

One novel feature of the quantum Hall system is that vortices in the φ field are "merons," carrying one half of the topological charge of skyrmions (see figure 5a). This implies that a meron carries *half* the fermion number of an ordinary fermion like an electron. The easy-plane anisotropy allows these "half skyrmions" to be topologically stable.

The onset of superfluidity below the Kosterlitz–Thouless temperature will manifest itself as an infinite antisymmetric conductivity between the two layers. One way to observe this would be to perform a drag experiment in which one sends current through one layer and then measures the voltage drop induced in the other layer. In ordinary fermi liquids, this drag is caused by collisions that transfer momentum between quasiparticles in different layers. Simple phase-space arguments show that this drag voltage should vanish like T^2 at low temperature. But in the superfluid phase, where the antisymmetric conductivity is infinite, the voltage drop must be exactly the same in both layers. That will lead to a very large drag that is not only opposite in sign to the usual drag effect, but actually increases in magnitude with decreasing tem-



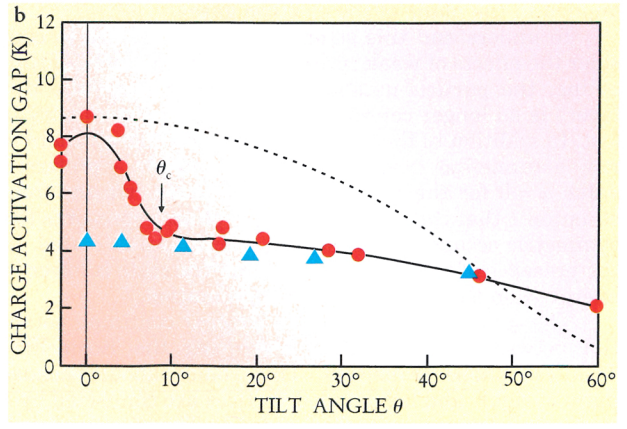


FIGURE 5. (a) MERON PAIR, formed by pseudospin orientation φ, is connected by a domain wall or "string." One half of an extra electron (or hole) resides in each defect.[12] (b) ENERGY GAP for charge activation, as a function of magnetic-field tilt angle in a weakly tunneling double-layer sample.[15] Red circles are for filling factor $\nu = 1$, blue triangles for $\nu = 2/3$. Arrow indicates critical angle θ_c. Dashed line is an estimate of the renormalization (which we neglect) of the tunneling amplitude by the parallel magnetic-field component at nonzero tilt angle.

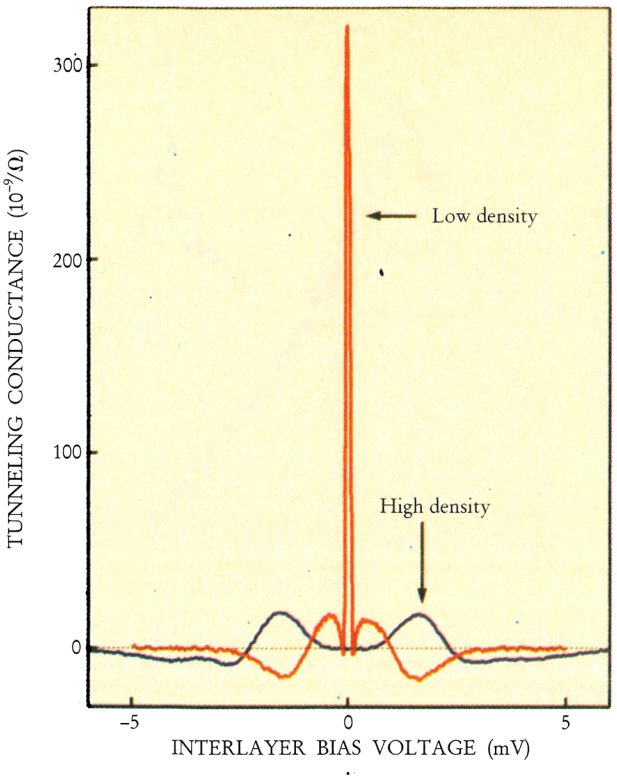


FIGURE 6. DIFFERENTIAL TUNNELING CONDUCTANCE between two adjacent two-dimensional electron gases. When the sample's electron density is high, the bilayer system is not in a phase-coherent state, and the tunneling shows a Coulomb pseudogap in the density of states. At lower electron density, the same sample goes into a phase-coherent state in which the electrons have strong interlayer correlations and the tunneling exhibits a huge anomaly at zero bias.[16]

perature. Thus, as the temperature is lowered through the Kosterlitz–Thouless point, the drag should change sign and *increase* in magnitude, providing a very clear experimental signature.

