The Origins of Mathematical Physics: New Light on an Old Question

A recently resurfaced tenth century manuscript, the Archimedes Palimpsest, includes the sole extant copy of Archimedes's treatise, the *Method*. As scholars begin study, new insights into Archimedes are emerging.

Reviel Netz

Imagine that you have to start science from scratch. Upon what disciplines should you draw? Philosophy, for instance, discusses the nature of time, space, and reality. Religion, too, tries to make sense of the world as a whole; and so, sometimes, does literature. Several disciplines—for example, biology and medicine—deal with special and highly significant features of the world. Such are the most natural ways to begin thinking about the world, and, in fact, most cultures make sense of their world through a combination of such intellectual resources. Mathematics, in comparison, appears like a non-starter. Here is a theory dealing with abstract objects, aiming at internal coherence rather than at connection to any external reality. All cultures develop some ways of dealing with calculation and measurement, and in some societies, a more abstract discipline, a "mathematics," may also emerge. But it is a peculiarity of the modern world to take this abstract discipline as the cornerstone for science.

In this respect, as in many others, modern science is Greek: The strange combination of mathematics and physics is a Greek invention, pioneered by Archimedes. Modern science is a mythical monster: half-goat, half-bird. The student of physics is led simultaneously to the laboratory, to face the phenomena of physical reality; and to the math course, to forget about the phenomena and to contemplate pure abstractions. That this hybrid existence is at all fertile is amazing: We use it, because we have discovered its effectiveness through experience.

But just what went through the head of the person who first tried to put this combination to work? Why marry the goat to the bird in the first place? In Syracuse, Sicily, in the third century BC, Archimedes set out in a series of works to combine physics and mathematics. How did he manage to do it? And why did he believe it was worth the try?

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In October 1998, a manuscript containing some of Archimedes's works, known to scholars as the Archimedes Palimpsest, resurfaced from long obscurity and was sold in New York for two million dollars. The private owner has, with great generosity, agreed to make it available for research and publication. This manuscript, shown in figure 1 and on this month's cover, is a unique source of evidence for Archimedes's thought. Among its many treasures is the only evidence we have for the treatise known as the *Method*, in which physics and mathematics are most intimately combined by Archimedes.

As seen in figure 1, the Archimedes manuscript has been overwritten by a twelfth century prayer book. (*Palimpsestos* is the Greek word for rescraped, or overwritten, parchment.) Work is only just beginning on uncovering and studying the original text. Many scholars in the field hope we are near a much better understanding of Archimedes. I have looked at the palimpsest, and I believe this hope is well founded. In this article, I delineate some of Archimedes's originality, give an example of the new information the Archimedes Palimpsest may provide us, and I suggest, tentatively, what Archimedes's mathematical physics may have meant.

Archimedes's originality

"Give me a place to stand, and I shall move the Earth"—Archimedes may indeed have said this: Among the wealth of anecdotes and legends surrounding the man, this is perhaps one of the more plausible.[1] He was referring to the law of the lever, which (in the variant form of the law of the balance) he had proved in his treatise, *Planes in Equilibrium*. One can say that Archimedes moved the Earth—in principle—*without standing anywhere*: Pure thought alone showed how the Earth must behave.

Ernst Mach, who in the beginning of this century offered a philosophy of science in which science was assumed to do no more than arrange sensory input, thought Archimedes's proof of the law of the lever was

FIGURE 1. ARCHIMEDES PALIMPSEST as it appears today. In the 12th century, the text of a prayer book (horizontal) was written over the original Greek text (vertical)—here, a section of Archimedes's treatise, *On Floating Bodies*. The cover of this month's issue shows the same pages, rotated 90° and digitally enhanced to highlight the underlying text. (Figure © Christie's Images, New York.)

flawed. Effectively—so Mach argued—Archimedes had reasoned in a circle, taking for granted his main result. Otherwise, how could he obtain a physical result without any experiment? However, Mach failed to see the way Archimedes's proof worked: No circular reasoning was involved.[1] The way in which Archimedes manages to have satisfactory physical proofs, based purely on conceptual considerations, may be neatly illustrated by a closely related proof, found in *Planes in Equilibrium* and presented in box 1: that the center of the weight of a triangle lies at the intersection of its medians. (The modern term "center of gravity" should be avoided for Archimedes, as it misrepresents both his language and his underlying thought.)

