# Consistent Histories and Quantum Measurements

**The traditional Copenhagen orthodoxy saddles quantum theory with embarrassments like Schrödinger's cat and the claim that properties don't exist until you measure them. The consistent-histories approach seeks a sensible remedy.**

Robert B. Griffiths and Roland Omnès

Students of quantum theory always find it a very difficult subject. To begin with, it involves unfamiliar mathematics: partial differential equations, functional analysis, and probability theory. But the main difficulty, both for students and their teachers, is relating the mathematical structure of the theory to physical reality. What is it in the laboratory that corresponds to a wavefunction, or to an angular momentum operator? Or, to use the picturesque term introduced by John Bell,[1] what are the "beables" (pronounced *BE-uh-bulls*) of quantum theory—that is to say, the physical referents of the mathematical terms?

In most textbooks, the mathematical structures of quantum theory are connected to physical reality through the concept of measurement. Quantum theory allows us to predict the results of measurements—for example, the probability that this counter rather than that one will detect a scattered particle. That the concept of measurement played an important role in the early development of quantum theory is evident from Niels Bohr's account of his discussions with Albert Einstein at the 1927 and 1930 Solvay conferences.[2] And it soon became part of the official "Copenhagen" interpretation of the theory.

But what may well have been necessary for the understanding of quantum theory at the outset has not turned out to provide a satisfactory permanent foundation for the subject. Later generations of physicists who have tried to make a measurement concept a fundamental axiom for the theory have discovered that this raises more problems than it solves. The basic difficulty is that any real apparatus in the laboratory is composed of particles that are presumably subject to the same quantum laws as the phenomenon being measured. So, what is special about the measuring process? Is not the entire universe quantum mechanical?

When quantum theory is applied to astrophysics and cosmology, the whole idea of using measurements to interpret its predictions seems ludicrous. Thus, many physicists nowadays regard what has come to be called "the measurement problem" as one of the most intractable difficulties standing in the way of understanding quantum mechanics.

ROBERT GRIFFITHS *is the Otto Stern Professor of Physics at Carnegie-Mellon University in Pittsburgh, Pennsylvania.* ROLAND OMNÈS *is a professor of physics at the University of Paris XI in Orsay, France.*

## Two measurement problems

There are actually two measurement problems that conventional textbook quantum theory cannot deal with. The first is the appearance, as a result of the measurement process, of macroscopic quantum superposition states such as Erwin Schrödinger's hapless cat. The second problem is to show that the results of a measurement are suitably correlated with the properties the measured system had before the measurement took place—in other words, that the measurement has actually measured something.

The macroscopic-superposition problem is so difficult that it has provoked serious proposals to modify quantum theory, despite the fact that all experiments carried out to date have confirmed the theory's validity. Such proposals have either added new, "hidden" variables to supplement the usual Hilbert space of quantum wavefunctions, or they have modified the Schrödinger equation so as to make macroscopic superposition states disappear. (For a discussion of two such proposals, see the two-part article by Sheldon Goldstein in PHYSICS TODAY, March 1998, page 42, and April 1998, page 38.) But even such radical changes do not resolve the second measurement problem.

Both problems can, however, be resolved without adding hidden variables to the Hilbert space and without modifying the Schrödinger equation. In a series of papers starting in 1984, an approach to quantum interpretation known as consistent histories, or decoherent histories, has been introduced by us and by Murray Gell-Mann and James Hartle.[3] The central idea is that the rules that govern how quantum beables relate to each other, and how they can be combined to form sensible descriptions of the world, are rather different from what one finds in classical physics.

In the consistent-histories approach, the concept of measurement is not the basis for interpreting quantum theory. Instead, measurements can be analyzed, together with other quantum phenomena, in terms of physical processes. And there is no need to invoke mysterious long-range influences and similar ghostly effects that are sometimes claimed to be present in the quantum world.[4]

