Modern Classical Physics THROUGH THE WORK OF G. I. TAYLOR

uring the spring of 1998 we co-taught a graduate course on modern classical physics that aimed to cover the fundamentals while also conveying the directions and sense of current research. As we talked about the subject. we realized that many of the important discoveries underlying a wide range of topics of current interest in physics and engineering were made

One scientist's work provides material for an entire course, covering topics ranging from hydrodynamic stability and turbulence to electrohydrodynamics and the locomotion of small organisms.

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by a single individual, the British scientist Geoffrey Ingram (G. I.) Taylor (1886-1975). Although many researchers are familiar with one or another of Taylor's contributions, few seem to be aware of the incredible breadth of his scientific publications and their relevance to important research questions today. The same person who is commonly remembered as the namesake for several basic fluid flow instabilities (Taylor-Couette, Rayleigh-Taylor, and Saffman-Taylor) also was the first to show experimentally that a diffraction pattern produced by shining light on a needle does not change when the intensity of light is decreased. And these topics are only the beginning. Taylor made fundamental contributions to turbulence, championing the need for developing a statistical theory, and performing the first measurements of the effective diffusivity and viscosity of the atmosphere. He wrote one of the first scientific papers using random walks; gave the first consistent theory of the structure of shocks in gases; and explained the importance of dislocations for determining the strength of solids. He also described the counterintuitive physics of fluid motion in a rotating environment, providing the basic principles for important aspects of atmospheric and oceanic dynamics.

Taylor studied all of these topics during the first 30 years of his career, between his 20th and 50th years. During the next 30 years, among other achievements, he quantitatively described dispersion of solute in fluid flow; elaborated the basic principles for how microorganisms can swim; and predicted, by dimensional analysis, the energy of the atomic bomb explosion from a series of US government publicity photographs. He also recognized that accelerating an interface between two fluids can lead to instability, and did seminal work on the interaction between fluids and electric fields, providing the foundation for electrohydrodynamics and the basic principles for a slew of present-day industrial processes and devices.

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Taylor did much of this research involving electric fields between his 70th and 80th years.

The remarkable depth and breadth of Taylor's research impacts in one way or another much of modern research in classical physics. Therefore, we decided that our ends would be well served by structuring the course exclusively around

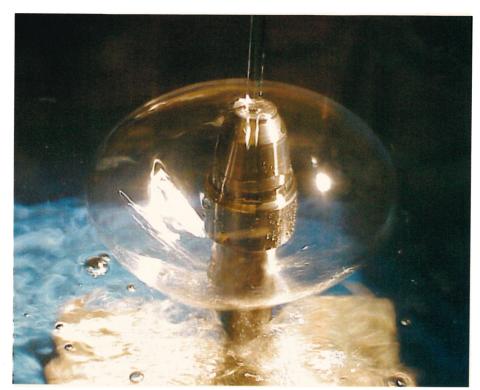
Taylor's scientific papers. In this article we summarize the structure and content of our course, and in the process describe a few of Taylor's discoveries that are perhaps not widely known outside of the disciplines that they impact most substantially.

Course structure

Throughout the semester, it became increasingly clear that there were many advantages to structuring a course around Taylor's published papers.1 First of all, Taylor's research interests provide an excuse to cover a much wider range of topics than is normally justifiable in a single course. Second, a careful study of his papers inevitably draws attention to his style, which is to compare theoretical arguments and scaling analyses directly and quantitatively with experimental results. The value of investigating science and engineering questions in this way, while on the one hand rather obvious, is on the other hand extremely difficult both to teach and to learn, especially when considering complicated nonequilibrium problems as Taylor routinely did.

As anyone who has tried to make a prediction about such a system knows too well, the greatest difficulty is posing questions that at the same time have simple quantitative answers and prove insightful. Taylor's great talent was to repeatedly find ways of extracting a simple feature from a complicated process or experiment. Not only did this lead to direct, quantitatively testable predictions, but later researchers tended to identify Taylor's extractions as the most important quantitative aspects for understanding the system. In "teaching Taylor," there are endless opportunities to draw attention to the value of this approach to scientific and engineering questions and to compare and contrast it with more modern, brute-force approaches such as direct computation of every aspect of a system. Although there is clearly much to be said for both approaches, it is vastly easier to teach the latter, as the examples of the former are few and far between.