This proof is one of the earliest and most simple applications of mathematics to physics. Archimedes went on to a backward application: using such physical results to derive results in pure mathematics.

Archimedes died in 212 BC, but what may be his most interesting work—the *Method*—came to the attention of modern readers only in 1906 AD, following the initial discovery of the Archimedes Palimpsest. The treatise is surely one of the longest-neglected pieces of intellectual legacy in the history of science. It is fascinating to speculate how the history of science might have looked with Galileo, say, aware of its existence. For it is in this work that Archimedes most explicitly connects the mathematical and the physical. He claims here that he has invented a procedure that allows him to use physics—in particular, mechanics—to derive mathematical results. Archimedes derives a wide range of results, including such highlights of his mathematical achievement as the volume of the sphere and the volumes of segments of solids of revolution. Box 2 presents a relatively simple case, the one that Archimedes himself took as a representative example for the method of the *Method*.

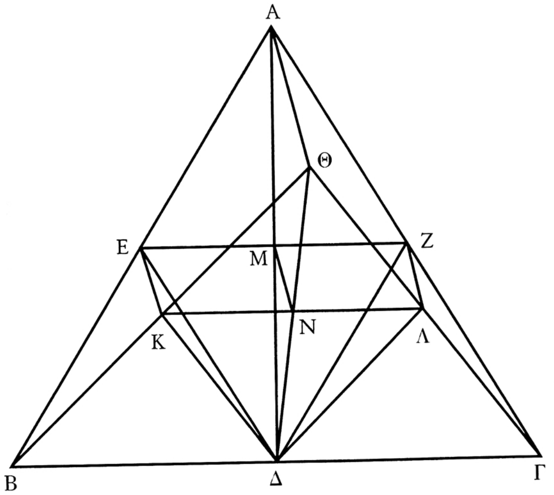
The reader will notice from box 2 that, besides anticipating mathematical physics, Archimedes further anticipates, perhaps, the integral calculus. The summation of areas through lines—and of solids through areas—is a feature of the *Method* that Archimedes may have considered to be less than rigorous. It is probably for this reason that he considered this treatise merely as heuristics, literally a "method."

The combination of the two types of proofs—from

Box 1. From Mathematics to Physics

This figure shows a triangle whose three sides are bisected at E, Z, and Δ, giving rise to four equal small triangles, each similar to the original triangle. We hypothetically assume that the center of the weight is *not* on the median AΔ but at Θ. Take Λ and K as centers of the weight of their respective small triangles, and bisect ΛK at N. Now, apply the postulate (explicitly stated by Archimedes) that, in similar figures, the centers of the weight are similarly situated. (Archimedes rigorously defines the concept of "similarly situated," but it may be intuitively understood as the congruence of the centers of the weight once the similar figures are made to scale). From this postulate, we have AΘ||ZΛ||EK, and a simple geometrical consideration shows that ZΛ||EK||MN as well, so that finally we have AΘ||MN. N must also be the center of the weight for the composite object made of the two small triangles centered at Λ and K (a direct result of another postulate: Equal weights balance at equal distances). The remainder of the triangle is the parallelogram AEΔZ, and Archimedes has shown in an earlier proof that the center of the weight of a parallelogram is on its intersection of the diagonals (rigorously proved by Archimedes, expanding what is essentially a symmetry intuition). The center of this remainder is, therefore, at the point M. Thus the center of the whole triangle, Θ, must be somewhere on the line MN, which is impossible (because of the parallels). Therefore, the center of the weight must be on the median line—that is, on the well-defined point of the intersection of all median lines (at one-third of each median line).

Since "center of the weight" is a concept with physical implications—it is the point from which the plane can be suspended and remain in balance—we have just given a mathematical proof of a physical result: If you take a triangle and suspend it from a median line, at its third, it will remain balanced.



mathematics into physics and then from physics into mathematics—closes a circle. By thinking of triangles and their symmetries and similarities, one finds the center of the weight of any triangle; by thinking of centers of the weight, one finds the area of a parabolic segment. But what did Archimedes himself think of, primarily: balances and weights, or triangles and segments? In several ways, the Archimedes Palimpsest may shed some light on this question. I now consider one of those ways.