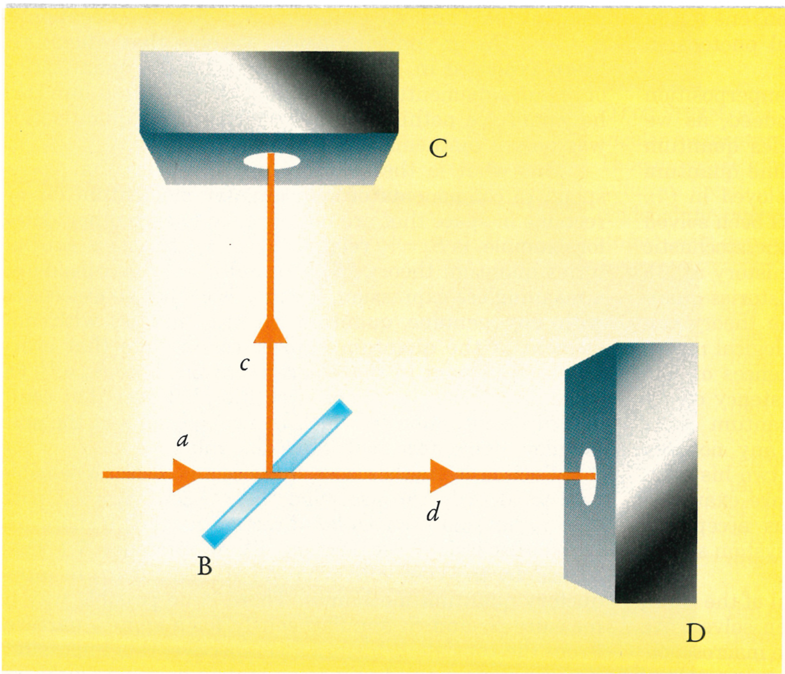



FIGURE 1. A SIMPLE *GEDANKEN* EXPERIMENT helps illustrate the authors' approach to macroscopic quantum superposition states like the hapless Schrödinger cat. A photon initially in channel $a$ passes through the beam splitter B and is eventually detected by one of the two detectors C and D. The struck detector undergoes an obvious macroscopic change of state, not unlike a poisoned cat.

## Quantum histories

The two measurement problems, and the consistent-histories approach to solving them, can be understood by referring to the simple *gedanken* experiment shown in figure 1. A photon (or neutron, or some other particle; it makes no difference) enters a beam splitter in the $a$ channel and emerges in the $c$ and $d$ channels in the coherent superposition:

$$|a\rangle \to |s\rangle = (|c\rangle + |d\rangle)/\sqrt{2}. \tag{1}$$

Here $|a\rangle$, $|c\rangle$, and $|d\rangle$ are wavepackets in the input and output channels, and $|s\rangle$ is what results from $|a\rangle$ by unitary time evolution (that is, by solving the appropriate Schrödinger equation) as the photon passes through the beam splitter.

The photon will later be detected by one of two detectors, C and D. To describe this process in quantum terms, we assume that $|C\rangle$ is the initial quantum state of C, and that the process of its detecting a photon in a wavepacket $|c\rangle$ is described by

$$|c\rangle|C\rangle \to |C^*\rangle, \tag{2}$$

where $|C^*\rangle$ is the triggered state of the detector after it has detected the photon. Once again, the arrow indicates the unitary time evolution produced by solving Schrödinger's equation. It is helpful to think of $|C\rangle$ and $|C^*\rangle$ as physically quite distinct: Imagine that a macroscopically large pointer, initially horizontal in $|C\rangle$, is moved to a vertical position in the state $|C^*\rangle$ when the photon has been detected.

By putting together the processes (1), (2), and the counterpart of (2) that describes the detection of a photon in the $d$ channel by detector D, one finds that the unitary time development of the entire system shown in figure 1 is of the form

$$|a\rangle|C\rangle|D\rangle \to |S\rangle = (|C^*\rangle|D\rangle + |C\rangle|D^*\rangle)/\sqrt{2}. \tag{3}$$

Ascribing some physical significance to the peculiar macroscopic-quantum-superposition state $|S\rangle$ in (3) poses the first measurement problem in our *gedanken* experiment. The difficulty is that $|S\rangle$ consists of a linear superposition of two wavefunctions representing situations that are visibly, macroscopically, quite distinct: The pointer on C is vertical and that on D is horizontal for $|C^*\rangle|D\rangle$, whereas for $|C\rangle|D^*\rangle$ the D pointer is vertical and the C pointer is horizontal. In Schrödinger's famously paradoxical example, the two distinct situations were a live and a dead cat. A great deal of effort has gone into trying to interpret $|S\rangle$ as meaning that either one detector or the other has been triggered, but the results have not been very satisfactory.[5]

The first measurement problem is an almost inevitable consequence of supposing that, in quantum theory, a solution of Schrödinger's equation represents a deterministic time evolution of a physical system, in the same way as does a solution of Hamilton's equations in classical mechanics. That was undoubtedly Schrödinger's point of view when he introduced his equation. The probabilistic interpretation now universally accepted among quantum physicists was introduced shortly thereafter by Max Born. Since then, chance and determinism have maintained a somewhat uncomfortable coexistence within quantum theory, with many scientists continuing to share Einstein's view that resorting to probabilities is a sign that something is incomplete.