The outline for our course is shown in the box on page 32. Typically there were two 90-minute lectures per week, in which we critically discussed a single paper, or sometimes a group of two or three papers. The papers were distributed in advance and students were expected to have



read them. In several instances, we distributed recent review articles or closely related research papers as well. We also organized a number of special seminars given by local faculty and visitors; we asked these lecturers to frame their remarks as: "subject X since Taylor."

Introductory ideas

To set the stage for Taylor's research, we used the first lecture to summarize the state of fluid mechanics in the early years of the 20th century, before Taylor became involved. We based this presentation on the excellent review by Sydney Goldstein, published as the first article in the first issue of Annual Reviews of Fluid Mechanics.2 Although much was known about fluid motion in the early years of the 20th century, much discord and debate existed over the relation of the theories to experiments. In 1916, Lord Rayleigh wrote a review for *Nature* of the fourth edition of Horace Lamb's Hydrodynamics, in which he said "Perhaps the time for [comparing theoretical hydrodynamics with experiments] has not yet come We may hope that before long [experiments may be] brought into closer relation with theoretical hydrodynamics."

A major problem at the time was that there was still uncertainty about the correct boundary conditions on the fluid velocity at solid surfaces, and whether these boundary conditions could be independent of the state of motion of the fluid. Although Ludwig Prandtl's 1904 work introducing viscous boundary layers pointed toward the resolution, his ideas were only gradually being disseminated and understood. Goldstein writes that by the mid-20th century these problems were largely resolved. "Several factors . . . contributed to this, but the greatest influence has been the example of G. I. Taylor."

We then turned to a discussion of Taylor's papers. Our choice of ordering, summarized in the box, was an attempt to be pedagogical. We started with Taylor's first two scientific papers, written when he was less than 25 years old, and proceeded to read his work on instabilities, turbulence, rotating flows, and so on.

A WATER BELL forms when a water jet hits the top of a closed cylinder. The impact of the jet creates a thin fluid sheet, which then wraps around the cylinder to form a beautiful "bell." This flow configuration was first analyzed by Felix Savart in 1833. Taylor provided a theoretical description of the shape of the bell. This photograph was taken by Robert Buckingham in the fluid dynamics laboratory at MIT's mathematics department, under the supervision of John Bush.

The rest of this article gives brief summaries of some of the topics. Taylor contributed so much to fluid and solid mechanics that it is both impossible and beyond our competence to do justice even to his qualitative ideas in a single course, much less

in a single article, and so in both cases there are egregious omissions. Our choice of topics for this article was motivated by our desire to show the breadth and continued relevance of Taylor's research, as well as to highlight those topics that we found to be the most useful pedagogically. For more detailed information about Taylor's work and life. we recommend George Batchelor's recent biography of Taylor,3 and recent review articles.4,5

Interference fringes by feeble light

We began our tour of Taylor's research by discussing his first scientific paper, which was published in 1909. This was his only paper that was not classical physics, but it nonetheless bore the experimental characteristics that were to appear throughout his later work. At the request of J. J. Thomson, Taylor performed an experiment (in the children's room of his parent's house!) to determine whether there was a qualitative change in a diffraction pattern when the intensity of the light is reduced greatly.3 Taylor indicates that Thomson believed that there would be a change in the pattern. Taylor took photographs of the shadow of a needle, varying the intensity of light by shielding the light source with smoked glass screens. When decreasing the intensity he increased the exposure time to keep the total amount of light on the photograph constant. The longest experiment took three months, corresponding to the intensity of a candle more than a mile away; some of the experiments even took place while Taylor was on a yachting trip. Taylor observed no change in the diffraction pattern, wrote a two-page paper describing this result, and then dropped this line of research.

Motion of discontinuities in gases

Taylor's second scientific paper, published in 1910 when he was 25 years old, was awarded the Smith Prize for senior mathematics students at Cambridge University. This paper solved a long-standing, fundamental problem in fluid mechanics. George Gabriel Stokes had noticed that there was the real possibility that the velocity in a gas



could form discontinuities in a finite time, if a slower region of gas were moving ahead of a faster region. Such discontinuities, now called "shocks," are easily predicted from the equations of ideal (inviscid) fluid dynamics. They represent singularities, in that velocity gradients diverge at the discontinuity. At the time, it was not known what happened after such shocks formed. Taylor demonstrated that in a real gas the discontinuity would be eliminated by dissipative effects (both viscosity and thermal heating). This solution (realized qualitatively in 1908 by Rayleigh, then 66 years old) is one of the most basic features in gas dynamics.