This superfluid response of a phase-coherent interlayer state has, in fact, not yet been directly observed. That's because it's hard to prevent tunneling between the layers when they are close enough to exhibit interlayer phase coherence. (A new generation of experiments is addressing this problem.) But long-range pseudospin xy order has been observed experimentally through the strong response of the system to a weak magnetic field applied in the plane of the electron gases.

To understand this strong response, one has to consider the effects of weak tunneling. In the presence of tunneling, the particle-number difference between the two layers is no longer conserved and the global symmetry is lost. In addition to the exchange potential energy, there is now a tunneling energy term, which yields a preferred value $\varphi = 0$ for the order-parameter phase. We see from equation 1 that the vanishing of this phase represents the symmetric occupation of the two quantum-well states. In the presence of tunneling, this symmetric state is lower in energy than the antisymmetric combination.

The tunneling term induces a *linear* confining potential between vortices, thus destroying the Kosterlitz–Thouless phase transition. This comes about because pairs of right- and left-handed vortices are connected by a "string" or domain wall (see figure 5a). The energy of such a composite object of length L is given by

$$E \approx WL + \frac{(e/2)^2}{L} + 2E_{\text{core}}, \qquad (4)$$

where W is the string tension (energy per unit length of the domain wall). The second term is the Coulomb repulsion between the half fermions bound to each vortex, and the third term is a constant governed by the ultraviolet details of the vortex cores.

The string tension for typical sample parameters is on the order of 0.1 kelvin per nanometer. That's 19 orders of magnitude weaker than the string tension that confines quarks inside nucleons and mesons! Furthermore, the string tension beween vortices, unlike that between quarks, is conveniently adjustable by simply tilting the magnetic field so that it has a component in the plane of electron gases (see figure 5b). This tilt causes tunneling particles to pick up a phase shift, making the order parameter prefer to tumble spatially. That, in turn, lowers the string tension and eventually drives it to zero, causing a phase transition to a deconfined phase in which domain walls proliferate.

In 1994, James Eisenstein and Sheena Murphy observed precisely this physics by exploiting the extreme sensitivity of the charge excitation gap to tilted magnetic fields.[12,15] As the string tension is lowered, the string stretches due to the Coulomb repulsion term in equation 4. That produces a readily observable rapid drop in the thermal activation energy needed to produce these charged objects.

The similarity between superconductivity and the physics of interlayer phase coherence has led to several suggestions of Josephson-like effects.[14] The equations of motion are indeed similar. But I believe that caution is required in their physical interpretation. For widely separated electron gas layers with no interlayer phase coherence, the tunneling current is extremely weak at small voltages. When an electron suddenly tunnels into an electron gas in a high magnetic field, it is very difficult for the other electrons to get out of the way of the newcomer, because the Lorentz force causes them to move in circular paths. Thus tunneling inevitably leaves the system in a highly excited state, with no ground-state overlap. Energy conservation then requires a finite voltage if there is to be any current.

By contrast, a system in a state with interlayer phase coherence has an indefinite number of particles in each layer, so that tunneling can leave the system in the ground state. Another way of saying this is to note that the tunneling operator that transfers an electron from one layer to the other is precisely the order parameter given by equation 2. Tunneling conductance is thus a spectroscopic probe of the order-parameter fluctuations. It should have a sharp peak at zero voltage in the broken-symmetry state, where the order parameter takes on a finite, nearly static value. This prediction, first made by Xiao-Gang Wen and Anthony Zee,[14] has recently received spectacular confirmation in some beautiful experiments carried out by Eisenstein's group at Caltech[16] (see figure 6).

Other examples of pseudospin order

So far we have only discussed the case of pseudospin order at filling factor $\nu = 1$ under the assumption that the real spins are fully aligned. Another very interesting situation at total filling factor $\nu = 2$, has recently been investigated theoretically by Sankar Das Sarma, Subir Sachdev and collaborators, and experimentally by Aron Pinczuk and his collaborators.[17] At $\nu = 2$, the situation is quite rich: There are four nearly degenerate levels (two spin and two

isospin) producing a novel mixing of the pseudospin and real-spin order parameters that leads to a "canted antiferromagnetic" state for the real spins. The low-frequency fluctuations in the resulting *xy* order parameter have been indirectly observed in light-scattering experiments.

In addition to the examples we have focused on here, there are several other examples where states of different Landau level, spin and/or electric-subband indices can be made degenerate by tuning tricks such as tilting the applied magnetic field. If the electron orbitals in question have little overlap, the pseudospin anisotropy tends to be of the easy-plane variety. But if the orbitals are fairly similar, the anisotropy tends to be of the Ising-like easy-axis type, leading to rather different physics, including the possibility of first-order phase transitions.[18]

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