## A stochastic theory

By contrast, the consistent-histories viewpoint is that quantum mechanics is fundamentally a stochastic or probabilistic theory, as far as time development is concerned, and that it is not necessary to introduce some deterministic underpinning of this randomness by means of hidden variables. The basic task of quantum theory is to use the time-dependent Schrödinger equation, not to generate deterministic orbits, but instead to assign probabilities to quantum histories—sequences of quantum events at a succession of times—in much the same way that classical stochastic theories assign probabilities to sequences of coin tosses or to Brownian motion. This perspective does not exclude deterministic histories, but those are thought of as arising in special cases in which the probability of a particular sequence of events is equal to 1.

For the *gedanken* experiment in figure 1, the consistent-histories solution to the first measurement problem consists of noting that a perfectly good description of what is happening is provided by assuming that the initial state is followed at a later time by one of two mutually exclusive possibilities: $|C^*\rangle|D\rangle$ or $|C\rangle|D^*\rangle$. They are related to each other in much the same way as heads and tails in a coin toss. That is to say, the system is described by one (and, in a particular experimental run, only one) of the two quantum histories:

$$|a\rangle|C\rangle|D\rangle \to |C^*\rangle|D\rangle \quad \text{or} \quad |a\rangle|C\rangle|D\rangle \to |C\rangle|D^*\rangle, \tag{4}$$

where the arrow no longer denotes unitary time development. Quantum theory assigns to each history a probability of ½. (Of course, to check this prediction, one would have to repeat the experiment using several photons in succession, each time resetting the detectors.)

The troublesome macroscopic quantum superposition state $|S\rangle$ of (3) appears nowhere in (4). Indeed, as we discuss below, the rules of consistent-histories quantum theory mean that $|S\rangle$ cannot occur in the same quantum description as the final detector states employed in (4). Therefore, the first measurement problem has been solved (or, at least it has disappeared) if one uses the stochastic histories in (4) in place of the deterministic history in (3).

The fundamental beables of consistent histories quantum theory—that is, the items to which the theory can ascribe physical reality, or at least a reliable logical meaning—are consistent quantum histories: sequences of successive quantum events that satisfy a consistency condition about which more is said below. A quantum event can be any wavefunction—that is to say, any nonzero element of the quantum Hilbert space. The two histories in (4), as well as the single history in (3), are examples of consistent quantum histories. They are thus acceptable quantum descriptions of what goes on in the system shown in figure 1.

At this point, the reader may be skeptical of the claim that the first measurement problem has been solved. We have simply replaced (3), with its troublesome macroscopic quantum superposition state, by the more benign pair of histories in (4). But as long as (3) is an acceptable history—as is certainly the case from the consistent-histories perspective—how can we claim that (4) is the correct quantum description rather than (3)? Or is it possible that both (3) and (4) apply simultaneously to the same system? Before attempting an answer, let us take a slight detour to introduce the concept of quantum incompatibility, which plays a central role in the consistent-histories approach to quantum theory.

## Quantum incompatibility

The simplest quantum system is the spin degree of freedom of a spin-½ particle, described by a two-dimensional Hilbert space. Every nonzero (spinor) wavefunction in this space corresponds to a component of spin angular momentum in a particular direction taking the value ½ in units of $\hbar$. Thus the quantum beables of this system, in the consistent-histories approach as well as in standard quantum mechanics, are of the form $S_w = ½$, where $w$ is a unit vector pointing in some direction in three-dimensional space, and $S_w$ is the component of spin angular momentum in that direction. (Actually, $S_w = ½$ corresponds to a whole collection of wavefunctions obtained from each other through multiplication by a complex number, and thus to a one-dimensional subspace of the Hilbert space.)

The nonclassical nature of quantum theory begins to appear when one asks about the relationship of these beables, or quantum states, for two different directions $w$. If the directions are opposite, for example $+z$ and $-z$, the states $S_z = ½$ and $S_{-z} = ½$ are two mutually exclusive possibilities, one of which is the negation of the other. Thus they are related in the same way as the results of tossing a coin: if heads ($S_z = ½$) is false, tails ($S_z = -½$) is true, and vice versa. This means, in particular, that the proposition "$S_z = ½$ *and* $S_z = -½$" can never be true. It is always false.