The Taylor-Couette paper

The first topic we treated in detail was Taylor's 1923 paper on instabilities of Couette flow-the flow between concentric rotating cylinders. An interesting feature is the paper's motivation. Taylor begins by observing that "A great many attempts have been made to discover some mathematical representation of fluid instability, but so far they have been unsuccessful in every case."6 The concept of stability had been well formulated by this time, and many authors (among them Lord Kelvin, Rayleigh, Heinz Hopf, and Arnold Sommerfeld) had attempted to predict the instability of a solution to the equations of fluid dynamics. Unfortunately, however, no calculation agreed with experiments. The failure to predict instabilities led to great consternation and confusion. For example, Hopf suggested that perhaps it was necessary to take account of the rigidity of the boundaries to explain the instability of shear flows in channels. (Taylor commented: "There seems little to recommend this theory as an explanation of the observed turbulent motion of fluids."6)

Taylor's paper is a major intellectual accomplishment, representing the first example where a stability calculation quantitatively matches an experiment. The fact that the comparison worked is due in large part to Taylor's insight that among the different possible experiments, the rotating cylinder apparatus is best suited for quantitative comparison between theory and experiment.

GEOFFREY INGRAM TAYLOR (right) at age 69, in his laboratory with his assistant Walter Thompson. (AIP Emilio Segrè Visual Archives.)

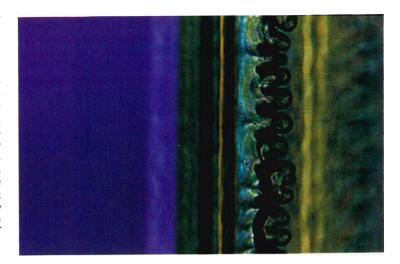
The work demonstrated unambiguously that both the approach used in the stability calculation, and its underlying assumptions (the boundary conditions), were correct. As Goldstein states in his review article, "Simplifications of the mathematics... were to follow, but there could be no [more] controversy."²

Taylor's paper was equally remarkable for its technical detail, both theoretical and experimental. The calculations leading to an instability threshold for inner and outer cylinders of arbitrary radii are tedious, producing formulas that are each about a page long, involving determinants of Bessel functions. (In lecture, we avoided the algebra by using the thin-gap limit, first introduced by Harold Jeffreys in 1928, and expanded on at length by Subrahmanyan Chandrasekhar.7) At the time, determining the numerical values of the formulas was itself a significant challenge. Designing an experiment consistent with the assumptions of the calculation was equally delicate-in particular, end effects of the cylinder could not influence the onset of the instability. The results for the instability boundary as a function of the rotation rates of the two cylinders were in beautiful agreement with the theory, as the figure on page 35 shows, and several of Taylor's photographs of the flow are still reproduced. Rather amusingly, Taylor actually measured more points on the stability boundary experimentally than he calculated theoretically, presumably due to the tediousness in evaluating the Bessel function determinants!

At the end of the paper, Taylor described his observations of the panoply of nonlinear states that exists in the rotating cylinder apparatus above the instability threshold. As the relative speed of the cylinders is increased, the flow

Course outline	
Introductory remarks	Overview of G. I. Taylor's research State of fluid mechanics in 1900
Taylor's first two papers	Diffraction at low light levels Regularization of shocks
Instabilities	Taylor-Couette flow Saffman-Taylor problem Rayleigh-Taylor instability
Turbulence	Eddy diffusivity in the atmosphere Diffusion by continuous movements Statistical theory of turbulence Vortex breakdown
Rotating flows	Taylor-Proudman theorem Particle motion and Taylor columns
Dispersion in laminar flows	Taylor-Aris dispersion Measurement of molecular diffusivities
Solid mechanics	Dislocations and the strength of solids
Swimming at low Reynolds numbers	
Drops and bubbles	Drop deformation and breakup Viscosity of mixtures; emulsions
Electrohydrodynamics	Leaky dielectric model Conical interfaces
Surface tension	Thin films, peeling, water bells
Shocks	
Explosions	

INSTABILITY during the peeling of adhesive tape. G. I. Taylor studied this problem in 1964 (at the age of 78), and demonstrated that viscous stresses in the adhesive fluid contribute significantly to its "stickiness." When the adhesive is peeled from a solid surface (the blue region), competition between applied pressure and surface tension leads to an instability with a well-defined wavelength (squiggles). Interest in the relevance of fluid mechanical instabilities to adhesion continues to this day. (For a review, see the article by Cyprien Gay and Ludwik Leibler, PHYSICS TODAY, November 1999, page 48.) (Image © Felice Frankel, Massachusetts Institute of Technology; from F. Frankel, G. M. Whitesides, On the Surface of Things, Chronicle Books, San Francisco, 1997.)



