A Mathematician Who Lived for Mathematics

My Brain is Open: The Mathematical Journeys of Paul Erdös

Bruce Schechter
*Simon and Schuster, New York, 1998. 224 pp. $25.00 hc
ISBN 0-684-84635-7*

The Man Who Loved Only Numbers: The Story of Paul Erdös and the Search for Mathematical Truth

Paul Hoffman
*Hyperion, New York, 1998. 302 pp. $22.95 hc
ISBN 0-7868-6362-5*

Reviewed by Peter D. Lax

When the mathematician Paul Erdös died on 20 September 1996, at age 83, his obituary appeared on page 1 of the *New York Times*. In the short time since, two full biographies, under review here, have been published. Since the public has scant interest in mathematics, and even scanter understanding of it, the explanation for such attention lies in the special qualities of Erdös's mathematical thinking and in his unusual lifestyle and personality.

For Erdös, the purpose of existence was to conjecture and prove new mathematical theorems; he arranged his life so as to spend as many hours of the day as possible pursuing his goal. To this end, he never acquired the baggage that, for most people, gives meaning to life: a home, a spouse, children, a car, possessions, a job; he was not interested in sex. In the last 30 years of his life he traveled incessantly, lecturing at universities all around the globe, staying with mathematician friends and collaborating with them on joint projects. As a result, during his lifetime he wrote an unprecedented 1500 papers, almost 500 of them with collaborators. (Mathematicians were accorded "Erdös numbers" depending on the length of the collaborative chain that linked them to Erdös: An Erdös collaborator had an Erdös number of 1; a colaborator with that collaborator had a 2, and so on. My Erdös number is 3.) To put his productivity into perspective, John von Neumann wrote a total of 154 papers, Hermann Weyl 167 and Bernhard Riemann 31.

PETER D. LAX *is professor of mathematics at the Courant Institute of New York University. He first met Erdös in 1942, and wrote his first paper in 1944 on a problem of Erdös's about polynomials.*

Sounds like an arid life! Actually it was far from arid. Erdös was good company. He had a capacity for friendship that, in some cases, went very deep. He had a lively, offbeat sense of humor that included a vocabulary, called Erdösese, wherein children were called "epsilons," marriage was "capture," wives "bosses," husbands "slaves," God the "Supreme Fascist," people who left mathematics were described as having "died," and so on; the slang was amusing, when you first heard it. Erdös had an interest in politics and literature. Although not at all athletic, he was a good ping-pong player and liked to take long walks—to be sure, another opportunity to think or talk about mathematics.

In spite of his brilliance and very great accomplishments in mathematics, there was not an ounce of arrogance in Erdös. Moreover, he was a good person, not just in his moral ideals but in his down-to-earth practices. Most of whatever money he had he gave to good causes. He would go out of his way to help a friend in trouble or to meet and encourage a budding young mathematician.

Both biographies—Bruce Schechter's *My Brain is Open* and Paul Hoffman's *The Man Who Loved Only Numbers*—describe Erdös's peregrinations, his mathematical accomplishments and his development into the kind of person he became. Very briefly, this is his story: Erdös was born in Budapest, where both of his parents were mathematics teachers. His two older sisters died of scarlet fever while his mother was in the hospital giving birth to Paul. The devastated mother pampered her remaining child, protecting him from all likely—and some unlikely—harm. The child turned out to be a mathematical prodigy. He was educated at home, first by his mother and later by his father, once the elder Erdös returned from Siberia, where he had spent six years as a prisoner of war in World War I. The pattern of Paul's life was set in his youth; so was his attachment to his mother. She spent the last ten years of her life accompanying Paul, circling the globe many times. She died, at age 91, in a foreign land. Paul never quite recovered from this loss.

Erdös's mathematical interests were in the theory of numbers, set theory, graph theory, probability, combinatorics and analysis. He made many outstanding discoveries:

▷ He and Mark Kac created the field of probabilistic number theory, the study of the distribution of values of number theoretic functions.

▷ In set theory, Erdös made far-reaching extensions of a basic result of Frank Plumpton Ramsey; this theory is aimed at finding hidden regularities in the random arrangement of objects.

▷ In graph theory, Erdös and Alfred Rényi discovered an astounding property of the evolution of random graphs: that there is a sharp threshold, proportional to the square root of the number of vertices such that, if the number of edges is less than this threshold, then, with probability very near one, the graph is highly disconnected. On the other hand, if the number of edges exceeds this threshold, then, with probability very near one, the graph is highly connected. This result has a bearing on phase transition in statistical mechanics and on the random evolution of organic compounds.

▷ In combinatorics, Erdös developed a probabilistic method that guarantees the existence of solutions of combinatorial problems by showing that the probability of there being a solution is positive. This still leaves the formidable task of actually finding a solution; the situation here is analogous to Claude Shannon's theorem, which gives no clue to the encoding of messages to achieve the maximum rate of transmission over a channel with given error rate.

Both books describe the elements of the mathematical subjects that were dear to Erdös's heart. Neither attempts to describe the details of Erdös's own contributions—a wise decision, I think, for they were on a very high technical level.

There are some differences between the two books. Hoffman gives a more detailed account of the political unrest in Hungary, following the first World War, that drove so many mathematicians abroad. Schechter points out that, when set theory was introduced into schools as part of the "new math," it was a bowdlerized version that omitted infinite sets; that is like leaving out the poetry when teaching Shakespeare.

Hoffman's description of an unfortunate controversy between Erdös and the great mathematician Atle Selberg is wrong; Schechter gets the story more or less right.