That this is a reasonable way of understanding the relationship between $S_z = ½$ and $S_z = -½$ is confirmed by the fact that if a spin-½ particle is sent through a Stern–Gerlach apparatus with its magnetic field gradient in the $z$ direction, the result will be either $S_z = ½$ or $-½$, as shown by the position at which the particle emerges. Precisely the same applies to any other component of spin angular momentum. Thus, for example, $S_x = ½$ is the negation of $S_x = -½$. (As an amusing aside, we note that when Otto Stern proposed in 1921 to demonstrate the quantization of angular-momentum orientation, Born assured him that he would see nothing, because such spatial quantization was only a mathematical fiction.[6])

But what is the relationship of beables that correspond to components of spin angular momentum for directions in space that are not opposite to each other? How, for example, is $S_x = ½$ related to $S_z = ½$? In consistent-histories quantum theory, "$S_x = ½$ and $S_z = ½$" is considered a meaningless expression, because it cannot be associated with any genuine quantum beable, that is, with any element of the quantum Hilbert space. Note that every non-zero element in that space corresponds to $S_w = ½$ for some direction $w$, so there is nothing left over that could describe a situation in which two components of the spin angular momentum both have the value ½.

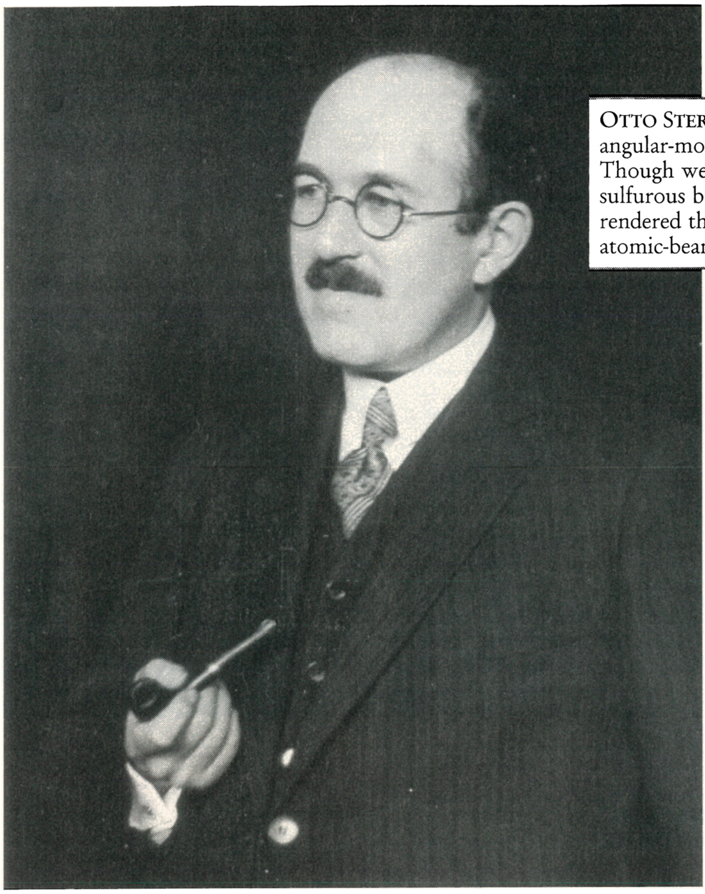
Putting it another way, there seems to be no sensible way to identify the assertion "$S_x = ½$ *and* $S_z = ½$," with $S_w = ½$ for some particular direction $w$. (For a more detailed discussion, see section 4A of reference 7.) That agrees, by the way, with what all students learn in introductory quantum mechanics: There is no possible way to measure $S_x$ and $S_z$ simultaneously for a spin-½ particle. From the consistent-histories perspective, this impossibility is no surprise: What is meaningless does not exist, and what does not exist cannot be measured.

## Meaningless or simply false?

It is very important to distinguish a meaningless statement from a statement that is always false. "$S_z = ½$ and $S_z = -½$" is always false, because $S_z = ½$ and $S_z = -½$ are mutually exclusive alternatives. The negation of a statement that is always false is a statement which is always true. By contrast, the negation of a meaningless statement is equally meaningless. The negation of the meaningless assertion "$S_x = ½$ and $S_z = ½$," following the ordinary rules of logic, is "$S_x = -½$ *or* $S_z = -½$." In consistent-histories quantum theory, this latter assertion is just as meaningless as the former. How, after all, would one go about testing it by means of an experiment?

This spin-½ example is the simplest illustration of quantum incompatibility: Two quantum beables $A$ and $B$, each of which can be imagined to be part of some correct description of a quantum system, have the property that they cannot both be present simultaneously in a meaningful quantum description. That is, phrases like "$A$ and $B$" or "$A$ or $B$," or any other attempt to combine or compare $A$ and $B$, cannot refer to a real physical state of affairs. Many instances of quantum incompatibility come about because of the mathematical structure of Hilbert space and the way in which quantum physicists understand the negation of propositions. Others are consequences of violations of consistency conditions for histories. In either case, the concept of quantum incompatibility plays a central role in consistent histories. Failure to appreciate this has, unfortunately, led to some misunderstanding of consistent-histories ideas.