goes from steady, to a time varying "barber-pole" pattern of vortices, to a turbulent irregular flow. As summarized by Richard Feynman in his lectures:

The main lesson to be learned from [Taylor's work] is that a tremendous variety of behavior is hidden in the [Navier–Stokes equations]. All the solutions are for the same equations, only with different values of the [rotation speed]. We have no reason to think that there are any terms missing from these equations. The only difficulty is that we do not have the mathematical power today to analyze them.... That we have written an equation does not remove from the flow of fluids its charm or mystery or its surprise.⁸

Diffusion by continuous movement

Taylor's work on turbulence centered on relentless attempts to describe turbulence by formulating mathematical theories that could be directly and quantitatively compared with experimental data. During the semester, we discussed five of Taylor's papers on turbulence, starting with his monumental (and largely unreadable) 1915 paper, "Eddy motion in the atmosphere," and ending with his 1939 paper introducing what is now known as the Taylor–Greene vortex. In the latter paper, Taylor constructs a solution to the Navier–Stokes equations that demonstrates the turbulent energy cascade.

In general terms, Taylor's contribution to our understanding of turbulence was his observation that "by analogy to the kinetic theory of gases" one should find a statistical description. He therefore aimed to find ways of predicting statistical properties of the flow. His most penetrating contribution was probably the formula (given in a 1923 paper)

$$\frac{\mathrm{d}\langle x^2\rangle}{\mathrm{d}t} = \langle v^2\rangle \int C(t-\xi) \,\mathrm{d}\xi,$$

where $\langle \cdot \rangle$ denotes a time average, x denotes position, and $C(t-\xi) = \langle v(t)v(t-\xi)\rangle/\langle v(t)^2\rangle$ is the velocity correlation function.

At one level this formula is a trivial mathematical identity and is independent of the details of how an actual fluid moves. However, the formula represents two different types of experimental measurements: The left-hand side gives the dispersion of tracers in the flow and can be measured by observing the diffusivity of dye in a turbulent flow; the right-hand side can be measured by sampling the velocity field at different times, and meas-

uring the correlations. Taylor demonstrated that the correlation function is sufficient to specify the statistical properties of a stationary random function, an idea that has had great influence beyond the realm of fluid mechanics. For example, Norbert Wiener writes, describing his beginning research on random functions:

I was an avid reader of the journals, and in particular of the *Proceedings of the London Mathematical Society*. There I saw a paper by G. I. Taylor, later to become Sir Geoffrey Taylor, concerning the theory of turbulence.... The paper was allied in my own interests, in as much as the paths of air particles in turbulence are curves and the physical results of Taylor's papers involve averaging or integration over families of curves.⁹

Wiener goes on to say that Taylor "represents a peculiarly English type in science: the amateur with a professional competence." The above formula has had tremendous impact on developing the theory of turbulence: To this day, it is believed that the fundamental quantities to be predicted from the governing equations are correlation functions.

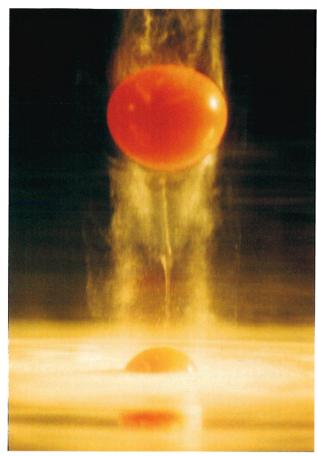
Taylor dispersion

One of Taylor's most useful results concerns the dispersion of a solute in a flowing fluid stream. The motivation for this project was to understand the manner in which drugs are dispersed in blood flow; other applications abound. The idea is to consider the steady laminar flow in a straight circular pipe of radius a, and understand how an initially localized solute disperses with time.