Hoffman's statement that Kurt Gödel tried but failed to prove the continuum hypothesis is misleading. In fact, Gödel succeeded in 1938 in showing that the continuum hypothesis is consistent with the axioms of set theory, and Paul Cohen showed in 1963 that the denial of the continuum hypothesis is consistent with the axioms.

Hoffman credits Ken Ribet with discovering that the Taniyama–Shimura conjecture implies Fermat's theorem; the first connection was, in fact, made by Gerhard Frey.

Hoffman correctly points out that today the distinction between pure and applied mathematics is more muddled than ever. Erdös was not interested in applications of mathematics; nevertheless, some of his most talented disciples have ended up in departments of computer science.

Hoffman goes on to quote John Tierney: "The remarkable paradox of mathematics . . . is that no matter how determinedly its practitioners ignore the world, they consistently produce the best tools for understanding it." So far so good. Unfortunately, Tierney then adds that "for no good reason, in 1854 a German mathematician, Bernhard Riemann, wonders what would happen if he discards one of the hallowed postulates of Euclid's plane geometry. His non-Euclidean geometry replaces Euclid's plane with a bizarre abstraction called curved space, and then, 60 years later, Einstein announces that this is the shape of the universe." This is at odds with what Riemann wrote. During his brief life, Riemann was deeply interested in science; a substantial number of his papers dealt with problems in physics. In his famous dissertation on the principles underlying geometry, he openly speculated on the physical meaning of curved space. So it would be more correct to say that in some general way he anticipated Einstein.

Back to Erdös: Because of his singular devotion to mathematics, his great contributions to it, the huge number of his collaborators, the goodness of his character, his disdain of worldly goods and honors and his eccentricity, Erdös has become a cult figure to those who knew and loved him. These books serve as a good introduction for those who did not have that privilege.

Fashionable Nonsense: Postmodern Intellectuals' Abuse of Science

Alan Sokal and Jean Bricmont
Picador (St. Martin's Press), New York, 1998. 300 pp. $23.00 hc
ISBN 0-312-19545-1

Many scholars who are not physicists or mathematicians appear to believe that the formal languages of contemporary physics and mathematics may fruitfully be employed in disciplines far from those for which they were originally developed. On the face of it, this is implausible. Those languages were constructed for such highly specialized purposes, and are characterized by such tight and intricate internal logical interconnections, that it would be a remarkable coincidence if, for example, the quantitative tools of the special theory of relativity had any relevance for understanding the structure of human societies or if the deep theorems of mathematical logic could be applied in psychoanalytic theory. Nevertheless, people have tried to make such connections.

Alan Sokal and Jean Bricmont share my prejudice that such efforts are futile. They are persuaded that little more has emerged from such attempts than a jumble of meaningless jargon and contradiction-ridden nonsense. To support their view, in *Fashionable Nonsense*, they offer many excerpts, ranging from a sentence to a few pages, from a dozen eminent authors such as Jacques Lacan, Julia Kristeva, Luce Irigaray and Jean Baudrillard. These passages do indeed sound like irredeemable rubbish to one who has learned to use in the original contexts the technical terms they employ. Not only is it impossible to extract from the excerpts any meaningful use of those terms, but it is clear that, if they are being used in anything like their conventional senses, then the authors of these excerpts have utterly failed to grasp their original meaning or purpose.

This raises questions: To what uses are the excerpted authors trying to put this apparently inappropriate language? To what extent has the broader setting from which the excerpts have been extracted loosened or shifted the conventional meaning of the technical terms? What apparently nontechnical terms in the apparently nonsensical passages have been elsewhere endowed by their authors with specialized meanings?

It is the great failing of this book not to address such questions. If the passages are read as excerpts from technical treatises in mathematics or theoretical physics, then they are indeed manifest nonsense on an almost lunatic scale. That is how they are read by Sokal and Bricmont, who confidently announce that the cited authors are not only ludicrously ignorant of the technical concepts they invoke but that their real aim is only to impress their nonscientist readers with a technical expertise they manifestly do not possess.

These are serious charges that carry a scholarly and, indeed, a moral obligation to make a serious effort to come to terms with the offending texts. Sokal and Bricmont do not even try. Perhaps this is because the passages they cite, if read in the only way physicists and mathematicians know, are so transparently absurd that it seems a waste of effort to explore alternative readings. If Sokal and Bricmont's only aim were to persuade their scientific colleagues that some very silly-sounding things are being passed off as profound, then one would have to count their book a roaring success.

But that was not and ought not to have been their aim. If, indeed, many of the luminaries of critical studies are promulgating pure rubbish when they turn their attention to matters of science and mathematics, then those nonscientists who take seriously their discourse on less technical matters deserve to be warned of this. But warning, in this case, requires persuasion. There is nothing persuasive in a barrage of jocular declarations that the cited authors have no idea what this or that isolated chunk of what they have written is supposed to mean.

Potentially more convincing are Sokal and Bricmont's many attempts to explain how, if the technical terms in these passages are taken at face value, then they are being grotesquely misused. But the crucial question of why the terms should, in fact, be so taken, is never seriously considered. One cited author, for example, is taken to task for misunderstanding the symbol "+." "As we all learned in elementary school," Sokal and Bricmont tell their readers, " '+' denotes the addition of two numbers. We are at a loss to explain how Irigaray got the idea that it indicates the 'definition of a new term.' " Unorthodox this abuse of "+" may be, but does it take an enormous leap of the imagination to see 2 + 3 as a definition of the new term 5? By being superficial in their more accessible jibes, Sokal and Bricmont badly undermine whatever confidence those readers who are technically unsophisticated might have had in their more