Now let us return to the discussion of the histories in (3) and (4). The two histories in (4) are mutually exclusive; if one occurs, the other cannot. Think of them as analogous to $S_z = ½$ and $S_z = -½$ for a spin-½ particle. On the other hand, each of the histories in (4) is incompatible, in the quantum sense, with the history in (3), which one can think of as analogous to $S_x = ½$. Indeed, the relationship between the state $|S\rangle$ in (3) and the states $|C\rangle|D^*\rangle$ and $|C^*\rangle|D\rangle$ in (4) is formally the same as that between the state $S_x = ½$ and the states $S_z = ½$ and $S_z = -½$. Consequently, the question of whether (3) occurs



AIP EMILIO SEGRÈ VISUAL ARCHIVES

OTTO STERN, who, with Walther Gerlach, showed in 1922 that the angular-momentum orientation of a silver atom is quantized. Though we see him here as an elegant pipe smoker, it was the sulfurous breath attributed to his customary cheap cigars that rendered the two faint telltale silver streaks on the Stern–Gerlach atomic-beam apparatus visible.[6]

rather than, or at the same time as, the histories in (4) makes no sense.

It may be helpful to push the spin analogy one step further. Imagine a classical spinning object subjected to random torques of a sort that leave $L_x$, the $x$ component of angular momentum, unchanged while randomly altering the other two components, $L_y$ and $L_z$. In such a case, a classical history that describes only $L_x$ will be deterministic; it will have a probability of 1. $L_z$, on the other hand, can be described by a collection of several mutually exclusive histories, each having a nonzero probability.

Of course, classical histories of this kind can always be combined into a single history, whereas the deterministic quantum history in (3), corresponding to the $L_x$ history in this analogy, cannot be combined with the stochastic histories in (4), the analogs of the $L_z$ histories. Nevertheless, the analogy has some value in that it suggests that (3) and (4) might be regarded intuitively as describing alternative aspects of the same physical situation. Although all classical analogies for quantum systems break down eventually, this one is less misleading than trying to think of (3) and the set of histories in (4) as mutually exclusive possibilities. It helps prevent us from undertaking a vain search for some "law of nature" that would tell us that (4) rather than (3) is the correct quantum description.

## The second measurement problem

Particle physicists are always designing and building their experiments under the assumption that a measurement carried out in the real world can accurately reflect the state of affairs that existed just before the measurement. From a string of sparks or bubbles, for example, they infer the prior passage of an ionizing particle through the chamber. Extrapolating the tracks of several ionizing particles backward, they locate the point where the collision that produced the particles took place. But according to many textbook accounts of the quantum measuring process, retrodictions that use experimental results to infer what the particle was doing before this kind of measurement was made are not possible. Should we conclude, then, that experimenters don't take enough courses in quantum theory?

The consistent-histories analysis shows that the experimenters do, in fact, know what they are doing, and that such retrodictions are perfectly compatible with quantum theory. It also provides general rules for carrying out retrodictions safely, without producing contradictions or paradoxes. The consistent-histories approach even offers some insight into why the textbooks have often regarded retrodiction as dangerous.

The basic idea can be illustrated once again by reference to figure 1. Suppose the photon has been detected by detector C. In which channel was it just prior to detection: channel $c$ or $d$? The very nature of the question tells us that (3) is of no help; we must resort to the histories in (4). But even they are inadequate, because they tell us nothing about what the photon is doing at intermediate times. To address that question, we must consider the following refinements of the histories in (4):

$$|a\rangle|C\rangle|D\rangle \to |c\rangle|C\rangle|D\rangle \to |C^*\rangle|D\rangle,$$
$$|a\rangle|C\rangle|D\rangle \to |d|C\rangle|D\rangle \to \rangle|C\rangle|D^*\rangle, \tag{5}$$

in which intermediate events have been added to describe the photon after it passes through the beam splitter, but before it is detected. The consistent-histories rules assign a probability of ½ to each of these histories. That means it is impossible, given the initial state, to predict whether the photon will leave the beam splitter through channel $c$ or $d$. But if the final detector state is $|C^*\rangle|D\rangle$, meaning that C has detected the photon, then the first history in (5), not the second, is the one that actually occurred. So, at the intermediate time, the photon was in state $|c\rangle$ rather than $|d\rangle$. That is to say, it was in the $c$ channel.