If there were no molecular diffusion, the solute would be spread out considerably by the flow, because of the large velocity gradient across the pipe. Taylor recognized that molecular diffusion actually impedes this dispersion: Molecular diffusion forces the solute in the center of the pipe to diffuse near the walls, where it moves more slowly. Taylor demonstrated that if the concentration is denoted c(r,z,t), where z lies along the pipe axis, and the area-averaged cross-sectional concentration is $\langle c \rangle(z,t)$, then the average concentration evolves according to the convective-diffusion equation

$$\frac{\partial \langle c \rangle}{\partial t} + \langle u \rangle \frac{\partial \langle c \rangle}{\partial z} = \mathcal{D} \frac{\partial^2 \langle c \rangle}{\partial z^2}, \quad \text{where } \mathcal{D} = D \left(1 + \frac{1}{48} \frac{\langle u \rangle^2 \alpha^2}{D^2} \right),$$

and D is the molecular diffusion constant.¹⁰ The solute center of mass moves with the mean velocity $\langle u \rangle$ and has a Gaussian spread about the mean that increases in pro-



portion to $\sqrt{\mathcal{D}t}$. The largest contribution to the dispersion typically comes from the ${}^{1}\!/_{48}(\langle u \rangle^{2}a^{2}/D)$ term, which is inversely proportional to the diffusion coefficient! Taylor even used this idea to *measure* the molecular diffusion constant, an approach that is used to this day. ¹¹

Viscous hydrodynamics

The subject of viscous hydrodynamics was popularized in the physics community by Edward Purcell's article, "Life at low Reynolds numbers," in which he describes his work with Howard Berg on understanding bacterial propulsion. 12 What is perhaps not so well known is that the first widely recognized work on this topic was Taylor's. 13 Purcell wrote

But at that time G. I. Taylor's paper in the *Proceedings of the Royal Society* could conclude with just three references: H. Lamb, *Hydrodynamics*; G. I. Taylor (his previous paper); G. N. Watson, *Bessel Functions*. That is called getting in on the ground floor.

Taylor's interest in this subject was apparently stimulated by his interaction with the zoologist James Gray of Cambridge University. The basic difficulty of low-Reynolds-number propulsion is that motion is reversible: By reversing kinematical motions one always ends up at the same starting place. Purcell popularized this idea through his "scallop theorem," which states that a scallop (an object with only one joint) in a viscous fluid cannot swim.

Taylor investigated simple swimming situations where reversibility is broken, to demonstrate how motion is possible. For example, through explicit calculation he demonstrated that transverse waves propagating along a sheet submerged in a fluid cause the sheet to translate

TAYLOR COLUMNS. When an object moves in a rotating flow, it drags along with it a column of fluid parallel to the rotation axis. This photograph shows the flow when a dyed drop of silicone fluid (radius 2 cm) rises through a large tank of water rotating at 56 rpm (From ref. 17.)

with uniform velocity. These ideas have found many recent applications, from the design of micromechanical machines to hypotheses about propulsion mechanisms in unusual organisms. Also, Taylor developed the still-available educational film "Low Reynolds Number Flows," which is familiar to many and recommended to all as a wonderful example of Taylor's creativity and clarity.

Swimming snakes

Gray also provoked Taylor's interest in the swimming of snakes. How do various types of deformations of the snake produce forward thrust? At first sight, this problem seems intractable, because the flow generated by a snake is typically turbulent, and so theories do not really exist. Taylor observed, however, that there is much experimental data regarding the forces on cylinders in a turbulent flow, and proceeded to use this data as the basis for his theory. By modeling the snake as a sum of cylinders, he computed the swimming velocity as a function of the deformation. This allowed him to explain quantitatively features of how snakes swim-for example, the wave amplitude of the snake that makes it move the fastest. Perhaps his most interesting discovery is that a snake with a rough surface can swim forward by sending waves in the forward direction. Taylor writes, "On showing [the result] to Professor Gray, [he] called my attention to a set of photographs he had taken of a marine worm Nereis diversicolor which does in fact swim in this way."14 And, as predicted, the worm has a rough surface.

Taylor columns

In a steady, rapidly rotating flow with angular velocity Ω , the dominant forces are pressure gradients and Coriolis forces, and the Navier–Stokes equations reduce to

$$2\rho\mathbf{\Omega}\times\mathbf{v}=-\nabla p,$$

where ρ is the fluid density and p is the pressure. Taking the curl of this equation, it follows that the velocity is independent of the coordinate along the rotation axis. The flow is therefore effectively two-dimensional. This result, first demonstrated by Joseph Proudman in 1915, is now called the Taylor–Proudman theorem.