What can the palimpsest tell us?

When asking what a scientist had in mind, we should examine all the available evidence. Even an unconscious doodle may offer some clues to what the author was thinking. It is, therefore, necessary to reproduce faithfully all the evidence. As yet, this has not been done for Archimedes. Refer to the figure in box 2, which is based on the works of Archimedes published by the Danish philologist Johan Ludvig Heiberg. This figure does not represent the actual diagram seen in the Archimedes Palimpsest. There, the diagram is more like figure 2. Whereas the box figure is technically "correct," figure 2 is "incorrect." For instance, the size relations $KB = \frac{1}{2}K\Gamma = \frac{1}{2}\Theta K$ may be seen in the box figure but not in figure 2. Furthermore, while the box figure has a bona fide parabolic segment, figure 2 has a segment of a circle, crudely drawn.

It is probably for just this reason—that figure 2 is "wrong"—that Heiberg chose to ignore the diagrams of the manuscript and instead produced his own, "correct" figures. In doing so, however, he may have suppressed an important piece of evidence about Archimedes.

Drawing on the corpus of diagrams in all the treatises by Archimedes in all the extant independent manuscripts, I believe the following claims can be made:

▷The diagrams in the Archimedes Palimpsest stem ultimately from antiquity, in great probability from Archimedes himself.

▷The diagrams display a consistent visual logic. While Heiberg aims to represent actual ratios and figures, Archimedes himself produced a schematic figure. Archimedes's diagrams show mainly the relations of configuration and identity: which are the objects participating in the proposition, and what are their relations of intersection and inclusion. There is little attempt to show the "real shape" that the objects have.

If these conjectures are the case, then it is no longer valid to think of the box 2 figure as "correct" and of figure 2 as "incorrect." The palimpsest gives us an insight into the particular way in which Archimedes visualized his objects.

This type of visualization is used throughout Archimedes's writings, independent of subject matter. The fact that the *Method* deals, in a sense, with physical objects does not make its diagrams any different from those in his strictly geometrical works, such as *On Sphere and Cylinder*. Indeed, the diagrams are strictly geometrical, not only in the works on the lever (on which this article concentrates), but also in his hydrostatic masterpiece, *On Floating Bodies*, whose physical objects are much more visibly discussed, with questions of weight and specific weight. A diagram from this work, as found in the palimpsest, appears on the cover of this issue.

The figures for *On Floating Bodies*, uniquely for Archimedes's works, are systematically different between the two extant manuscripts for this treatise. One may compare the diagram on the cover to figure 3, which essentially reproduces the figure of the alternative tradition preserved in a Latin translation from the 13th century. We are not yet in a position to identify the correct diagrams for *On Floating Bodies*, but we know their visual logic, which is always schematic rather than pictorial.

Obviously, the way in which a scientist represents an object may throw some light on the way in which that object is conceived. I now move on to offer, tentatively, an interpretation of Archimedes's conception of his objects.

Box 2. From Physics to Mathematics

This figure represents the first proposition of the *Method*. The curved area ΑΒΓ is a closed parabolic segment. We have ΑΔ = ΔΓ, ΔΒ a diameter, ΓΖ tangent to the segment, and ΑΖ parallel to ΔΒ. Let ΞΜ be an arbitrary line parallel to ΔΒ and ΑΖ. Through the properties of the parabola, we get the key result that ΓΑ:ΞΑ::ΜΞ:ΞΟ, that is, ΓΚ:ΚΝ::ΜΞ:ΞΟ.