Why has this rather obvious way of solving the second measurement problem been overlooked for so long? Probably because a quantum physicist who grew up with the standard textbooks will describe the situation in figure 1 by means of a pair of histories

$$|a\rangle|C\rangle|D\rangle \to |s\rangle|C\rangle|D\rangle \to |C^*\rangle|D\rangle,$$
$$|a\rangle|C\rangle|D\rangle \to |s\rangle|C\rangle|D\rangle \to |C\rangle|D^*\rangle, \tag{6}$$

in which, at the intermediate time, the photon is in the superposition state $|s\rangle$ defined in (1). He will wait until the measurement takes place and then "collapse" the wavefunction for reasons that he may not understand very well. But at least they make more sense to him than does the *macroscopic* quantum superposition state $|S\rangle$ of (3).

## Consistency Conditions: An Application

The consistency conditions as formulated in reference 9 are obtained by associating with each of the histories in a particular family a "weight" operator on the Hilbert space, and then requiring that the weight operators for mutually exclusive histories be orthogonal to each other—the operator inner product being generated by the trace. This somewhat abstract prescription is best understood by working through simple examples, such as the one in section 6C of reference 8. Here, we give an application of the consistency conditions to a situation of some physical interest.

Consider the Mach–Zehnder interferometer illustrated in figure 2. A wavepacket of light passing through the first beam splitter $B_1$ is reflected by a pair of mirrors, C and D, onto a second beam splitter $B_2$ preceding the output channels *e* and *f*. The effect of $B_1$ on the wavepacket $|a\rangle$ of a photon in the initial *a* channel at time $t_1$ is to produce, at a slightly later time $t_2$, the same kind of superposition $|s\rangle$ of wavepackets $|c\rangle$ and $|d\rangle$ in the *c* and *d* arms of the interferometer as we had in equation (1). The effect of the second beam splitter is given by

$$|c\rangle \rightarrow (|e\rangle + |f\rangle)/\sqrt{2}$$
$$|d\rangle \rightarrow (-|e\rangle + |f\rangle)/\sqrt{2}, \qquad (7)$$

where $|e\rangle$ and $|f\rangle$ are wavepackets in the output channels at $t_3$. The optical paths have been so arranged that the two $|e\rangle$ components in (7) appear with opposite phases.

Therefore, when we combine (1) and (7), we see that the photon entering at *a* must emerge in channel *f*, corresponding to the three-time history

$$|a\rangle \rightarrow |s\rangle \rightarrow |f\rangle, \qquad (8)$$

which satisfies the consistency conditions simply because it is a solution of Schrödinger's equation.

On the other hand, the pair of mutually exclusive histories

$$|a\rangle \rightarrow |c\rangle \rightarrow |f\rangle \text{ and } |a\rangle \rightarrow |d\rangle \rightarrow |f\rangle, \qquad (9)$$

in which the particle passes through either the *c* or *d* arm at the intermediate time $t_2$ and then emerges in the *f* channel, are *not* consistent, because the corresponding weight operators are not orthogonal. The reader may check this by the methods of reference 9, but it will require some work.

Consequently, it makes no sense to say that the particle passes through the *c* or the *d* arm and then emerges in the *f* channel. However, the two histories

$$|a\rangle \rightarrow |c\rangle \rightarrow (|e\rangle + |f\rangle)/\sqrt{2}$$
$$|a\rangle \rightarrow |d\rangle \rightarrow (-|e\rangle + |f\rangle)/\sqrt{2} \qquad (10)$$

*are* consistent, because here the weight operators are orthogonal. Again we leave the proof as an exercise. Thus it makes perfectly good sense to say that the photon passes through the *c* arm and emerges in a certain coherent superposition of states in the two output channels, or through the *d* arm to emerge in a different superposition.

This Mach–Zehnder example is analogous to the canonical double-slit experiment, if one regards passing through the *c* or *d* arm as analogous to passing through the upper or lower slit, and emerging in *e* or *f* as analogous to the particle arriving at a point of minimum or maximum intensity in the double-slit interference zone.

From the standpoint of consistent histories, such a physicist is, in effect, employing the histories in (6), which are perfectly good quantum beables, as part of a stochastic quantum description. However, if the photon is in the superposition state $|s\rangle$ at the intermediate time, quantum incompatibility implies that it makes no sense to ask whether it is in the *c* channel or the *d* channel. That question can be asked only in the context of the histories in (5).