Taylor's name got attached because he addressed the question of what happens if one tries to disturb the twodimensionality of the flow. In a paper published in 1923, he reported placing a short cylinder in a rotating tank of fluid and dragging the cylinder relative to the flow. Without rotation, of course, the motion of the short cylinder would disturb the flow in all directions. How can this be reconciled with Proudman's result? The experiments demonstrated what Taylor called a "remarkable" conclusion: The flow remains two-dimensional! The solid object (nearly) immobilizes an entire column of fluid parallel to the rotation axis. Thus, in a rotating environment, a slowly moving object behaves nearly as a solid cylinder extended parallel to the rotation axis. There are numerous applications of this idea to motions in atmospheres and oceans, because surface topographic features produce "columnar" disturbances that interfere with, or block, the flow at substantial elevations.

Electrohydrodynamics

Taylor spent much of his later life studying the interaction of fluids with electric fields. His most important contribution—made at age 80—is the realization that the idealization of perfect conductors or perfect dielectrics is misleading for electrically dominated flows. There is always some residual free charge present, typically residing on the interfaces between different fluids. Thus, any electric field tangential to the interface results in a tangential stress, and this stress can be balanced only by a viscous flow.

Taylor discovered this basic notion when trying to explain an experimental anomaly in the observed shapes of dielectric drops in a uniform external electric field. Simple energetics predicts that such a drop should elongate in the direction

of the field, whereas for some fluids the drop actually shortened in the field direction. Because the tangential electrical stresses described above require a steady viscous flow for balance, the drop shape cannot be obtained by energy minimization. The characterization of liquids using both a conductivity and a dielectric constant is referred to as the "leaky dielectric model." ¹⁵

Nuclear explosions

No article about Taylor would be complete without including the often told story about his calculation of the energy in a nuclear blast. Fables of this story abound. As told by Taylor himself, ¹⁶ during the early years of World War II he was told by the British government about the development of the atomic bomb, and was asked to think about the mechanical effect produced by such an explosion. He realized that the energy released from the bomb would quickly lose memory of its initial shape and distribution, and would produce a strong shock in the air. The structure of the shock far from the ground would be well-approximated as spherical.

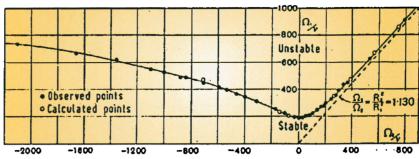
With these simplifications Taylor recognized that the parameters in the problem are the energy E, the density ρ of air, the pressure p in the air, the radius R(t) of the blast wave, and the time t since the blast. Because the blast is very strong, the air pressure will not affect the wave very much, and so p is not a relevant parameter. Taylor realized that this implies that there is a single dimensionless number characterizing the process; the reader can verify that $Et^2/\rho R^5$ is dimensionless.

Because this quantity does not depend on any aspect of the problem, it must be a constant. This implies that the radius of the blast wave is given by

$$R(t) = c(Et^2/\rho)^{1/5}$$

where c is a constant. In fact, it turns out that for air $c \approx 1.033$ according to a calculation. Therefore, given a picture that shows the radius of the blast, a reference length scale, and the time since the blast, one can deduce the energy.

Much after the fact, Taylor analyzed photographs taken by J. E. Mack of the first atomic explosion in New Mexico. These pictures were taken at precise time intervals from the instant of the explosion, and Taylor confirmed that the scaling law agrees very nicely with the



TAYLOR-COUETTE STABILITY DIAGRAM. This plot, from Taylor's 1923 paper on the instability of flow between two coaxial rotating cylinders, was the first example of a theoretical calculation of a fluid-flow instability that quantitatively agreed with experiments. The stability boundary as a function of the rotation speed of the outer cylinder (ordinate) and inner cylinder (abscissa) is shown. The dashed line $\Omega_1 R_1^2 = \Omega_2 R_2^2$ is a previous theory by Lord Rayleigh. The solid points represent experimental measurements; the open points, theoretical calculations of the stability boundary. Due to the complexity of evaluating numerically the formulas from the theoretical calculations, there are more experimental data points than theoretical points.

data. It is interesting to note that, of the papers written in the early 1950s reporting independent discoveries of the blast scaling law (authors including John von Neumann and Leonid Sedov), only Taylor's paper took publicly available data to show that the above equation agrees with experiments.

There are many topics that we have not been able to cover in this article, or in our course—among them, the bulk of Taylor's contributions to solid mechanics. Another effort to design a course around Taylor's papers would likely arrive at a completely different list of topics. We encourage interested readers to browse Taylor's collected works and to design their own course using his papers as a gateway to the modern literature.

Many people provided constructive criticism of an early draft of this article. We thank Herbert Huppert for many helpful suggestions that improved the final manuscript.

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