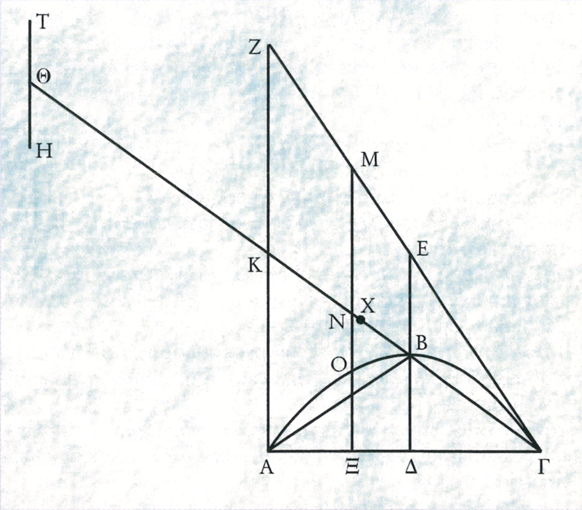
We now set ΘΚ = ΓΚ, so we also have the relation ΘΚ:ΚΝ::ΜΞ:ΞΟ.

We imagine ΟΞ positioned as ΤΗ, with its center at Θ, and we have the line ΜΞ, positioned with its center at Ν. From the relation above, the two lines ΜΞ and ΞΟ are to each other, reciprocally, as their distances from the point Κ. If we imagine a balance with its fulcrum at Κ, carrying ΜΞ on one side and ΤΗ (that is, ΟΞ) on the other side, then the balance will be at equilibrium.

The above procedure deals with an arbitrary line; repeating it for all parallel lines, we balance each and every line of the triangle with its correlated line from the parabolic segment. The entire triangle is now at equilibrium with the parabolic segment: the triangle where it is right now, the parabolic segment relocated so that it is centered on the point Θ.

But wait—we actually know where the center of the weight of a triangle is! Namely, it is at the point Χ, one-third of the way along the medial line ΚΓ. The triangle and the parabolic segment balance around the point Κ, and their centers of the weight are, respectively, Χ and Θ.

Since the two geometrical objects, the triangle and the parabolic segment, balance each other at distances whose ratio is 1:3, then this must be, reciprocally, the ratio of the two geometrical objects themselves: The parabolic segment is one third of its enclosing triangle. Thus one of the major quests of Greek mathematics—finding the ratios between rectilinear and curvilinear figures—has been accomplished.



A tentative conclusion

Let us go back to the proposition showing the location of centers of the weight in triangles (box 1). All we needed to do was to make a few assumptions: that equal weights balance at equal distances and that the centers of the weight of similar objects are similarly situated. Both assumptions are plausible, and so we may take them on board, at least tentatively. This is all Archimedes ever does, because he explicitly postulates those assumptions. The structure of the application of mathematics to physics by Archimedes, then, is this: by making explicit, clear assumptions, one draws the logical implications of the assumptions, which then have to hold for the world—as long as the assumptions themselves do. Archimedes did not prove that the centers of the weight of triangles are physically located one-third of the way along the median line. All he proved is that this result follows from some plausible assumptions. As a correlate, one may say that Archimedes proved that if the centers of the weight of triangles in the actual physical world are not at the point mentioned, then at least one of the plausible assumptions is, in fact, wrong. One can imagine Archimedes trying to balance extra-thin triangular slices to find their centers of weight, and in this way to find the fundamental structure of the universe of weights. This, indeed, is rather like what Gauss was to do, two thousand years later, when he finely measured the sum of angles in physical triangles to determine the geometry of the universe.

Yet I do not believe Archimedes did anything of the kind. My suggestion is that Archimedes was largely indifferent to the question of where physical triangles balance. There are three main reasons for thinking this was the case:

▷The straightforward reading of Archimedes's text strongly suggests his indifference to actual physical properties. Once the physical presuppositions are explicitly set out, all the arguments are strictly geometrical. Archimedes plunges directly into speaking about triangles, lines, and segments—using a purely geometrical jargon and making no reference to the idealizations involved (objects have thickness, weight is not uniform, and so on). This, I suggest, is because no idealization takes place: The discussion is not about idealized physical objects, but directly about geometrical triangles, whose "weight" is simply their area.