The existence of a quantum description employing the set of histories in (6), in which the question of the relationship between the measurement result and the location of the photon *before* the measurement is meaningless, does not invalidate the conclusion reached by means of the histories in (5), which provide a definite answer to that question. It is a quite general feature of quantum reasoning that various questions of physical interest can be addressed only by constructing an appropriate quantum description. That is quite unlike classical physics, where a single description, such as specifying a precise point in the phase space of a mechanical system, suffices to answer all meaningful questions.

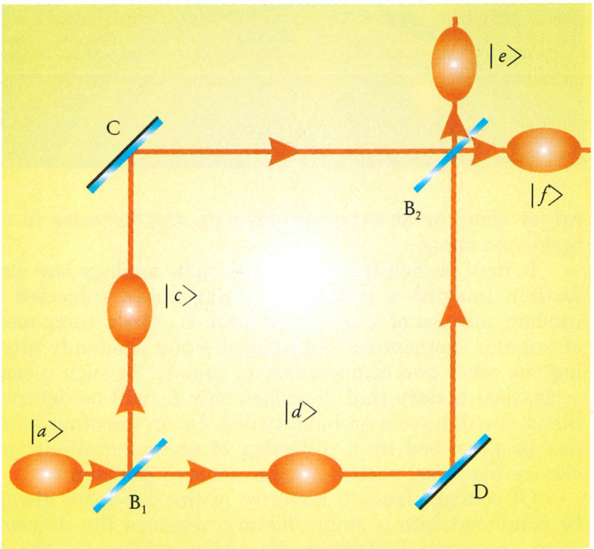



FIGURE 2. A MACH–ZEHNDER INTERFEROMETER, in which the first beam splitter $B_1$ transforms an initial photon wavepacket $|a\rangle$ into a superposition of $|c\rangle$ and $|d\rangle$ wavepackets in the two arms. The second beam splitter turns each of these wavepackets into a superposition of wavepackets $|e\rangle$ and $|f\rangle$ in the output channels. (See equations (7).)

## Consistency conditions

The beables in consistent-histories quantum theory are a collection of mutually exclusive histories to which probabilities are assigned by the dynamical laws of quantum mechanics (Schrödinger's equation). If the histories involve just two times, as in (4), these probabilities are given by the usual Born rule—namely, the absolute square of the inner product of the time-evolved initial state and the final state in question. Histories involving three or more times, as in (5), require a generalization of the Born rule and additional consistency conditions to assure that the probabilities make physical sense.

Not all collections of mutually exclusive histories satisfy the mathematical conditions of consistency. The consistent-histories approach ascribes physical meaning only to histories that satisfy the consistency conditions. Other cases are regarded as meaningless; that is to say, they are rather like trying to simultaneously ascribe values for $S_x$ and $S_z$ to a spin-½ particle. (See the box above for additional remarks on consistency conditions.)

Consistency conditions are needed for a consistent discussion of the quantum double-slit experiment,[8] in which a wavepacket approaches the slits at time $t_1$, it passes through one or the other slit just before $t_2$, and it arrives at $t_3$ at some point in the interference zone, where waves from the two slits interfere with each other. It turns out that histories in which the particle passes through *a particular slit* and then arrives at a particular point in the interference zone do not satisfy the consistency conditions, and thus do not constitute acceptable quantum beables. That will come as no surprise to generations of students who have been taught that asking which slit the particle passes through is not a sensible question. In this respect, the consistency conditions support the physicist's usual intuition at the same time as they provide a precise mathematical formulation applicable in other situations where intuitive arguments are not sufficient for precise analysis.

On the other hand, if there are detectors just behind the two slits, one's physical intuition says that it should be sensible to say which slit the particle passes through. Such intuition is used all the time in designing experiments in which collimators are placed in front of detectors. In that case, the relevant histories, which are the analogs of (5), turn out to be consistent. Furthermore, even if there are no detectors behind the slits, there are consistent histories in which the particle passes through a particular slit and then arrives in a spread-out wavepacket in the interference zone, rather than at a particular point. (See the box for more details in an analogous situation involving a Mach–Zehnder interferometer.)

The physical consequences of consistency conditions are still being explored, and there is not yet complete agreement even on their mathematical form. However, the different formulations one finds in references 9, 10, and 11 do not seem to make any significant difference in most physical applications.

## Classical limit

Because classical mechanics provides an excellent description of the motion of macroscopic objects in the everyday world, one would expect that quantum theory, in an appropriate limit, would yield the laws of classical physics to very good approximation. This conclusion is supported by Paul Ehrenfest's argument, which one finds in elementary textbooks, to the effect that average values of certain quantum observables satisfy equations similar to those of classical mechanics. But that is not a satisfactory solution to the problem of the classical limit, for two reasons: One wants to know how individual systems behave, not just the ensemble to which such an average applies. Furthermore, such an average, in the usual textbook understanding of quantum theory, refers to the results of measurements, and is not valid when measurements are not made.

In the consistent-histories approach, the classical limit can be studied by using appropriate subspaces of the quantum Hilbert space as a "coarse graining," analogous to dividing up phase space into nonoverlapping cells in classical statistical mechanics. This coarse graining can then be used to construct quantum histories. It is necessary to show that the resulting family of histories is consistent, so that the probabilities assigned by quantum dynamics make good quantum mechanical sense. Finally, one needs to show that the resulting quantum dynamics is well approximated by appropriate classical equations.

Demonstrating all this in complete detail is a difficult problem. But so is the analogous problem of finding the behavior of a large number of particles governed by classical mechanics. Indeed, the problem of showing that a system of classical particles will exhibit thermodynamic irreversibility, a typical macroscopic phenomenon, has not yet been settled to everyone's satisfaction, despite a continuing effort that goes back to Ludwig Boltzmann's work a century ago. (See the articles by Joel Lebowitz in PHYSICS TODAY, September 1993, page 32, and by George Zaslavsky in this issue, page 39.)

Nonetheless, calculations carried out by one of us,[11,12] and by Gell-Mann and Hartle,[10] indicate that, given a suitable consistent family, classical physics does indeed emerge from quantum theory. Of course the classical equations are only approximate. They must be supplemented by including a certain amount of random noise, as one would expect from the fact that quantum dynamics is a stochastic process. In many circumstances, this quantum noise will not have much influence, but it can be amplified in systems that exhibit (classical) chaotic behavior. Even so, because the classical dynamics of such systems is noisy for all practical purposes, even if it is deterministic in principle, they are not likely to exhibit distinctive quantum effects.

The consistency of a family of histories for a macroscopic system is often ensured by quantum decoherence, an effect closely related to thermodynamic irreversibility. (See the article by Wojciech Zurek in PHYSICS TODAY, October 1991, page 36.) Demonstrating that quantum systems actually exhibit irreversible behavior in the thermodynamic sense, on the other hand, is not trivial. There are conceptual and computational difficulties similar to those that arise when one considers a classical system of many particles. Nonetheless, there seems at present to be no difficulty, in principle, that prevents us from understanding macroscopic phenomena in quantum terms, including what happens in a real measurement apparatus. Thus, by interpreting quantum mechanics in a manner in which measurement plays no fundamental role, we can use quantum theory to understand how an actual measuring apparatus functions.

*We are grateful to Todd Brun, Sheldon Goldstein, James Hartle, and Wojciech Zurek for comments on the manuscript. One of us (Griffiths) acknowledges financial support from the National Science Foundation through grant PHY 9602084.*

## References

1. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge U. P., Cambridge, England (1987).
2. N. Bohr, in *Albert Einstein: Philosopher-Scientist*, P. A. Schilpp, ed., Tudor, New York (1951), p. 201. Reprinted in ref. 4, p. 9.
3. R. Griffiths, J. Stat. Phys. **36**, 219 (1984). R. Omnès, J. Stat. Phys. **53**, 893 (1988). M. Gell-Mann, J. B. Hartle, in *Complexity, Entropy and the Physics of Information*, W. Zurek ed., Addison Wesley, Reading, Pennsylvania (1990) p. 425.
4. *Quantum Theory and Measurement*, J. A. Wheeler, W. H. Zurek, eds., Princeton U. P., Princeton, N. J. (1983). This book reprints a number of important papers on quantum foundations and paradoxes.
5. J. S. Bell, in *Sixty-Two Years of Uncertainty*, A. I. Miller, ed., Plenum, New York (1990), p. 17.
6. B. Friedrich, D. Herschbach, Daedalus **127**, No. 1, 165 (1998).
7. R. B. Griffiths, Phys. Rev. A **57**, 1604 (1998).
8. For a superb discussion, see R. P. Feynman, R. B. Leighton, M. Sands, *The Feynman Lectures on Physics,* Addison-Wesley, New York (1963), vol. 1, chap. 37, or vol. 3, chap. 1.
9. R. B. Griffiths, Phys. Rev. A **54**, 2759 (1996).
10. M. Gell-Mann, J. B. Hartle, Phys. Rev. D **47**, 3345 (1993).
11. R. Omnès, Rev. Mod. Phys. **64**, 339 (1992).
12. R. Omnès, *Understanding Quantum Mechanics*, Princeton U. P., Princeton, N. J. (1999). ■