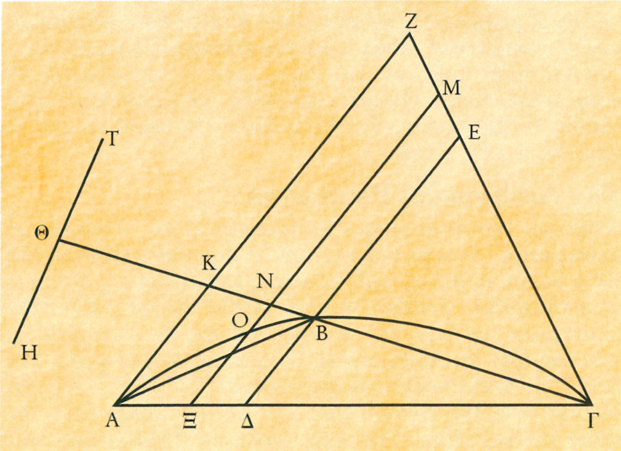


FIGURE 2. ARCHIMEDES'S FIGURES are schematic, rather than precise, illustrations. Thus this figure—from his treatise, the *Method*, found in the palimpsest—is not as accurate as the one shown in box 2. This diagram has not yet been digitally enhanced. (Adapted from a drawing prepared by Dr. William Noel, manuscript curator at the Walters Art Gallery, Baltimore, Maryland. My thanks go to him and to the owner of the manuscript for permission to reproduce the drawing.)

Box 3. The Eurekas of Archimedes

Archimedes made many discoveries. Some, perhaps most, he committed to writing, and some of these writings, perhaps most, survived. The most remarkable of them, the *Method*, survives only thanks to the Archimedes Palimpsest.

The palimpsest contains, in more or less fragmentary form, seven works by Archimedes. The first three form a natural sequence in mathematical physics:

▷*Planes in Equilibrium*. Archimedes proves the law of the balance and derives results for centers of gravity in planes.

▷*On Floating Bodies*. Here he proves his law of buoyancy and derives results for the flotation of solids of geometrically interesting shapes.

▷*Method*. As illustrated in the main text, the law of the balance is used to derive geometrical properties.

Next come four studies in pure geometry:

▷*Spiral Lines*. Spirals are first defined and their lengths and areas are measured.

▷*On Sphere and Cylinder*. Archimedes provides the ratios for the surface area and volume of a sphere and then solves a series of problems concerning spheres.

▷*Measurement of the Circle*. An approximation of the value of π is obtained using a method that can, in principle, be extended indefinitely.

▷*Stomachion*. Only a fragment survives. Apparently, this is a study in a tangram-like game, where areas are covered by given geometrical figures.

Three further works of Archimedes have survived in Greek in other manuscripts:

▷*Quadrature of the Parabola*. Related in certain ways to the *Method*, this is an exploration of the applicability of the law of the balance to geometry.

▷*Sand-Reckoner*. In this complex miscellany, Archimedes sets out to measure how many grains of sand it would take to fill up the universe, in the process contributing to astronomy as well as to calculation techniques.

▷*Conoids and Spheroids*. Archimedes introduces the solids of revolution of conic sections, and provides several measurements for those figures.

Archimedes may also have been the author of the *Cattle Problem*, a numerical problem comparable in spirit to the *Sand-Reckoner*, although the attribution is nowhere directly founded. An Arabic text, *On Lemmas*, showing various configurations of circles and measurements concerning them, may be derived from Archimedes; the same may be said, with even less confidence, of an Arabic treatise on the *Construction of the Regular Heptagon*. We know for sure, on the authority of the knowledgeable commentator Pappus, that Archimedes had produced a work (now lost) on *Semi-Regular Solids* (the faces of which are all regular, though not identical).

One may go on counting, beyond these 14 works, well into the realm of myth, as recounted by Greek and Arabic stories on Archimedean feats of engineering and proof. We cannot know how much the Archimedean corpus originally contained. However, we do have a relatively large body of surviving works—more representative, probably, than for any other mathematician from antiquity: None of the others appears to us with as well-defined a scientific personality.

▷Further, this is the implication of the contexts in which the theory is used. We never see Archimedes deducing anything about the physical world (as is standard in modern physical writings). On the other hand, the theory is indeed applied—but in geometrical contexts, such as the *Method*.

▷Finally, I suggest we use the evidence of the diagrams as further indication of the thinking behind the propositions. In all of Archimedes's proofs, whether strictly geometrical or more "physical," the same visual logic is in use. Nowhere is an attempt made at pictorial accuracy. In modern mechanical writings, beginning with the Italian Renaissance, precise and specific drawings often accompany the mathematical discussions, underscoring the intended application of mathematics to the physical world. For Archimedes, however, the object in its visual reality is of lesser importance. The diagram is not a picture, but instead provides the arena for geometrical proof.

If this is true, we have found a simple answer to our original question regarding how mathematical physics was originally conceived—namely, because it was mathematics.

The basis for this conclusion is extremely simple: The one common denominator for all of Archimedes's writings, whether "physical" or "mathematical," is that they all provide proofs. Proof was the real passion of Archimedes—and that of his culture in general. The Greeks were forever arguing, refuting, and attempting to provide irrefutable arguments. Out of this consummately argumentative society came that unique form of expression that is characterized by its stress on argument, and on argument alone: the Greek mathematical, deductive argument.[3] And if what counts is to have a correct argument, it becomes of minor significance to know where triangles actually balance, especially because, if you attempt to balance physical triangles, you open yourself to all sorts of objections. So why even bother messing with the physical? In geometry, Archimedes could be irrefutable. My sense is that this is where he preferred to remain.

If this assessment is correct, we may also see why mathematical physics is such a good idea. It embodies the principle that one should aim for the best possible arguments, using the discipline in which the highest standards of proof are available. Mathematics may have little to say, directly, about the physical world, but it is the only way to say anything at all with any certainty. The bet of modern science—following on Archimedes—is that we are willing to say very little, as long as what we say is well argued. Good arguments are good starting points for truly productive discussion, and so it is not surprising that the mathematical route has been so productive in modern science.

But is this interpretation true? It is only a possibility, suggested by the writings of Archimedes. He explicitly says very little about his goals and conceptions. When—as the legend goes—he cried "Eureka," sallying forth from the bath, this may have been because he had discovered truths of physics. Or he may have discovered new properties of geometrical solids. Like the citizens of Syracuse, we cannot really tell, but can only gape at his discoveries with amazement. We are extremely fortunate that now, thanks to the Archimedes Palimpsest, we shall be able to gape from a bit closer.

The Archimedes Palimpsest

Although the Archimedes Palimpsest is an "Archimedes manuscript," it was not written by the man himself. Far from being an autograph from the third century BC, it is a manuscript written in the tenth century AD. The palimpsest is only one among six extant independent manuscripts for Archimedes. It contains seven works (see box 3 on Archimedes's achievement). Of these, only the *Method* and the *Stomachion* are not available from the other manuscripts. The palimpsest is fragmentary and

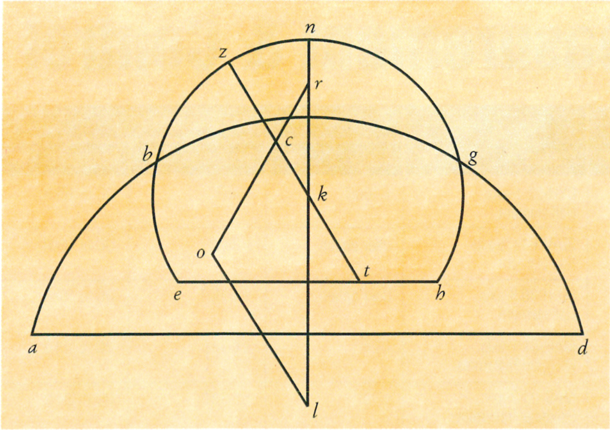


FIGURE 3. *ON FLOATING BODIES* has two different styles of figures. Compare those shown here, as found in a Latin transcription, and those from the Archimedes Palimpsest, seen on the cover and in figure 1. (Adapted from M. Clagett, *Archimedes in the Middle Ages*, American Philosophical Society, Philadelphia, 1978.)

contains many obvious mistakes. It is an ugly piece of parchment, scorched and seriously suffering from mold.

Such characteristics are typical for ancient works. Very little evidence for ancient authors survives from before the 9th and 10th centuries AD: The palimpsest is, by a long stretch, the earliest evidence we have for Archimedes. It is uncommon to have but a single independent manuscript for an author. All manuscripts are riddled with errors, and most are, to some extent, incomplete. And while many manuscripts are things of beauty, their significance lies elsewhere.

What makes a manuscript significant? Being independent (that is, not copied from any other surviving manuscript), and unique (no other parallel manuscript with the same texts exists). The Archimedes Palimpsest is fully independent of all other Archimedes manuscripts, and it is the only one to be a unique source for any of his works. It is thus the most important Archimedes manuscript.

A manuscript is rather like a planetary probe. The results of a single probe are tantalizingly incomplete, yet they are also uniquely significant. The comparison becomes precise in that a manuscript is like a probe sent to us: a time capsule from Archimedes.

The travels undergone by this particular capsule were especially arduous. It was put together in the tenth century, but, judging from the total absence of marginal notes or corrections, it seems never to have been read by any mathematician. That it fell into disuse is clear from its fate: Two hundred years later, Greek monks used it as scrap parchment. They cut each page into two and discarded some pages. They scraped each page as clean as they could. Finally they wrote a prayer book on the scraped leaves, making this a re-scraped manuscript, literally a "palimpsest."

(The monks should not be considered villains. They have very much saved Archimedes, inadvertently, by recycling him, and they can not be blamed for seeing no value in a work which, possibly, no one alive then could read and follow. Had it not been for the Greek Church, practically nothing would have survived from Greek antiquity.)

The adventures of the palimpsest in the ensuing seven hundred years are more difficult to follow. An ex libris, once present in the manuscript but since disappeared, hailed from the Mar Saba monastery near the Dead Sea, in today's Palestine. The palimpsest may have then passed to the Church of the Holy Sepulchre in Jerusalem, and certainly, by the mid-19th century, it reached the church of the same name in Constantinople (now Istanbul). There it lay for the remainder of the century.

Meanwhile, the Danish scholar Heiberg began to publish, almost single-handedly, ancient Greek mathematics, starting with his first edition of the works of Archimedes in 1880. Twenty-six years later, his attention was brought to a library catalogue mentioning "some mathematics in a palimpsest" and quoting a few words. A glance sufficed: This was Archimedes. Visiting Constantinople, Heiberg managed to read much of the palimpsest using only a magnifying glass. Anyone who has looked at the palimpsest today (see figure 1) must admire the genius and patience shown in Heiberg's second edition,[2] published in the years 1910–1915.

In the aftermath of the First World War, in which the Greeks were largely expelled from present-day Turkey, many works were cast in all directions. The manuscript's fate during this time is shrouded in mystery, but clearly, no scholar since Heiberg himself had had access to this manuscript, which was privately and secretively owned.

This state of affairs came to a dramatic end in 1998, when suddenly the manuscript appeared for sale at Christie's in New York. Legally contested by the Greek Orthodox Church, the sale was allowed to proceed by a last-minute court decision. The Greek government took the challenge and sent a representative to the sale. At around one million dollars, all the contenders dropped out, with two exceptions: the Greek representative and the representative of a private collector. The private collector held steady for two million dollars, which Greece was unable to match.

The new owner (who wishes to keep his anonymity) made clear from the start that the manuscript would be made available for scholarly study. The manuscript was publicly exhibited in American museums and is currently being conserved at the Walters Art Gallery in Baltimore, Maryland.[4] Plans for the future include a path-breaking technological effort to produce a text based on digital image analysis and enhancement. Indeed, while Heiberg's edition is remarkably accurate given the means available to him, much can be clarified with today's technologies. This prospect alone is certain to make the recent resurfacing of the Archimedes Palimpsest a historical moment for the study of ancient science. At long last, we are in a position to discharge our duty to Archimedes: to publish the best possible edition of his works—to recover, in the fullest detail, the time capsule he has sent us.

References

1. An excellent source for information on Archimedes, including evidence for and against the anecdotes and legends, is E. J. Dijksterhuis, *Archimedes*, Princeton U. P. (1987).
2. J. L. Heiberg, *Archimedis Opera* (2nd ed.), Teubner, Leipzig (1910–1915).
3. See G. E. R. Lloyd, *Demystifying Mentalities*, Cambridge U. P. (1991), and R. Netz, *The Shaping of Deduction in Greek Mathematics*, Cambridge U. P. (1999).
4. The history of the Archimedes Palimpsest, as well as biographical information about Archimedes, can be found at the website of the Walters Art Gallery: http://www.thewalters.org/archimedes/index.html. ■